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February 1994**Order- α^2 leading-logarithm corrections in Bhabha scattering at LEP/SLC energies.****M. Caffo**

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Abstract

We discuss the leading logarithmic corrections of order α^2 to Bhabha scattering cross section on the basis of order- α^2 factorizable parts known at present. We implement these corrections in BHAGEN94, the new version of our event generator. We investigate the influence of these corrections and we estimate the size of smaller leftover corrections which are still unknown. We conclude that they can give an error of 0.1% at small angle, but up to 1% at large angle.

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High-precision Bhabha scattering predictions are required for the high-precision measurements at LEP/SLC in two kinematical regions: at small scattering angle for the luminosity monitoring and at large angle for the measurements of the Z^0 parameters.

The experimental uncertainty in luminosity monitoring, now about 0.5%, is expected to decrease to a few per mille with the forward monitors, while the experimental error on large-angle measurements will be less than 1%.

For not spoiling the exploitation of such experimental efforts the uncertainty in the theoretical calculation should reach a better precision, say 0.1% at small angle and 0.5% at large angle.

Along this line, the purpose of the present work is to include in the non-perturbative approach outlined in [1] (and based on the radiative corrections of order α) the leading terms at order α^2 , which in a perturbative calculation would be expected to be proportional to $\alpha^2 L^2$, although separate contributions can have $\alpha^2 L^4$ terms. Here, as usual, $L = \ln(\frac{s}{m_e^2})$, where s is the Mandelstam variable equal to the square of the total energy, m_e is the electron mass and α the fine structure constant.

The improvements of the present approach are implemented in BHAGEN94, an improved version of our event generator BHAGEN [2].

In the known results up to order α , after cancellation of terms proportional to αL^2 , leading terms proportional to αL are present in corrections coming from vacuum polarization and vertex diagrams and also from real photon emission diagrams.

In photon vacuum polarization corrections of order α^2 , the dominant term $\alpha^2 L^2$ appears only in the contribution from one-particle reducible vacuum polarization insertions [3], therefore these terms from order α^2 vacuum polarization were already included in our

approach [1], through the use of running α (Dyson resummation).

The leading perturbative corrections from vertex and soft real emission diagrams at order α^2 to the e^+e^- scattering in the s -channel were obtained in [4] using the order- α^2 vertex form factors [5] and the double-emission function, and verified in [6] by a complete calculation. Furthermore, in [7], by using the Yennie-Frautschi-Suura method, the properly factorized infrared limit in the structure function approach (also used in [1]) is obtained.

The Bhabha scattering has been widely investigated and implemented in computer numerical programs and event generators, and in some cases also the higher-order leading terms have been examined; see for instance [1], [2], [8]–[15].

In the present work we write a properly factorized expression for the Bhabha scattering cross section, up to corrections of order $\alpha^2 L^2$, by using the existing perturbative results, providing a few newly evaluated contributions and discussing explicitly the assumptions, the approximations of the terms that are still missing.

By taking as a starting point the factorization of the vertex form factors and of the soft emission function, we try to account for all the corrections expected to contribute to leading order $\alpha^2 L^2$, including soft e^+e^- pair production in the s and t channels, but we disregard terms next to leading (i.e. $\alpha^2 L$), which are not yet entirely known, even with the assumptions that we take for the leading terms. Particular care is devoted to an examination of the behaviour of the corrections in the t -channel at small scattering angle.

