Dynamics of Apparent and Event Horizons

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The dynamics of apparent and event horizons of various black hole spacetimes, including those containing distorted, rotating and colliding black holes, are studied. We have developed a powerful and efficient new method for locating the event horizon, making possible the study of both types of horizons in numerical relativity. We show that both the event and apparent horizons, in all dynamical black hole spacetimes studied, oscillate with the quasinormal frequency.

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Introduction. Black holes are among the most fascinating predictions in the theory of General Relativity. During the past 20 years there have been intense research efforts on black holes and their effect on the astrophysical environment. With the exciting possibility of detecting gravitational wave signals from black holes by the gravitational wave observatories under construction (LIGO and VIRGO\(^{(1)}\)), we have seen an ever increasing surge of interest. The black hole events likely to be observed by the gravitational wave observatories involve highly dynamical black holes, e.g., two black holes in collision. The most powerful tool in studying such highly dynamical and intrinsically non-linear events is probably numerical treatment. In recent years, there has been significant progress in numerical relativity in this direction (See, e.g.,\(^{(2)}\)). In particular, long evolutions of highly dynamical black hole spacetimes are now possible\(^{(3,4)}\), opening up the opportunity of many interesting studies.

The essential characteristics of a black hole in relativity are its horizons, in particular, the apparent horizon (AH) and the event horizon (EH). Singularity theorems\(^{(5)}\) link the formation of black hole singularities to the formation of the AH. The membrane paradigm\(^{(6)}\) characterizes black holes by the properties of its EH, which is regarded as a 2D membrane living in a 3D space, evolving in time and endowed with many everyday physical properties like viscosity, conductivity, entropy, etc. We believe such a point of view is a powerful tool in providing new insight into the numerical studies of black holes.

The apparent horizon (AH) at each instant of time is generically a spacelike surface defined as the outermost trapped surface of a region in space, whereas the event horizon (EH) is a null surface defined as the boundary of the causal past of the future null infinity (For a rigorous description, see, e.g.\(^{(5)}\)). The AH is a local object, readily obtained in a numerical evolution by searching a given time slice for closed surfaces whose outgoing null rays
have zero expansion. In contrast, the EH is a globally defined object which makes it harder to find in numerical studies. Its location, and even its existence cannot be determined without knowledge of the complete four geometry of the spacetime.

Despite the formal distinctions between the two kinds of horizons, there are intriguing relations between them. For example, the cosmic censorship conjecture [7] (together with the singularity theorems) suggests that the formation of an AH is usually accompanied by the formation of an EH, and for all stationary spacetimes [8] the AH and EH coincide with each other. How far can these similarities be extended? Can the shape and surface area of the two horizon surfaces be very different? Can some membrane-like [6] properties be associated with the AH? These are some of the many interesting questions concerning the properties of apparent and event horizons that we touch upon in this paper and shall investigate further in future publications.

The central development that makes possible our study of horizon dynamics is that we can now find both the AH and the EH in numerical relativity. In particular, we have developed a powerful and efficient method to locate the EH in dynamic black hole spacetimes. In this paper, we outline this method and apply it to study and compare the dynamical evolutions of the AH and EH for various spacetimes, including single black holes interacting with external gravitational fields and black holes in collision. In particular we show that the EH and the AH oscillate with the same frequency (the quasinormal frequency as determined using black hole perturbation theory [9]), even though the early stages of the spacetime dynamics are highly nonlinear in the cases studied.

Finding the Horizons. It is by now a routine exercise to find apparent horizons in numerical relativity (see, e.g., Refs. [10]) and we will not discuss AH methods further in this paper. The actual EH of a black hole can in principle be found by tracing the path of null rays through time. Outward going light rays emitted just outside the EH will diverge away from it, escaping to infinity, and those emitted just inside the EH will fall away from it, towards the singularity. In a numerical integration it is difficult to follow accurately the evolution of an actual null horizon generator forward in time, as small numerical errors cause the ray to drift and diverge rapidly from the true EH. It is a physically unstable process. But we can actually use this property to our advantage by considering the time-reversed problem. Any outward going photon that begins near the EH will rapidly be attracted to the horizon if integrated backward in time. In integrating backwards in time, the initial guess of the photon does not need to be very precise as it will converge to the correct trajectory after only a short time. Therefore, we can integrate backward starting from a position we expect to be near the EH, e.g., the location of the AH at a time when the black hole has settled down to an approximate
stationary state after some dynamical evolution.

