BARI-TH 94-166
SW 54 10

CORRELATION FUNCTIONS AND EINSTEIN LOCALITY

Augusto Garuccio,¹ and Liberato De Caro²

¹Dipartimento di Fisica
Università di Bari
Via Amendola 173
I-70126 Bari, Italy

²Centro Nazionale Ricerca e Sviluppo Materiali
S.S. 7 Appia Km. 712
I-72100 Brindisi, Italy

INTRODUCTION

Among the problem concerning the foundation of quantum mechanics (QM), the disagreement between this theory and relativity is the main and most difficult one to solve. The argument was posed in 1935 by Einstein, Podolsky and Rosen in a famous paper¹ in which they proved the incompatibility among three hypotheses: a) quantum mechanics is correct; b) quantum mechanics is complete; c) local "elements of reality" exist associated with the atomic system that determine the results of performed measurements.

This paper opened a long and as yet unsettled debate about which one of the three hypotheses should be discarded. For example, Einstein suggested to abandon the completeness of QM, while Bohr² refused to suppose that "elements of reality" exist.

The most important step forward in this debate was taken in 1965 by Bell³, who proved that the assumption (b) is unessential and that there is an incompatibility between the hypothesis that QM is correct and the hypothesis that local "elements of reality" exist. The paper of Bell opened the way to an experimental check of the validity of assumption (a) or (c), and in the last years many attempts have been carried out in order to solve this debate using the experimental results.

Unfortunately the inequality proved by Bell is based on two hypotheses hard to realise experimentally. The first one is that an experimental configuration yielding total correlation exists; the second one is that it is possible perform measurements on every physical system and that such measurements always yield well defined results.

The aim of research since 1964 was directed on one hand toward new experimental designs able to better approximate the ideal configuration, on the other hand toward the definition of new experimentally testable inequalities. The papers by Clauser, Horne, Shimony and Horne⁴, Clauser and Horne⁵, and Garuccio and Rapisarda⁶ represent a step toward that goal. In all these papers some "supplementary assumptions" have been introduced in order to compare the experimental data with Bell's inequality obtained in the ideal case.

In the present paper we will introduce a different approach⁷ to the problem: starting from a general causal and local theory we will deduce a new Bell-type inequality for the case of real experiments. The mathematical formalism, which we will introduce in order to describe the most general experimental set-up, will include all the pervious supplementary assumptions as particular cases.

BELL'S INEQUALITY FOR THREE-VALUED OBSERVABLES.

Schematically, an experiment on the EPR paradox based on polarization correlation measurements of optical photons consists of a source emitting polarized photon pairs travelling in opposite directions and two measuring apparatus, each composed of a polarizer followed by one or two photomultipliers.

Let A and B be two three-valued observables which can assume the values $\{\pm 1, 0\}$. For example, using a source of correlated photon pairs in a typical EPR experimental set-up, the values $A=\pm 1$ ($B=\pm 1$) can be associated with the transmission through the ordinary and extra-ordinary channels of polarizer 1 (2) with axis orientation a (b), respectively, and subsequent detection; whereas $A=0$ ($B=0$) can be associated with the case of no count in the detector 1 (2), which means either absorption in the polarizer or lack of detection. Let the result of measurements of A and B be dependent on the arguments a and b, respectively, which are assumed to be experimental parameters, and on the element of reality λ . Let λ be variable over the set L, with density $\rho(\lambda)$ such that

$$\int_{\Lambda} d\lambda \rho(\lambda) = 1. \quad (1)$$

The expectation value of a function A is, by definition

$$E[A] = \int_{\Lambda} d\lambda \rho(\lambda) A(a, \lambda) \quad (2)$$

and, analogously, the expectation value for the product of the observables A and B is:

$$E[A, B] = \int_{\Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) \quad (3)$$