As usual, for studying the leading-logarithmic part of the cross section in the soft photon-emission limit, we distinguish the various contributions of the s , t and s - t interference channels, and we factorize the vertex form factors F and the soft emission function B , as the leading-logarithmic terms are expected from these contributions only. We therefore

write

$$\begin{aligned}
d\sigma^{(s)} &= d\hat{\sigma}_0^{(s)} |F^2(s)|^2 B , \\
d\sigma^{(t)} &= d\hat{\sigma}_0^{(t)} \{F^2(t)\}^2 B , \\
d\sigma^{(st)} &= d\hat{\sigma}_0^{(st)} \{\text{Re} F^2(s)\} F^2(t) B ,
\end{aligned} \tag{1}$$

where $d\hat{\sigma}_0^{(s)}, d\hat{\sigma}_0^{(t)}, d\hat{\sigma}_0^{(st)}$ are the lowest-order cross sections in the s , t and s - t interference channels, times the infrared factor ($\Delta^{2\beta}$, see Eq. (15) below), which accounts for the compensation of the terms with $\ln(\lambda/m_e)$ (where λ is the usual fictitious photon mass), and the exponentiation of the terms with $\ln \Delta$, where $\Delta = \Delta E/E_b$ is the ratio between the radiated energy ΔE and the beam energy E_b . As this factor is well understood, it is not further discussed here.

The F functions are therefore the finite relics of the vertex corrections in the s , $|t| \gg m_e^2$ limit, once the $\ln(\lambda/m_e)$ and $\ln \Delta$ terms are removed:

$$\begin{aligned}
\text{Re} F(s) &= 1 + \left(\frac{\alpha}{\pi}\right) \left(-\frac{L^2}{4} + \frac{3}{4}L + 2\zeta(2) - 1\right) \\
&\quad + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{L^4}{32} - \frac{31}{144}L^3 + \left(\frac{229}{288} - \frac{5}{4}\zeta(2)\right)L^2 + \dots\right] + \dots , \\
\text{Im} F(s) &= \left(\frac{\alpha}{\pi}\right) \pi \left(\frac{L}{2} - \frac{3}{4}\right) + \left(\frac{\alpha}{\pi}\right)^2 \pi \left(-\frac{L^3}{8} + \frac{31}{48}L^2 + \dots\right) + \dots , \\
F(t) &= 1 + \left(\frac{\alpha}{\pi}\right) \left(-\frac{L_t^2}{4} + \frac{3}{4}L_t + \frac{1}{2}\zeta(2) - 1\right) \\
&\quad + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{L_t^4}{32} - \frac{31}{144}L_t^3 + \left(\frac{229}{288} - \frac{1}{8}\zeta(2)\right)L_t^2 + \dots\right] + \dots ,
\end{aligned} \tag{2}$$

where $\zeta(2) = \frac{\pi^2}{6}$. Note that here the F functions contain also the contributions from vacuum polarization insertion in the vertex.

The B function is the soft-photon emission function, again considered without the infrared $\ln(\lambda/m_e)$ and $\ln \Delta$ terms,

$$B = 1 + \left(\frac{\alpha}{\pi}\right) B^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 (B_1^{(2)} + B_2^{(2)}) + \dots . \tag{3}$$

For the process under consideration $e^-(p_1)e^+(p_2) \rightarrow e^-(q_1)e^+(q_2)$ with the emission of photons $\gamma(k_i)$, the soft photon-emission functions for one and two photons are given by the integrals

$$\begin{aligned} I_1 &= -\frac{1}{(2\pi)} \int_0^{\Delta E} \frac{d^3 k}{2\sqrt{k^2 + \lambda^2}} S(k) , \\ I_2 &= \frac{1}{2} \frac{1}{(2\pi)^2} \int_0^{\Delta E} \frac{d^3 k_1}{2\sqrt{k_1^2 + \lambda^2}} S(k_1) \int_0^{\Delta E - |k_1|} \frac{d^3 k_2}{2\sqrt{k_2^2 + \lambda^2}} S(k_2) , \end{aligned} \quad (4)$$

with

$$S(k) = \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} - \frac{q_1}{q_1 k} + \frac{q_2}{q_2 k} \right)^2 .$$

