THE q^2 DEPENDENCE OF W AND Z COUPLING
CONSTANTS IN THE INTERVAL $0 \leq |q^2| \leq m_Z^2$

V.A. Novikov^{*)} and L.B. Okun^{*)}

Theoretical Physics Division, CERN
CH-1211 Geneva 23

and

M.I. Vysotsky

ITEP, 117259 Moscow, Russia

Abstract

The gauge coupling constants of W and Z bosons, α_W and α_Z , are not running unless $|q^2| \gg m_Z^2$. Their dependence on q^2 is given by simple expressions and is very weak in the interval $0 \leq q^2 \leq m_Z^2$ compared to that of the photon.

^{*)} Permanent address: ITEP, 117259 Moscow, Russia.

The phenomenon of "running" (logarithmic dependence on the momentum transfer q^2) has been known for about four decades [1], [2] as regards $\alpha(q^2)$, the coupling constant of photons to charged particles. In a number of papers this property was also attributed to the gauge couplings of W and Z bosons, $\alpha_W(q^2)$ and $\alpha_Z(q^2)$. Even a running Fermi coupling constant $G_F(q^2)$ has been discussed in the literature.

It was stressed in Ref. [3] that $\alpha_W(q^2)$ and $\alpha_Z(q^2)$, contrary to $\alpha(q^2)$, do not run, but "crawl", and that they are therefore, up to corrections of the order of α , the same at $q^2 \simeq m_Z^2$ and at $q^2 \simeq 0$. As was also stressed in Ref. [3] the $\nu_\mu e$ scattering amplitude does not contain logarithmically large contributions to the neutrino electromagnetic radius: these contributions practically cancel each other. Hence one can extract the coupling of the Z boson to the muon neutrino, $g^{\nu\mu}$, by combining CHARM II data on the $\nu_\mu e$ scattering amplitudes $g_{A,V}^{\nu\mu}(0)$ and the LEP data on the coupling of the Z boson to the electron, $g_{A,V}^e$.

Indeed if we neglect non-logarithmic corrections of the order of α/π the amplitudes for Z decay can be written in the form

$$T(Z \rightarrow \nu_\mu \bar{\nu}_\mu) = \frac{1}{2} f^{\nu\mu} Z_\alpha \bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu , \quad (1)$$

$$T(Z \rightarrow e \bar{e}) = \frac{1}{2} Z_\alpha \bar{e} \gamma_\alpha (f_V^e + f_A^e \gamma_5) e$$

where $f^{\nu\mu} = f g^{\nu\mu}$, $f_{A,V}^e = f g_{A,V}^e$, and axial and vector couplings $g_{A,V}$ are equal in this approximation to

$$g_A = T_3 , \quad g_V = T_3 - 2Qs^2 ,$$

where T_3 is the third projection of the isospin and $s^2 = 0.2312$ [4]. In all our previous papers the Z -boson coupling constant f was defined at $q^2 = m_Z^2$ by the equation:

$$f^2 = 4\sqrt{2}G_\mu m_Z^2 = 0.54864(8) . \quad (2)$$

In this paper we will consider $f(q^2)$ and will use eq. (2) as the definition of $f(m_Z^2)$. We will denote by eQ the coupling of the photon to the charge Q , and the coupling of the W boson to the charged current by $g/\sqrt{2}$. It is convenient to consider: $\alpha(q^2) = e^2(q^2)/4\pi$, $\alpha_W(q^2) = g^2(q^2)/4\pi$, $\alpha_Z(q^2) = f^2(q^2)/4\pi$.