Let us note that usually in the literature on this subject the expectation value (3) is referred to as correlation function. This is not generally correct, since the correlation function is expressed by formula (3) if and only if the expectation values (2) of the two observables A and B equal zero and the variances equal one, i.e. if we are dealing with a dichotomic case. Nevertheless, Bell's inequality has been always defined as a linear combination of the functions $E[AB]$. Therefore, in order to obtain results directly comparable with those of the dichotomic case, also for the three-valued case we will define the Bell's observable as a function of the expectation values $E[AB]$. Thus, let define the Bell's observable as follows:

$$E(a, a'; b, b') = |E[A(a)B(b)] - E[A(a)B(b')]| + |E[A(a')B(b)] + E[A(a')B(b')]|. \quad (4)$$

From the properties of the integrals, from the definitions of $E(a, a'; b, b')$ and $E[AB]$, and from the property $A(a, \lambda) \in \{\pm 1, 0\} \forall \lambda \in \Lambda$, it immediately follows:

$$\begin{aligned} E(a, a'; b, b') &\leq \int_{\Lambda} d\lambda \rho(\lambda) [|A(a, \lambda)| |B(b, \lambda) - B(b', \lambda)| + |A(a', \lambda)| |B(b, \lambda) + B(b', \lambda)|] \leq \\ &\leq \int_{\Lambda} d\lambda \rho(\lambda) [|B(b, \lambda) - B(b', \lambda)| + |B(b, \lambda) + B(b', \lambda)|]. \end{aligned} \quad (5)$$

Now, let us define $\Lambda_0(b)$ and $\Lambda_0(b')$ the two subset of Λ in which $B(b, \lambda)=0$ and $B(b', \lambda)=0$, respectively. Thus, we can subdivide the set Λ into two subsets: the subset $\Lambda_0(b) \cap \Lambda_0(b')$ of l's such that both $B(b, \lambda)$ and $B(b', \lambda)$ are zero; and the subset $\Lambda' = \Lambda - \Lambda_0(b) \cap \Lambda_0(b')$ of l's such that either $B(b, \lambda)$ or $B(b', \lambda)$ or both are different from zero. As in the subset $\Lambda_0(b) \cap \Lambda_0(b')$ the function $|B(b, \lambda) - B(b', \lambda)| + |B(b, \lambda) + B(b', \lambda)|$ is always equal to zero, we can calculate the integral of Eq. (5) only over its complementary subset Λ' .

As we always have $B(b, \lambda) \in \{\pm 1, 0\} \quad \forall \lambda \in \Lambda'$, it is a simple matter to show that, in this subset, $|B(b, \lambda) - B(b', \lambda)| + |B(b, \lambda) + B(b', \lambda)| = 2$.

Thus, we obtain:

$$E(a, a'; b, b') \leq 2 \int_{\Lambda'} d\lambda \rho(\lambda) \equiv 2\mu(\Lambda'), \quad (6)$$

where $\mu(\Lambda')$ denotes the measure of Λ' . From Eq.(1) and from the definition of Λ' it follows that we can put:

$$\mu(\Lambda') = 1 - \mu(\Lambda_0(b) \cap \Lambda_0(b')) \equiv 1 - \mu_0(b, b'), \quad (7)$$

and, therefore, we have:

$$E(a, a'; b, b') \leq 2(1 - \mu_0(b, b')). \quad (8)$$

We wish to note that the local upper limit given by Eq. (8) is stronger than the analogous limit valid in the dichotomic case. Only if $\mu_0(b, b') = 0$ the upper limit becomes 2. In the most general case $\mu_0(b, b')$ is different from zero and it will be necessary to evaluate its value in each case.

A local inequality for probabilities for three-valued observables.

As the original Bell's inequality cannot be actually applied to a concrete experiment Clauser, Horne, Shimony and Holt (CHSH) in the 1969 introduced a deterministic approach in order to obtain a stronger inequality⁴. The CHSH inequality, given by the following equation, follows from the hypothesis of local realism and from the famous CHSH no-enhancement assumption:

$$-D(\infty, \infty) \leq \text{CHSH}(a, a'; b, b') \equiv D(a; b) - D(a; b') + D(a'; b) + D(a'; b') - D(a', \infty) - D(\infty, b) \leq 0, \quad (9)$$

where $D(a, b)$ denotes the joint transmission and detection probability with the linear single-channel polarizers set to the angles a and b , and ∞ the physical situation with the correspondent polarizer removed.