In Eq. (3), $B^{(1)}$ is the non-infrared part of the integral I_1 for the emission of one soft photon in the s , $-t$, $-u \gg m_e^2$ limit [16]:

$$B^{(1)} = L^2 - 4\zeta(2) + L(L_t - L_u) + f , \quad (5)$$

where θ is the scattering angle,

$$\begin{aligned} a &= \sin \frac{\theta}{2} , \quad b = \cos \frac{\theta}{2} , \\ s &= 4E_b^2 , \quad t = -sa^2 , \quad u = -sb^2 , \\ L &= \ln \left(\frac{s}{m_e^2} \right) , \quad L_t = \ln \left(\frac{-t}{m_e^2} \right) , \quad L_u = \ln \left(\frac{-u}{m_e^2} \right) , \\ f &= 2(\ln^2 a - \ln^2 b) + \text{Li}_2(b^2) - \text{Li}_2(a^2) , \end{aligned} \quad (6)$$

and Li_2 is the Euler dilogarithm.

The last terms in Eq. (5), $L(L_t - L_u) + f$, are due to initial-final interference.

Going back to Eq. (3), $B_1^{(2)}$ comes from the integral I_2 for the emission of two soft photons in Eq. (4), when $\ln(\lambda/m_e)$ and $\ln \Delta$ terms are dropped:

$$B_1^{(2)} = \frac{1}{2} (B^{(1)})^2 - 8\zeta(2) [L - 1 + L_t - L_u]^2 , \quad (7)$$

again in the limit $s, -t, -u \gg m_e^2$.

The presence of the last two terms $(L_t - L_u)$ accounts for initial-final interference. Note that, if the soft emission factorization is taken separately for each fermion line, the result is slightly different: for example in the s -channel case with initial and final separate factorization, Eq. (5) and the first term in Eq. (7) would be the same, but the second term in Eq. (7) would be $-4\zeta(2)[L - 1]^2$, which is half the result obtained in the present case, disregarding the interference terms $(L_t - L_u)$.

We then have $B_2^{(2)}$, the contribution due to e^+e^- soft pair production, which will be stated explicitly later on by approximating for particular configurations. The definition of these explicit approximations to be used for small and large angle deserves some comments.

In order to determine the factorized function B , we recall that at large angle the s -channel prevails, while at small angle the t -channel is dominant.

Starting our considerations from the s -channel, we observe that the s -channel vertex form factors in Eq. (2) do not include the contributions (as for instance the box diagrams) which account for initial-final interference. Indeed we know that the term $L(L_t - L_u)$ in $B^{(1)}$ is exactly compensated by an opposite term coming from box diagrams at order α ; therefore we drop this term in $B^{(1)}$ for the s -channel and also the function f , which has a small value at large angle. Assuming a similar behaviour at order α^2 we omit also in Eq. (7) the term $L_t - L_u$, which is due to the interference.

The above considerations do not affect of course the order- α result, which is included completely, but only the determination of the order- α^2 leading result.

With these assumptions the expressions for the B functions in the s -channel, which

dominates at large angle, are

$$\begin{aligned}
B^{(1)}(s) &= L^2 - 4\zeta(2) , \\
B_1^{(2)}(s) &= \frac{1}{2} \left[B^{(1)}(s) \right]^2 - 8\zeta(2)(L-1)^2 , \\
B_2^{(2)}(s) &= \frac{1}{9}L^3 - \frac{5}{9}L^2 + \dots .
\end{aligned} \tag{8}$$

The expression for $B_2^{(2)}(s)$, the soft pair production, is taken from [6], where it is used for initial radiation in the s -channel.