In principle one could consider finding the EH by tracing a collection of photons backwards in time through the numerically generated spacetime. This necessitates integrating the geodesic equation for each photon. However, because the geodesic equation requires taking derivatives of the metric functions, this procedure is extremely sensitive to inaccuracies in the numerically generated metric. Also (and more importantly) although photons will be attracted in the normal direction to the EH, there is no such attractive property in the tangential direction. As the geodesic equation is 2nd order, the initial directions of the photon must be specified. The trajectory is sensitive to the initial choice in the tangential direction, and it may further drift tangentially due to inaccuracy in integration. An initial surface of photons is not guaranteed to be surface forming after integration due predominantly to such tangential drifts.

Rather than independently tracking all individual photons starting on a surface, we follow the entire null surface itself. A null generator of the null surface is guaranteed to satisfy the geodesic equation [8]. A null surface defined by \( f(t, x^i) = 0 \) satisfies the condition

\[
g^{\mu\nu} \partial_\mu f \partial_\nu f = 0.
\]  

Hence the evolution of the surface can be obtained by a simple integration,

\[
\partial_t f = \frac{-g^{tt} \partial_t f - \sqrt{(g^{tt} \partial_t f)^2 - g^{ij} \partial_i f \partial_j f}}{g^{tt}}
\]  

Notice that this equation contains only derivatives of the surface and not of the metric components themselves and is therefore less susceptible to the numerical inaccuracies present in the metric data.

The advantages to integrating an entire surface include: (i) Tangential drifting is not a source of error, because the only direction that a surface can move is normal to itself; Once the surface becomes the EH, it cannot drift away from it. (ii) Unlike integrating null geodesics, this method is guaranteed to be surface forming. (iii) It is simpler and more accurate than evolving individual photons.

Using this method we are able to trace accurately the entire history of the EH in a short period of time. It takes just a few minutes to trace the EH on a computer workstation for an axisymmetric spacetime representing a black hole interacting with a gravitational wave (the first case detailed below) resolved on a grid of 200 radial by 53 angular zones and evolved to \( t = 75M \) (where \( M \) is the mass of the black hole). We contrast our backward surface method with another method [11] that uses forward integration of individual photons to find the EH.

**Horizons of Black Hole Spacetimes.** The first case we discuss consists of a non-rotating black hole surrounded by an axisymmetric gravitational wave initially at a finite
distance away from the hole. The system was evolved with a code described in Refs. [12,13]. The black hole becomes distorted as the incoming wave hits. In time, it settles down and returns to a Schwarzschild hole with a larger mass. Fig. (1) shows the areas of the horizons vs. time. Six different integrations of the EH starting at different places are shown. In one case the AH was used as an initial surface for the integration. Because the AH is inside the correct location of the EH, the surface expands outward as it is attracted to the correct location. In other cases, surfaces larger or smaller than the AH are chosen as initial guesses. Note that in all cases the surfaces are attracted to the same surface in precise detail, as they should be. The insert shows an expanded view of the early time. All surfaces computed are shown, but they are completely indistinguishable in spite of their extremely different starting positions, clearly showing the power and stability of this method. At \( t = 0 \) the AH and EH practically coincide with each other. Then the EH foresees the coming of the wave and expands. As the wave is falling in, after about \( t = 15M \), the AH starts to expand and catch up. The behavior of the AH and EH are exactly as expected. (The area curves exaggerate the effect of very small differences in coordinate location of the horizons, as they are located in a region of the spacetime where the metric functions have very steep gradients. The numerically generated data of the spacetimes do not generally resolve these gradients accurately, leading to the spurious growing of the horizon area at late time [14].) We have also studied the EH and AH of distorted, rotating black holes. We find the dynamical evolution of these horizons to have similar properties.

In Fig. (2) we show a geometric embedding of the coalescing horizons for the head-on collision of two black holes, as discussed in Ref. [4,15]. The embedding, which preserves the proper surface areas of the horizons, shows not just the topology but also the geometric properties of the horizon. Although such a picture of the embedding is familiar, this is the first time it has actually been computed. There have been a number of attempts to estimate the critical separation parameter \( \mu \) beyond which these initial data sets contain two separate black holes [16], and with our method we can now say that for \( \mu \) greater than about 1.8 there are two holes. The horizon shown in Fig. (2) corresponds to \( \mu = 2.2 \), i.e., an initial proper separation of \( 8.92M \), where \( M \) is the mass of each hole.