Next we will present a generalisation of the CHSH approach to the three-valued case. First we will obtain a general local inequality with no further additional assumptions. Later, we will discuss how this inequality changes as a function of the additional hypotheses.

If we define $p(a_+, b_+)$, $p(a_+, b_-)$, $p(a_+, b_0)$ as the probabilities that the first photon of a pair will be detected in the ordinary channel, while the other photon is, respectively, detected in the ordinary channel, the extraordinary channel or absorbed, and if analogous definitions hold for $p(a_-; b_+)$, $p(a_-; b_-)$, $p(a_-; b_0)$, $p(a_0; b_+)$, $p(a_0; b_-)$ and $p(a_0; b_0)$, the following relations hold:

$$\begin{aligned} \sum_{ij} p(a_i; b_j) &= 1, \quad \sum_j p(a_i; b_j) = p_1(a_i), \quad \sum_i p(a_i; b_j) = p_2(b_j), \\ p(a_i; b_+) + p(a_i; b_-) &= p(a_i; \infty), \quad p(a_+; b_j) + p(a_-; b_j) = p(\infty; b_j), \\ p(a_+; b_+) + p(a_+; b_-) + p(a_-; b_+) + p(a_-; b_-) &= p(\infty; \infty), \end{aligned} \quad (10)$$

where: the indexes $i, j \in \{+, -, 0\}$; and the symbol ∞ , differently from the one of Eq.(9), means the sum of detected counts in the ordinary and extra-ordinary channels of the correspondent polarizer.

It is important to stress that in Eqs. (10) we have assumed the rotational symmetry of the quantum state emitted by the source. This hypothesis is analogous to the one made by

CHSH ; moreover, the validity of the above assumption can be easily verified with an experimental test, in which the coincidences $p(a+, \infty)$, $p(\infty, b+)$ and so on, should be measured at different values of a and b .

Using Eqs. (10) it is a simple matter to show that the expectation value $E(a; b)$, given by

$$E(a; b) \equiv p(a_+; b_+) - p(a_+; b_-) - p(a_-; b_+) + p(a_-; b_-), \quad (11)$$

and $p(\infty; \infty)$ can be expressed, respectively, by the following equations:

$$\begin{aligned} E(a; b) &= 4p(a_+; b_+) - 2p(a_+; \infty) - 2p(\infty; b_+) + p(\infty; \infty), \\ p(\infty; \infty) &= 1 - p(a_0; \infty) - p(\infty; b_0) - p(a_0; b_0) = 1 - p_1(a_0) - p_2(b_0) + p(a_0; b_0). \end{aligned} \quad (12)$$

Let us observe that in the first Eq. (12) we can simplify the notation neglecting the $+$ symbols. Finally, inserting Eq. (12-a) in the definition of Bell's observable for the three-valued case, given by the Eq. (8), we obtain:

$$L(b, b') \leq \text{CHSH}(a, a'; b, b') \leq U(b, b'), \quad (13)$$

where

$$\begin{aligned} L(b, b') &= [-1 - p(\infty, \infty) + \mu_0(b, b')]/2, \\ U(b, b') &= [1 - p(\infty, \infty) - \mu_0(b, b')]/2. \end{aligned} \quad (14)$$

The above inequality is quite different from the original CHSH one. This is due to the fact that the inequality (13) has not been obtained via the additional CHSH assumption.

In fact, if we put $\mu_0(b, b') = 0$, as in the dichotomic Bell's inequality, the limits that bound $\text{CHSH}(a, a'; b, b')$ are too high in order to being experimentally violated, due to the small value that $p(\infty; \infty)$ can actually reach. When we consider $\mu_0(b, b') \neq 0$, as it is a measure of the number of photons which have been lost, we should expect that the smaller is $p(\infty; \infty)$ the larger should be $\mu_0(b, b')$. Thus, in the lower and upper limits of the above inequality there are two terms that have opposite behaviour as a function of the quantum efficiency. In the next sections we will evaluate the lower and the upper limits of the inequality (13) for very important physical situations.