In the limit of small scattering angle, where the t -channel dominates the cross section, the initial-final interference contributions are present in the t -channel vertex form factors, which relate the initial fermion line to the final one. They should then also be included in the soft emission function. In this limit $b \rightarrow 1$, $f \rightarrow \frac{1}{2} \ln^2 a^2 + \zeta(2)$, $-u \rightarrow s$, and the B functions become

$$\begin{aligned}
B^{(1)}(t) &= L_t^2 - 2\zeta(2) , \\
B_1^{(2)}(t) &= \frac{1}{2} \left[B^{(1)}(t) \right]^2 - 8\zeta(2)(L_t - 1)^2 , \\
B_2^{(2)}(t) &= \frac{1}{9}L_t^3 - \frac{5}{9}L_t^2 + \dots .
\end{aligned} \tag{9}$$

The expression for $B_2^{(2)}(t)$ in the t -channel is obtained similarly to the derivation for the s -channel presented in [18], and then calculated in the approximation $\sqrt{-t}/2 \gg \Delta E \gg 2m_e$. The details of the derivation are too long and technical to be presented in this letter, and will be reported elsewhere.

Collecting results and keeping only the leading terms in the contributions of order α^2 , we get

$$\begin{aligned}
d\sigma^{(s)} &= d\hat{\sigma}_0^{(s)} \left[1 + \frac{3}{2}\beta_e(s) + \left(\frac{\alpha}{\pi}\right) (4\zeta(2) - 1) + \left(\frac{\alpha}{\pi}\right)^2 (5 - 8\zeta(2)) L^2 + \dots \right] , \\
d\sigma^{(t)} &= d\hat{\sigma}_0^{(t)} \left[1 + \frac{3}{2}\beta_e(-t) - \left(\frac{\alpha}{\pi}\right) + \left(\frac{\alpha}{\pi}\right)^2 (5 - 8\zeta(2)) L_t^2 + \dots \right] ,
\end{aligned} \tag{10}$$

where $\beta_e(s) = 2 \left(\frac{\alpha}{\pi} \right) \left(\ln \frac{s}{m_e^2} - 1 \right)$. These results show the desired leading-logarithmic dependence at order α^2 , where the coefficients of L^4, L^3 or L_t^4, L_t^3 vanish. The order $\frac{\alpha}{\pi}$ terms present are those coming from the factorization, the other terms of the same order are accounted for elsewhere (in $\tilde{C}^{(i)}$, see Eqs. (12) and (20)) without any loss in the desired precision, moreover the dots recall that $\left(\frac{\alpha}{\pi} \right)^2$ non-leading terms are disregarded.

When the approximations are used in regions where they are not well suited for, an error occurs. At small angle, this happens for the leading contribution $\alpha^2 L^2$ to $d\sigma^{(s)}$, which however contributes only very little in that region, so that the error actually resulting is smaller than other unaccounted for terms (much lower than 0.01%). At large angle, it happens for the cross-section part $d\sigma^{(t)}$, which, although smaller, is not negligible, but affects only the subleading term $\alpha^2 L$; the effect is therefore again negligible (lower than 0.05%).

To our knowledge, the behaviour of the order- α^2 leading corrections to the s - t interference part of the cross section cannot be determined a priori on assumptions such as the ones already used. Indeed neither the small-angle approximation, nor neglecting the initial-final state interference is allowed; as a consequence it is impossible to ignore the presumably sizeable corrections from box-like diagrams in second-loop approximation.

Fortunately at small angle (say less than 150 mrad) the s - t lowest-order part contributes less than 1% of the total cross section, therefore its α^2 leading-logarithmic contribution is expected to be less than 0.01% of the total and can thus be disregarded.

On the contrary, at large angle the s - t lowest-order part contributes a large fraction of the cross section and its α^2 leading-logarithmic contribution is therefore not negligible (it can amount to 1%). We cannot use for the s - t interference part of the cross section in Eq.

(1) the approximation used for the s -channel, since there are initial-final interference terms in $F(t)$; however, performing a naïve substitution, the coefficients of higher-logarithmic powers vanish and the value of the coefficient of $\alpha^2 L^2$ is $(5 - 11\zeta(2))$.