The statement that $\alpha_Z(q^2)$ does not run in the region $|q^2| \leq m_Z^2$ means that

$$\alpha_Z(0) \simeq \alpha_Z(m_Z^2) . \quad (3)$$

The $\nu_\mu e$ scattering is described by phenomenological amplitudes $g_{V,A}^{\nu\mu}(q^2)$.

$$T_{e\nu_\mu} = \frac{G}{\sqrt{2}} \bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu \bar{e} [g_V^{\nu\mu} \gamma_\alpha + g_A^{\nu\mu} \gamma_\alpha \gamma_5] e . \quad (4)$$

If we neglect small non-logarithmic one-loop corrections and take into account only those corrections that potentially could push $\alpha_Z(q^2)$ to run, we get for $T_{e\nu_\mu}$

$$T_{e\nu_\mu} = \frac{1}{4}f^2(q^2)\frac{1}{q^2 - m_Z^2}\bar{\nu}\gamma_\alpha(1 + \gamma_5)\nu \times \quad (5)$$

$$\times \bar{e}[g_V^e g^{\nu_\mu} \gamma_\alpha + g_A^e g^{\nu_\mu} \gamma_\alpha \gamma_5]e .$$

If there is no substantial $\alpha_Z(q^2)$ evolution from $q^2 = 0$ to $q^2 = m_Z^2$ (see eq. (3)) we get

$$g_{V,A}^{e\nu_\mu}(0) \simeq 2 \left[\frac{f^{\nu_\mu}(m_Z^2)}{f(m_Z^2)} \right] \left[\frac{f_{V,A}^e(m_Z^2)}{f(m_Z^2)} \right] = 2g^{\nu_\mu} g_{V,A}^e . \quad (6)$$

By using their own data and, in addition, LAMPF data on $\nu_e e$ scattering and the total invisible width of the Z boson, the CHARM II collaboration succeeded [5] in determining the couplings of Z to all three types of neutrinos:

$$2g^{\nu_\mu} = 1.00(3) , \quad (7)$$

$$2g^{\nu_e} = 1.02(14) , \quad (8)$$

$$2g^{\nu_\tau} = 0.98(15) . \quad (9)$$

In view of the importance of this test of the universality of the neutrino couplings, it seems necessary to derive explicit equations describing the dependence of α_Z and α_W on q^2 in order to estimate the accuracy of equation (6) and of similar equations for $g_{A,V}^{e\nu_\mu}$ on which the results (7)–(9) are based [5].

In what follows we will neglect masses of fermions by considering $q^2 \gg m_f^2$. We will also omit the W and Z boson loop contributions to $\alpha_Z(q^2)$ and $\alpha_W(q^2)$, because they are not logarithmically enhanced for $|q^2| \leq m_Z$.

As usual, we will start with a bare gauge coupling g_0 and bare boson mass m_0 and by summing up a geometric progression with self-energy insertions $\Sigma(q^2)$ will obtain the q^2 -dependent physical gauge coupling:

$$g^2(q^2) = \frac{g_0^2}{1 + g_0^2 \frac{\Sigma(q^2) - \Sigma(m^2)}{(q^2 - m^2)}} , \quad (10)$$

where m is the physical mass. It is possible to get rid of g_0 by subtraction, e.g. $1/g^2(q^2) - 1/g^2(0)$. Hence:

$$\frac{1}{\alpha_Z(q^2)} - \frac{1}{\alpha_Z(0)} = b_Z F(x) , \quad \text{where } x = \frac{q^2}{m_Z^2} , \quad (11)$$

$$\frac{1}{\alpha_W(q^2)} - \frac{1}{\alpha_W(0)} = b_W F(y) \quad , \quad \text{where} \quad y = \frac{q^2}{m_W^2} \quad , \quad (12)$$

$$F(x) = \frac{x}{1-x} \ln |x| \quad . \quad (13)$$

One can easily check that

$$F(-1) = F(0) = 0 \quad , \quad F(+1) = -1 \quad , \quad (14)$$

and

$$F((-0.2785)) = +0.2785 \quad (15)$$

is the maximal value of $F(x)$. As for the coefficients b_Z and b_W , they contain contributions from quarks and leptons:

$$\begin{aligned} b_Z &= \frac{1}{48\pi} \{ 2 \cdot 3 [1 + (1 - 8s^2/3)^2] + \\ &+ 3 \cdot 3 [1 + (-1 + 4s^2/3)^2] + 3 [1 + (1 - 4s^2)^2] + 3 \cdot 2 \} = \\ &= \frac{1}{16\pi} \{ 2 \cdot 1.147 + 3 \cdot 1.149 + 1.006 + 2 \} = 0.195 \quad , \end{aligned} \quad (16)$$

where the first term is the up-quarks contribution, the second the down-quarks one, the third one comes from the charged leptons, while the fourth comes from neutrinos:

$$b_W = \frac{1}{24\pi} \{ 2 \cdot 3 \cdot 2 + 3 \cdot 2 \} = \frac{3}{4\pi} = 0.239 \quad , \quad (17)$$

where the first term is the quark and the second is the lepton contribution.

Let us now remind [4] that

$$\alpha_Z(m_Z^2) = \alpha(m_Z^2)/c^2 s^2 = \frac{\sqrt{2}G_\mu m_Z^2}{\pi} = 1/22.905(4) \quad , \quad (18)$$

$$\alpha_W(m_Z^2) = \alpha(m_Z^2)/s^2 = 1/28.74 \quad , \quad (19)$$

$$\alpha(m_Z^2) = 1/128.87(10) \quad , \quad (20)$$

$$c^2 s^2 = \frac{\pi \alpha(m_Z^2)}{\sqrt{2}G_\mu m_Z^2} = 0.17774(2) \quad , \quad (21)$$

$$s^2 = 0.2312(3) \quad , \quad c^2 = 0.7688(3) \quad . \quad (22)$$

Thus

$$1/\alpha_Z(m_Z^2) - 1/\alpha_Z(0) = -0.195 \quad , \quad (23)$$

$$1/\alpha_Z(0) = 22.905 + 0.195 = 23.1 \quad , \quad (24)$$

$$\begin{aligned} 1/\alpha_W(m_Z^2) - 1/\alpha_W(0) &= 0.239F\left(\frac{1}{c^2}\right) = \\ &= 0.239F(1.3) = -0.239 \cdot 1.137 = -0.272 \quad , \end{aligned} \quad (25)$$

$$1/\alpha_W(0) = 28.74 + 0.272 = 29.01 \quad . \quad (26)$$

One can see that from $q^2 = 0$ to $q^2 = m_Z^2$ α_Z increases by 0.85%, while α_W by 0.95%. This should be compared with the running of the photon coupling [1]:

$$\frac{1}{\alpha(m_Z^2)} - \frac{1}{\alpha(0)} = 128.87 - 137.04 = -8.17 \quad , \quad (27)$$

which represents an increase by 6.34%. For large q^2 ($q^2 \gg m_f^2$) the q^2 dependence of $1/\alpha(q^2)$ is approximately logarithmic:

$$\frac{1}{\alpha(q^2)} - \frac{1}{\alpha(0)} \simeq -8.17 + \frac{20}{9\pi} \ln \frac{m_Z^2}{q^2} \quad . \quad (28)$$

Here

$$\frac{20}{9\pi} = \frac{1}{3\pi} \sum_f Q_f^2 = \frac{1}{3\pi} \left[3 + \frac{4}{9} \cdot 3 \cdot 2 + \frac{1}{9} \cdot 3 \cdot 3 \right] \quad . \quad (29)$$

Neglecting fermion masses, especially m_b , m_τ , m_c , leads to an artifact: the right-hand side of eq. (28) vanishes at $q^2 = (0.28 \text{ GeV})^2$, not at $q^2 = 0$.

From the 1% difference between $\alpha_Z(0)$ and $\alpha_Z(m_Z^2)$ one would naïvely expect that coefficient 2 in Eq. (6) should be multiplied by 0.99, which would change 1.00 in eq. (3) to 1.01. However, other radiative corrections may compensate this 1% change.

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