Evaluation of the boundary limits.

In this section, we will evaluate the functions $L(b, b')$ and $U(b, b')$ in two important physical situations in which these functions assume the maximum and the minimum possible values.

The two physical situations considered will differ only in the additional hypotheses. Therefore, we will prove that lower and upper limits of the inequality (13-14) depend of the additional hypotheses. Thus, our approach is able to include any additional hypotheses in the deduction of inequalities from locality.

The first important physical case concerns the case in which $\Lambda_0(b)$ and $\Lambda_0(b')$ are completely random.

Let us denote with $\beta(b)$ and $\beta(b')$ the measure of the set $\Lambda_0(b)$ and $\Lambda_0(b')$, respectively. The function $\beta(b)$ is the probability for a photon to have its reality element λ belonging to the subset $\Lambda_0(b)$ of Λ . Moreover, the function $\mu_0(b, b')$ is the joint probability that λ will belong to $\Lambda_0(b)$ and to $\Lambda_0(b')$. Therefore, the conditional probability $p(b'|b)$ that a λ , belonging to $\Lambda_0(b)$, will belong also to $\Lambda_0(b')$, is given by (if $\beta(b) \neq 0$):

$$p(b'|b) = \mu_0(b, b') / \beta(b). \quad (15)$$

Thus, in the case of a $\Lambda_0(b)$ and a $\Lambda_0(b')$ completely randomly determined, the conditional probabilities are $p(b'|b) = \beta(b')$, $p(b|b') = \beta(b)$ and

$$\mu_0(b, b') = \beta(b)\beta(b'). \quad (16)$$

From now on we will refer to this physical situation with the phrase: "random selection of the subset of coincidences in the set of photon pairs"; or more briefly with: "random selection".

Moreover, from the assumed rotational symmetry property of the quantum state emitted by the source, we have that the sum of counts in the ordinary and in the extra-ordinary channels does not depend on b . Thus, if we denote with $\Lambda_+(b)$ and $\Lambda_-(b)$ the subsets of λ in which $B(b, \lambda) = 1$ and $B(b, \lambda) = -1$, respectively, the above condition is equivalent to say that $\mu(\Lambda_+(b)) + \mu(\Lambda_-(b))$ does not depend on b . Therefore, from the obvious condition $\Lambda_+(b) + \Lambda_-(b) + \Lambda_0(b) = \Lambda$ we can also conclude that, even if $\Lambda_0(b)$ depends on b , its measure does not depend on b . In other words, there are different reality elements which contribute to $\Lambda_0(b)$ and $\Lambda_0(b')$, but the probability β of no detection is always the same, no matter the values of b and b' . The above conclusion permits us to consider the functions L and U , defined in eqs. (13-14), as not depending on b and b' , but only on β . Therefore, under the above symmetry conditions, we have:

$$\mu_0(b, b') = \beta^2. \quad (17)$$

In order to evaluate the functions U and L in the inequality (13) we have to calculate the values of $p(\infty; \infty)$ for the considered physical situation, or equivalently, given (12-b) the probabilities $p_1(a_0)$, $p_2(b_0)$ and $p(a_0, b_0)$. But the probability $p_1(a_0)$ of having no single count at the polarizer 1 is given by the measure of the subset $\Lambda_0(a)$, denoted with α . Analogously, $p_2(b_0)$ is given by β , and $p(a_0, b_0)$ is given by $\mu_0(a, b) = \mu(\Lambda_0(a) \cap \Lambda_0(b)) = \alpha\beta$. Therefore:

$$p(\infty; \infty) = 1 - \alpha - \beta + \alpha\beta = (1 - \alpha)(1 - \beta). \quad (18)$$