We prefer to observe that in the s - and t -channel in Eq. (10) the coefficients of the leading terms turn out to be equal and the same is true for the s - t -channel at order α . Assuming this universality also for the order α^2 , we write

$$d\sigma^{(st)} = d\hat{\sigma}_0^{(st)} \left[1 + \frac{3}{2}\beta_e(s) + \left(\frac{\alpha}{\pi}\right)(\zeta(2) - 1) + \left(\frac{\alpha}{\pi}\right)^2 (5 - 8\zeta(2))L^2 + \dots \right], \quad (11)$$

where now also the $\left(\frac{\alpha}{\pi}\right)$ term is arbitrarily extracted from the complete order α contribution (also in this case the correct $\left(\frac{\alpha}{\pi}\right)$ term is recovered through the terms $\tilde{C}^{(i)}$ in Eq. (20)).

To help comparing our results with others in the literature, we recall that our factorized vertex form factor F contains the contribution from the vacuum polarization insertion, while the factorized soft emission function B is obtained for the whole process (differences arise in the second term of $B_1^{(2)}$, when taking the factorization for initial and final emission separately) and contains the contribution for soft e^+e^- pair production. Moreover we perform different approximations for the different channels, because of their different relevance at small or large angle.

We are aware that only a true leading-order α^2 calculation can give these corrections without any doubt; as a matter of fact, several groups are at work in this field, and some have obtained partial results in the small-angle approximation [17]. In the absence of a complete calculation, we can only rely on the above assumptions and results.

To extend the results of Eq. (10) and Eq. (11) to the structure-function approach, as implemented in [1], we write the soft limit of the differential cross section contributions

for each possible combination ($i = 1, \dots, 10$) with the Z^0 and γ exchange in the s - and t -channel

$$d\sigma^{(i)} = \int_0^\Delta d\omega_1 \omega_1^{\frac{\beta}{2}-1} \int_0^{\Delta-\omega_1} d\omega_2 \omega_2^{\frac{\beta}{2}-1} \int_0^{\Delta-\omega_1-\omega_2} d\omega_3 \omega_3^{\frac{\beta}{2}-1} \int_0^{\Delta-\omega_1-\omega_2-\omega_3} d\omega_4 \omega_4^{\frac{\beta}{2}-1} \left(\frac{\beta}{2}\right)^4 (1 + \delta_N^{(i)})^2 d\bar{\sigma}_0^{(i)} [1 + \tilde{C}^{(i)}] . \quad (12)$$

The factor $(1 + \delta_N^{(i)})^2$, still to be determined, contains the leading-logarithmic contributions, αL (or αL_t) and $\alpha^2 L^2$ (or $\alpha^2 L_t^2$).

The term $\tilde{C}^{(i)}$ is what is left of the order- α correction $C^{(i)}$, introduced in [1], after extraction of the terms already included in $\delta_N^{(i)}$. It has no longer logarithmic singularity in any of the photon energies ω_j ($j = 1, 2, 3, 4$), so that in the soft limit it can be disregarded. The lowest-order cross-section contributions $d\bar{\sigma}_0^{(i)}$ with running coupling constant α were defined in [1]; β is given by

$$\begin{aligned} \beta &= \beta_e(s) + \beta_{int}(\theta) , \\ \beta_{int}(\theta) &= 4 \left(\frac{\alpha}{\pi}\right) \ln \left(\tan \frac{\theta}{2}\right) , \end{aligned} \quad (13)$$

with $\beta \simeq \beta_e(s)$ at large angle and $\beta \simeq \beta_e(-t)$ at small angle.

