The Green's function in the Bloch-Nordsieck model for $QED_3$.

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The Green's function is defined as the solution to the equation

\[ G(x, y; a, b) = \int_a^b \frac{1}{x - t} dt \]

subject to the boundary condition

\[ G(a, y; a, b) = 0. \]

The Green's function satisfies the homogeneous equation

\[ \frac{d^2 G}{dx^2} + \frac{1}{x} \frac{dG}{dx} = 0. \]

The solution to this equation is given by

\[ G(x, y; a, b) = C_1 \ln |x - a| + C_2 \ln |x - b|. \]

The constant coefficients are determined by the boundary conditions.

The Green's function is used to solve the inhomogeneous equation

\[ \frac{d^2 u}{dx^2} + \frac{1}{x} \frac{du}{dx} = f(x). \]

The solution is given by the convolution of the Green's function with the source term:

\[ u(x) = \int_a^b G(x, y; a, b) f(y) dy. \]
Solving Eq.(4) by means of a Fourier transformation, yields

\[
\mathcal{K}(\nu|A) = \frac{e}{(2\pi)^{3/2}} \int d^3k \, (u \cdot A(k)) e^{-i k \cdot x} \int_0^\infty dv' e^{i v' \nu A(k)} ,
\]

(6)

while, substituting Eq.(3) into Eq.(2), gives

\[
G(x,y|A) = \frac{i}{(2\pi)^3} \int_0^\infty dv \exp[-i \nu(x-y) - i v(m-u,p-i\epsilon) + i \mathcal{K}(\nu|A)] .
\]

(7)

The electron Green's function is the functional average

\[
G(x,y) = \frac{i}{S_0(A)} \langle G(x,y|A) S_0(A) \delta A \rangle
\]

where \(S_0(A)\) is the expectation value of the S-matrix in a Fermi vacuum. In this case it is equal to unity, since the RN model does not include the vacuum polarization. So, Eq.(8) simplifies to

\[
G(x-y) = \frac{1}{(2\pi)^{3/2}} \int d^3p \, G(p) e^{-i \nu \cdot x} .
\]

(9)

\[
G(p) = i \int_0^\infty dv e^{-i \nu \cdot x - i v - i\epsilon} \int e^{i \nu \cdot A} \delta A .
\]

(10)

In the present case, of course, \(n = 3\).

The functional integral in the above expression can be converted into an integral of the Gaussian type with respect to \(A_\alpha\). We thus obtain

\[
G(p) = i \int_0^\infty dv \exp -i \nu \cdot (m-u,p-i\epsilon) + f(\nu)
\]

(11)

where

\[
f(\nu) = -\frac{i e^2}{2(2\pi)^{3/2}} \int d^3k [n^\nu D_{\nu\alpha}(k) n^\alpha] \int_0^\infty dv' e^{i \nu' \nu A(k)} \int_0^\infty dv'' e^{-i \nu'' A(k)} .
\]

(12)

In the last equation, \(D_{\nu\alpha}\) stands for the free photon propagator,

\[
D_{\nu\alpha} = \frac{1}{k^2 + i\epsilon} \delta_{\nu\alpha} + \frac{1}{k^2 + i\epsilon} \delta_{\nu\alpha} ,
\]

(13)

where \(a\) is the gauge parameter. So, substituting Eq.(13) into Eq.(12),

\[
f(\nu) = -\frac{i e^2}{(2\pi)^{3/2}} \int_0^\infty dv' \int_0^\infty dv'' \int d^3k e^{i \nu A(k)} F(k) ,
\]

(14)

\[
F(k) = \frac{1}{k^2 + i\epsilon} + (a-1) \frac{\langle u(k) \rangle^2}{(k^2 + i\epsilon)^2} .
\]

(15)

It is convenient for our purposes to rewrite \(f(\nu)\) in the form

\[
f(\nu) = -i e^2 \int_0^\infty dv' \int_0^\infty dv'' [F^{(1)}(\nu') + (1-a) \frac{\partial}{\partial \nu''} F^{(2)}(\nu'')] .
\]

(16)

where

\[
F^{(1)}(\nu') = \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i \nu' A(k)} ,
\]

(17)

\[
F^{(2)}(\nu'') = \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i \nu'' A(k)} .
\]

(18)

We have left open the dimension in Eqs.(9), (17) and (18), to anticipate that we are going to use dimensional regularization for the integrals \(F^{(1)}\) and \(F^{(2)}\) above. One of the advantages of using dimensional as well as analytic regularization is the fact that, in this particular case, it is not necessary to attribute a fictitious mass to the photon, in order to take care of possible infrared divergences, in contrast with the Pauli-Villars regularization. For a given \(n\) we find

\[
F^{(1)}(\nu') = (-1)^{n-1} \frac{\Gamma(n/2-1)(\nu' - i\epsilon)^2-n}{2^{2-n/2}(3-n)} ,
\]

(19)

\[
F^{(2)}(\nu'') = (-1)^{n-1} \frac{\Gamma(n/2-2)(\nu'' - i\epsilon)^3-n}{2^{3-n/2}(3-n)} ,
\]

(20)

so that

\[
f(\nu) = e^2 \frac{(-1)^{n-1}}{2^{3-n/2}(3-n)} \Gamma(n/2-1) [2 + (1-a)(n-3)] P(\nu) ,
\]

(21)

with

\[
P(\nu) = \frac{1}{(4-n)[(\nu - i\epsilon)^{4-n} - (\nu - i\epsilon)^{4-n} - (\nu - i\epsilon)^{4-n}] .
\]

(22)
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References