Inserting Eq.(18) into Eqs.(14), for a "random selection" we have:

$$\begin{aligned} L(b, b') &= L(\alpha, \beta) = -1 + (\alpha + \beta + \beta^2 - \alpha\beta)/2, \\ U(b, b') &= U(\alpha, \beta) = (\alpha + \beta - \beta^2 - \alpha\beta)/2. \end{aligned} \quad (19)$$

In Eqs. (19) we have the lower and the upper limit of the local inequality for three-valued observables as a function of two variables: α and β . At this point, in order to make a quantitative evaluation of the functions L and U , we make the simplifying hypothesis that $\alpha = \beta$ (symmetrical measuring apparatuses of the two observers). In this case

$$p(\infty, \infty) = (1 - \beta)^2, \quad (20)$$

and the inequality (13-14) becomes:

$$-(1 - \beta) = -\sqrt{p(\infty, \infty)} \leq \text{CHSH}(a, a'; b, b') \leq (\beta - \beta^2) = \sqrt{p(\infty, \infty)} - p(\infty, \infty). \quad (21)$$

Thus, when we make the additional assumption that there is a "random selection of the

subset of coincidences", not only the lower but also the upper limit for the observable $\text{CHSH}(a,a';b,b')$ depends on $p(\infty,\infty)$.

The second case considered is when all the elements of reality λ (and, therefore, all the photon pairs) which give no count, are always the same, no matter the value of a , a' and b , b' for the first and second apparatus, respectively. This can be expressed by the following mathematical requirements:

$$\begin{aligned}\Lambda_0(a) \cap \Lambda_0(a') &\equiv \Lambda_0(a) \equiv \Lambda_0(a') \quad \forall a, a', \\ \Lambda_0(b) \cap \Lambda_0(b') &\equiv \Lambda_0(b) \equiv \Lambda_0(b') \quad \forall b, b', \\ \Lambda_0(a) \cap \Lambda_0(b) &\equiv \min\{\Lambda_0(a), \Lambda_0(b)\} \quad \forall a, b,\end{aligned}\tag{22}$$

where $\min\{\Lambda_0(a), \Lambda_0(b)\}$ denotes the smaller subset between $\Lambda_0(a)$ and $\Lambda_0(b)$. From now on, we will indicate this physical situation with the phrase: "polarizer-orientation independence of the selected subset of coincidences"; or more briefly with: "parameter-independent selection".

Then, denoting with $\beta(b)$ and $\beta(b')$ the measure of the set $\Lambda_0(b)$ and $\Lambda_0(b')$, respectively, from Eq. (22-b) we have: $\beta(b)=\beta(b')=\beta$. Thus, as in the previous sub-section, the probability for a photon to have the element of reality λ belonging to the subset $\Lambda_0(b)$ of Λ , does not depend on b . But, contrary to the previous case, if Eq. (22-b) holds, the conditional probability $p(b'|b)$ that a λ , belonging to $\Lambda_0(b)$, will belong also to $\Lambda_0(b')$, now is equal to one. Therefore, we have:

$$\mu_0(b,b')=p(b'|b)\beta(b)=p(b|b')\beta(b')=\beta\tag{23}$$

In order to evaluate the functions U and L in the inequality (13) we have to calculate the value of $p(\infty;\infty)$ for the considered physical situation. From Eqs. (12-b) and (23), it follows that, denoting with $\mu_0(a,b)$ the measure $\mu(\Lambda_0(a) \cap \Lambda_0(b))$, we have:

$$p(\infty,\infty)=1-\alpha-\beta+\mu_0(a,b).\tag{24}$$

Moreover, from the condition (22-c) it follows:

$$\mu_0(a,b)=\min\{\alpha,\beta\} \equiv m.\tag{25}$$

Inserting Eqs. (23), (24) and (25) into Eqs. (14), it leads to:

$$\begin{aligned}L(b,b') &= L(\alpha,\beta) = -1 + (\alpha + 2\beta - m)/2, \\ U(b,b') &= U(\alpha,\beta) = (\alpha - m)/2.\end{aligned}\tag{26}$$