The integration of Eq. (12) can be easily done, giving

$$d\sigma^{(i)} = d\hat{\sigma}_0^{(i)} \left[1 - \frac{3}{2} \zeta(2) \beta^2 \right] (1 + \delta_N^{(i)})^2 , \quad (14)$$

with the (already referred to) $d\hat{\sigma}_0^{(i)}$ given by

$$d\hat{\sigma}_0^{(i)} = \Delta^{2\beta} d\bar{\sigma}_0^{(i)} . \quad (15)$$

Comparing Eq. (14) with Eq. (10) and Eq. (11), we have

$$\begin{aligned}
\delta_N^{(s)} &= \frac{3}{4}\beta_e(s) + \left(\frac{\alpha}{\pi}\right) \left(2\zeta(2) - \frac{1}{2}\right) + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{11}{8} - 4\zeta(2)\right] L^2 + \frac{3}{4}\zeta(2)\beta^2 + \dots, \\
\delta_N^{(st)} &= \frac{3}{4}\beta_e(s) + \left(\frac{\alpha}{\pi}\right) \frac{1}{2}(\zeta(2) - 1) + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{11}{8} - 4\zeta(2)\right] L^2 + \frac{3}{4}\zeta(2)\beta^2 + \dots, \\
\delta_N^{(t)} &= \frac{3}{4}\beta_e(-t) - \frac{1}{2}\left(\frac{\alpha}{\pi}\right) + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{11}{8} - 4\zeta(2)\right] L_t^2 + \frac{3}{4}\zeta(2)\beta^2 + \dots,
\end{aligned} \tag{16}$$

To simplify the comparison, we recall that the coefficient often used [7] for the resummed initial s -channel radiator is

$$\frac{\exp\left[\frac{\eta}{2}\left(\frac{3}{4} - \gamma_E\right)\right]}{\Gamma(1 + \eta/2)}, \tag{17}$$

whose square (to be used in the e^+e^- collision) expanded in η is

$$1 + \frac{3}{4}\eta + \frac{1}{4}\left[\frac{9}{8} - \zeta(2)\right]\eta^2 + \dots. \tag{18}$$

The method leaves some ambiguity in the choice of the expansion parameter η ; by taking

$$\eta = -6 \ln\left(1 - \frac{\alpha}{3\pi}L\right) \simeq \left(\frac{\alpha}{\pi}\right) 2L + \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 L^2 + \dots, \tag{19}$$

the result of $1 + \delta_N^{(s)}$ in Eq. (16), related to initial-state emission in our approach, is recovered for the $\left(\frac{\alpha}{\pi}\right)^2 L^2$ terms.

Accounting for the channel dependence of the $1 + \delta_N^{(i)}$ factors, the main formula introduced in [1] is modified as follows:

$$\begin{aligned}
\Delta\sigma(e^+e^- \rightarrow e^+e^-) &= \int_{\Delta\Omega_-} d\Omega_- \int_{E_{min}^0}^{E_b} \frac{dE_-^0}{E_b} \int_{E_{min}^0}^{E_b} \frac{dE_+^0}{E_b} F(E_b, E_-^0, \theta_-) F(E_b, E_+^0, \theta_+) \\
&\quad \left(\frac{d\Omega^*}{d\Omega_-}\right) \sum_{i=1}^{10} \frac{d\bar{\sigma}_0^{(i)}(E^*, \theta^*)}{d\Omega^*} \left[1 + \delta_N^{(i)}(E_b, \theta^*)\right] \left[1 + \tilde{C}^{(i)}(E^*, \theta^*)\right] \\
&\quad \left[1 + \delta_N^{(i)}(E^*, \theta^*)\right] \Delta_-^{\frac{1}{2}\beta(E^*, \theta^*)} G(E^*, \theta^*, \Delta_-) \Delta_+^{\frac{1}{2}\beta(E^*, \theta^*)} G(E^*, \theta^*, \Delta_+).
\end{aligned} \tag{20}$$

The initial emission function F has been modified to include the second photon emission, using the lepton-energy distribution given in [7], and is

$$F(E_b, E_\pm^0, \theta_\pm) = D(E_b, E_\pm^0, \theta_\pm) \left\{ \frac{1}{2} \left(1 + \left(\frac{E_\pm^0}{E_b} \right)^2 \right) + A_m + \frac{\beta(E_b, \theta_\pm)}{8} \left[-\frac{1}{2} \left(1 + 3 \left(\frac{E_\pm^0}{E_b} \right)^2 \right) \ln \left(\frac{E_\pm^0}{E_b} \right) - \left(1 - \frac{E_\pm^0}{E_b} \right)^2 \right] \right\}. \quad (21)$$