We see that initially the two black hole horizons are separated. They coalesce at about \( t = 6M \), forming a single large black hole. Various properties of the horizons in the process of coalescence will be analyzed elsewhere.

The ability to determine the AH and EH for dynamical holes opens up the possibility for the first time of using the horizons as a tool to study black hole physics in numerical relativity. As a first example of this, we compute the ratio of the polar to equatorial circumference, \( C_p/C_e \), of both the AH and EH for the first case discussed above.
In Fig. (3) we show this ratio as a function of time for both the AH and the EH. [Note that the horizon begins and ends as a sphere, as its gaussian curvature is constant to within 1 part in $10^6$, which shows clearly that the hole was “Schwarzschild” at both times. This also provides a stringent test for our event horizon finder, since it must trace a surface backward in time as it undergoes a period of distortion, and then return to a sphere.] We see that the AH and the EH oscillate in precisely the same manner, despite the fact that one is a null surface while the other is spacelike. These oscillations of the horizons are caused by the “quasinormal mode ringing” of the gravitational waves generated by the potential barrier in the black hole spacetime. As the waves leak out to infinity and down the hole, those going down cause the horizons to oscillate. In the membrane point of view, the oscillations are dissipated into viscous heating of the horizon membrane, causing the horizon surface area to increase during the oscillations, as we have seen in our calculations. We show a fit of the EH oscillation to the two lowest $\ell = 2$ quasinormal modes of the black hole as determined in linear perturbation theory (the initial incoming Brill wave is predominantly in the $\ell = 2$ mode). The fit is remarkable, showing conclusively that both the AH and EH oscillate at the natural frequency of the black hole. (The horizons oscillate with the quasinormal frequency in coordinate time, as they should.) We observe similar oscillations in both the distorted rotating black holes and the black hole collisions, establishing the generic nature of these results. This oscillation is also just visible in the horizon embedding diagram (Fig. (2)) for the two black hole collision after the holes have coalesced, where about one wavelength of the quasinormal mode is shown.

We have also found that for highly distorted holes, both AH and EH can have $C_p/C_v$ very different from 1, making some circumference of the horizon rather bigger than $4\pi M$ at times. The details of this and its possible implication on the hoop conjecture (vacuum version [17]) will be discussed elsewhere.

Conclusions. We have developed a powerful method for finding black hole event horizons in dynamic spacetimes based on the ideas of (i) backward integration and (ii) integrating the entire null surface. This opens up the possibility of studying the dynamics of event horizons in numerical relativity. We studied and compared the behavior of both the apparent and event horizons for various dynamical spacetimes. In all cases studied, we show that the event and apparent horizons oscillate with the quasinormal frequency of the black hole.

Our method can find event horizons without knowledge of the apparent horizon, so it should be a useful tool for analyzing spacetimes even in cases where the apparent horizon cannot be found (e.g., if the time slicing does not intersect the apparent horizon). Its impact on the numerical investigations of the cosmic censorship conjecture and the hoop conjecture could prove interesting.
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FIG. 1. The area of the event horizon is traced through time for different initial surfaces (solid lines), and compared to the area of the apparent horizon (dashed line). The attracting nature of the event horizon is dramatic, as all of our backward surface integrations trace the same path, although they start from very different initial locations. The insert shows an expanded view of the early time results. All surface integrations are shown, and are completely indistinguishable.

FIG. 2. The geometric embedding of the event horizon for two black holes colliding head on is shown. The $z$ coordinate marks the symmetry axis, and $t$ is the coordinate time. Initially the two holes are separate, and by about $t = 6M$ they coalesce into a single black hole. The distance between the initial horizons is arbitrary in this embedding space, but is based on the proper distance between the holes.

FIG. 3. The ratio $C_p/C_e$ of the polar circumference to the equatorial circumference of both the event horizon (solid line) and apparent horizon (dashed line) is shown versus coordinate time $t$ for a black hole which is hit by a gravitational wave. The dot-dashed line shows the fit of the two lowest $\ell = 2$ quasinormal modes to the event horizon. The hole oscillates at its quasinormal frequency when hit by the wave.