Being $\Lambda_0(a) \cap \Lambda_0(b) \subseteq \Lambda_0(a)$, we always have $U(\alpha,\beta) \geq 0$. In particular, the sign of equality holds if

$$\Lambda_0(a) \cap \Lambda_0(b) \equiv \Lambda_0(a) \equiv \Lambda_0(b) \quad \forall a, \forall b,\tag{27}$$

because in this case we have surely $m=\alpha=\beta$. Let observe that the above condition can be satisfied if and only if the two analysing experimental apparatuses have polarizers, detectors and so on, with the same identical characteristics.

Thus, when we make the additional hypothesis of "parameter-independent selection",

and we are dealing with symmetrical experimental apparatuses of the two observers ($\alpha=\beta$), we have:

$$-(1 - \beta) = -p(\infty, \infty) \leq \text{CHSH}(a, a'; b, b') \leq 0. \quad (28)$$

In other words, we re obtain an inequality very similar to the CHSH inequality, given by Eq. (9). When $\alpha \neq \beta$, we obtain its generalisation given by the Eqs. (13) and (26).

It is very important to stress that the boundary limits in the inequality (28) are much stronger than the corresponding ones in the inequality (21). But, these two inequalities have been obtained starting from the local inequality given by the Eqs. (13-14), and making different "ad-hoc" additional assumptions. This result simply shows the great influence of the additional hypotheses on the boundary limits of the CHSH-type inequalities. In particular, the value zero for the upper limit of the CHSH inequality is obtained when:

- 1) the joint detection probability $p(\infty; \infty)$ is equal to 1, i. e. in the dichotomic case;
- 2) we assume that Λ is the sum of two disjoint subsets, Λ' and Λ_0 , such that for all $\lambda \in \Lambda'$ both photons of a correlated pair are detected, and for all $\lambda \in \Lambda_0$ both photons are not detected, no matter the value of a, a' and b, b' .

In fact, in this latter case, and only in this case, we can take into account only the photon pairs with $\lambda \in \Lambda'$. For this subset we can redefine a new density function

$$\rho'(\lambda) = \rho(\lambda) / \mu(\Lambda'), \quad (29)$$

and reduce our three-valued observables A and B to dichotomic ones.

QUANTUM MECHANICS VERSUS LOCAL REALISM.

In order to make a quantitative comparison between the quantum mechanical predictions and the local realistic ones, we need to make some important considerations.

Let denote with η the quantum efficiency of the photomultipliers. Quantum Mechanics predicts for the joint and the single detection probability with no polarizers: $D(\infty; \infty) = \eta^2$ and $D_1(\infty) = D_2(\infty) = \eta$, respectively, if the detection apparatuses are symmetric. Therefore, Quantum Mechanics predicts that the conditional probability, that a photon of a pair will be detected from the detector 2 when the first has just been detected from detector 1, is given by:

$$p(\infty | \infty) \equiv D(\infty; \infty) / D_1(\infty) = \eta. \quad (30)$$

Thus, Quantum Mechanics, when there are no polarizers, assumes a "casual selection of the subset of coincidences from all photon pairs"! This assumption is also considered valid when there are polarizers. In fact, in order to obtain the quantum mechanical predictions for the joint detection and transmission probabilities, it is sufficient to multiply the joint transmission probabilities by η^2 . In other words, for a photon pair which emerges from the polarizers, Eq. (30) is still valid. Therefore, it should be correct to compare the quantum-mechanical results for the EPR paradox with the inequality (21), which has been obtained under the same additional physical hypotheses, concerning the independence of the two events of photon detection at the respective detectors.

On the other hand, we cannot compare the quantum-mechanical results with the inequality (28). In fact, this latter stands on the additional assumption of "parameter-independent selection" which is not compatible with the predictions of the Quantum Theory for single detection rates.