The radiator D , after extraction of the $1 + \delta_N^{(i)}$ term, reads

$$D(E_b, E_\pm^0, \theta_\pm) = \frac{1}{2} \beta(E_b, \theta_\pm) \left(1 - \frac{E_\pm^0}{E_b} \right)^{\frac{1}{2} \beta(E_b, \theta_\pm) - 1}. \quad (22)$$

The integrated final emission function G then becomes

$$G(E^*, \theta^*, \Delta_\pm) = 1 - \frac{\beta(E^*, \theta^*)}{2 + \beta(E^*, \theta^*)} \Delta_\pm + B_m \frac{\beta(E^*, \theta^*)}{8 + 2\beta(E^*, \theta^*)} \Delta_\pm^2 - \frac{\beta(E^*, \theta^*)}{4} \left[\ln(1 - \Delta_\pm) \left(1 - \frac{3}{2} \frac{\beta(E^*, \theta^*)}{(2 + \beta(E^*, \theta^*))} \Delta_\pm + \frac{3}{4} \frac{\beta(E^*, \theta^*)}{(4 + \beta(E^*, \theta^*))} \Delta_\pm^2 \right) + \frac{2\Delta_\pm}{2 + \beta(E^*, \theta^*)} + \frac{8 + \beta^2(E^*, \theta^*)}{2(2 + \beta(E^*, \theta^*))(4 + \beta(E^*, \theta^*))} \Delta_\pm^2 + \frac{32 + 6\beta(E^*, \theta^*) + \beta^2(E^*, \theta^*)}{4(2 + \beta(E^*, \theta^*))(4 + \beta(E^*, \theta^*))} \Delta_\pm^3 \sum_{k=0}^{\infty} \frac{\Delta_\pm^k}{3 + k + \frac{1}{2} \beta(E^*, \theta^*)} \right], \quad (23)$$

where $\Delta_\pm = (E_b - E_{min})/E_\pm$ as discussed in [1]; $E_\pm$ is the energy deposited in the electromagnetic calorimeter by the final electron or positron with its accompanying photons, and E_{min} is the minimum accepted of this energy (the maximum allowed value for $\Delta_\pm$ is imposed to be $1 - m_e/E_b$).

Here we neglect the next-order contributions from hard photons, at variance with [7], as they are negligible for the cross-section precision specified above. The expressions for A_m and B_m are order- α corrections, accounting for the finite electron mass. They were obtained using the expression for one photon emission radiative Bhabha amplitude in the

collinear approximation

$$A_m = \frac{1}{2} \left(1 - \frac{E_{\pm}^0}{E_b} \right)^2 \frac{1}{\ln \left(\frac{s}{m_e^2} \right) - 1} , \quad (24)$$

and

$$B_m = \frac{\ln \left(\frac{s}{m_e^2} \right)}{\ln \left(\frac{s}{m_e^2} \right) - 1} . \quad (25)$$

For $m_e \rightarrow 0$ the limiting values $A_m = 0$ and $B_m = 1$ are recovered. These corrections are negligible at the required precision, although slightly larger than the previously mentioned (but neglected) next-order contributions.

All the higher-order corrections discussed above have been implemented in BHA-GEN94, the new version of our event generator.

Furthermore, in this new version the generation constraints, present in the previous version, are relaxed. The new mapping allows for an $E_{\min}$ range from E_b down to 1 GeV (still avoiding electron-mass corrections).