The quantum-mechanical prediction for the maximum value of observable $\text{CHSH}(a, a'; b, b')$ for a singlet state with positive parity as a function of $p(\infty; \infty)$ is:

$$\text{CHSH}(a,a';b,b')_{\text{MQ}} = \frac{\sqrt{2}-1}{2} p(\infty, \infty). \quad (31)$$

In the simplifying hypotheses of ideal one-channel polarizers and identical detectors h , the comparison between the above value and Eq. (21) shows that quantum-mechanical predictions (31) can violate the inequality (21) if and only if $\eta > 0.828$. Unfortunately this limit is too higher with respect to the actual values of the quantum efficiency of the photomultipliers, in order to do an experimental test of the inequality (21).

Let observe that all the experimental tests⁸ made using atomic cascade photon sources and, therefore, photon pairs in singlet state have been compared with the CHSH inequality or, equivalently, with Eq. (28). The experimental results are almost all in agreement with the violation of the upper limit equal to zero of this inequality and with the QM predictions, therefore they have been considered as the experimental proof of the violation of locality and as the confirm of the validity of the quantum mechanical predictions concerning correlated atomic systems. But, in the previous section we have shown that the inequality (28) stands on an additional hypothesis which is in contradiction with Quantum Mechanics also for single detection predictions. Thus, the fact that Quantum Mechanics violates inequality (28) and the experimental results violate its upper limit, simply means that the additional hypothesis of "parameter-independent selection" is wrong, and nothing can be concluded about the validity of the hypothesis of locality because, in order to have a meaningful test of the locality, we need to have detectors with a quantum efficiencies greater than 0.828.

On other hand, the operation of normalization of the measured joint probabilities with respect the subset of coincidences, in order to raise the values of the observable $\text{CHSH}(a,a';b,b')$, is possible only if we assume a "polarizer-orientation independence of the selected subset of coincidences", while if we assume a "casual selection of the subset of coincidences from all photon pairs" the subset of coincidence will not be always the same for each choice of the polarizer orientations (a,b) , (a,b') , (a',b) and (a',b') . In fact, in the previous section we have stressed that, even if the probability β of no detection is always the same, the reality elements λ of Λ , which contribute to $\Lambda_0(b)$ and $\Lambda_0(b')$, could be different.

Therefore, if we assume, coherently with Quantum Mechanics, a "casual selection of the subset of coincidences from all photon pairs", we cannot normalise the joint probabilities $p(a,b)$ with respect to $p(\infty, \infty)$ in the framework of a meaningful local and realistic theory. In other words, we cannot redefine a new dichotomic observable.

A similar conclusion can be draw if we analyse the experiments on EPR paradox based on parametric down-conversion sources⁷

REFERENCES

1. A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* 47:777 (1935).
2. N. Bohr, *Phys. Rev.* 48:696 (1935).
3. J. S. Bell, *Physics* 1:195 (1964).
4. J. F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, *Phys. Rev. Lett.* 23: 880 (1969).
5. J. F. Clauser and M.A. Horne *Phys. Rev. D* 10:526 (1974)
6. A. Garuccio and V.A. Rapisarda *Nuovo Cim. A* 65:269 (1981).
7. L. De Caro and A. Garuccio Bell's inequalities for three-valued observables, preprint (1993)
8. S.J. Freedman and J.F. Clauser, *Phys. Rev. Lett.* 28: 938 (1972).
E.S. Fry and R.C. Thompson, *Phys. Rev. Lett.* 37: 465 (1972).
J.F. Clauser, *Phys. Rev. Lett.* 36, 1223 (1976).
A. Aspect, P. Grangier and G. Roger, *Phys. Rev. Lett.* 47: 460 (1981);
A. Aspect, J. Dalibard and G. Roger, *Phys. Rev. Lett.* 49: 1804 (1982).
A.J. Duncan, W. Perrie, H.J. Beyer and H. Kleinpoppen, in: *Fundamental processes in atomic collision physics*; 555 (Plenum, New York, 1985).
For a complete review on this subject see: D. Home and F. Selleri, *La Rivista del Nuovo Cimento*, 14: 9 (1991),