Because of this enlargement, the formula of ref. [19] used in the previous version of BHAGEN to account for final collinear hard photons in a calorimetric measurement, valid only in the limit $E_{\min} \simeq E_b$, is substituted by the one presented in Eq. (8) of ref. [20], valid for any $E_{\min}$ with $(\delta E_{\min}/m_e) \gg 1$, where δ is the usual half opening angle of the cone inside which the photon is not resolved. The formula is split to permit the exponentiation so that in the last line of Eq. (20) each final fermion has the factor

$$\begin{aligned} & \left[1 + \delta_N^{(i)}(E^*, \theta^*) \right]^{\frac{1}{2}} \Delta_{\pm}^{\frac{1}{2} \beta_H(E^*, \theta^*, \delta, \Delta_{\pm})} \left\{ G(E^*, \theta^*, \Delta_{\pm}) \right. \\ & + \left(\frac{\alpha}{\pi} \right) \left[\frac{1}{4} - \left(1 - \frac{\Delta_{\pm}}{2} \right)^2 \right] \ln \left[1 + \left(\frac{\delta E^*}{m_e} \right)^2 (1 - \Delta_{\pm})^2 \right] \\ & \left. + \left(\frac{\alpha}{\pi} \right) \left[\frac{9}{4} - \frac{5}{2} \Delta_{\pm} + \frac{1}{4} \Delta_{\pm}^2 - 2Li_2(1 - \Delta_{\pm}) \right] \right\} , \end{aligned} \quad (26)$$

where

$$\beta_H(E^*, \theta^*, \delta, \Delta_{\pm}) = -2 \left(\frac{\alpha}{\pi} \right) \ln \left[\frac{\left(\frac{m_e}{E^*} \right)^2 + \delta^2 (1 - \Delta_{\pm})^2}{4} \right] + \beta_{int}(\theta^*) . \quad (27)$$

The proposed exponentiation behaves as previously at small values of the emitted energy, and does not differ numerically from the original (not exponentiated) formula of ref. [20] at large emitted energy.

The new cross-section results have been compared with the results obtained with our previous version of BHAGEN [1]: we find that, due to the corrections discussed above, they typically grow up by 0.7% at most for small angle, while decreasing by about 0.3% for large angle.

After the present improvement, the discrepancy of our results (larger than the estimated precision) with those of some other existing programs is reinforced. A detailed comparison will be presented in a forthcoming paper [21], since it is a complex task to illustrate the differences of the various approaches at the requested precision.

We can estimate the error due to the uncertain or lacking contributions by putting some possible values and by making them vary.

At small angle ($1^\circ < \theta < 5^\circ$) the error of the arbitrary $\alpha^2 L^2$ correction to the s - t interference part is less than 0.01%; the estimate of the omitted $\alpha^2 L$ total correction is about 0.03% and that of the next $\alpha^3 L^3$ leading-order correction comes out smaller than 0.06%.

At large angle ($40^\circ < \theta < 140^\circ$), as mentioned above, the s - t interference part is sizeable and the error on its unknown $\alpha^2 L^2$ correction can be estimated to be up to 1%. This is, in our opinion, the present indetermination on Bhabha scattering at large angle!

The estimation of the error due to the omitted $\alpha^2 L$ correction is about 0.05% and,

for the next $\alpha^3 L^3$ term, it is 0.2%.

The weak corrections of order α are known [8], [12], [22], and have been implemented in BHAGEN94, at the accuracy required for large angle: details and comparison will be presented in a forthcoming paper [21].

In conclusion of this survey the theoretical error, after inclusion of what is now known to be of order $\alpha^2 L^2$ from soft-photon and electron-pair production corrections, is of 0.1% at small angle and of 1% at large angle.

Another source of error is due to the approximate kinematics used in the forward emission limit, not discussed here, but at present under investigation. We expect that the error due to this approximation will be safely under 0.5%.

The event generator BHAGEN94 includes the CERN-Library-supported random number generator RANLUX, so that BHAGEN94 can be used as a stand-alone program.

The FORTRAN code of the Monte Carlo event generator in which all discussed corrections are implemented is available on request from the authors.

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