AN APPROACH TO FINITE-SIZE PARTICLES WITH SPIN

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Abstract

A short review of the theory of relativistic spin particles is presented. The diverse ways
of approach to the equations of motion of classical spin particles are described. The prop-
erties of classical spin particles are discussed and the similarities to quantum-mechanical
properties of Dirac's electron are stressed. Then the attempts of the quantization of the
theory of spin particles are sketched.

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We assume the pseudovector and pseudoscalar potentials of the form:

\[ \mathbf{V} = \frac{1}{\mathbf{r}} + \mathbf{A} \mathbf{r} + \mathbf{B} \mathbf{r}^2 \]

(1)

where \( \mathbf{r} \) is the position vector, \( \mathbf{A} \) and \( \mathbf{B} \) are constants.

The equations (1) reduce to

\[ 0 = \mathbf{D} \]

(2)

We shall restrict ourselves to the case of free particles. Then

\[ \mathbf{D} \cdot \mathbf{n} - \mathbf{D} \cdot \mathbf{n} = \frac{\mathbf{S} \cdot \mathbf{n}}{p} \]

(3)

where \( \mathbf{S} \) is the spin operator.

These three principles can be simultaneously satisfied. There were

III show the results, which should agree with experimental data.

II be gauge-invariant

I be Lorentz-invariant

1 Introduction

2 Classical equations of motion of dipole particles

Key words: Relativistic spin particles.
In order to save the law of the conservation of angular momentum we introduce the bivector \( S_{\mu\nu} \), satisfying

\[
\dot{S}_{\mu\nu} = u_{\nu} G_{\sigma} - u_{\sigma} G_{\nu}.
\]  

(4)

Then

\[
\frac{d}{ds} (x_{\nu} G_{\mu} - x_{\mu} G_{\nu}) + \dot{S}_{\mu\nu} = 0.
\]

(5)

Multiplying (4) by \( u^\nu \) we get

\[
G_{\mu} = m u_{\mu} + \frac{1}{c^2} S_{\mu\nu} \dot{u}^\nu.
\]

(6)

where

\[
m = \frac{1}{c^2} G_{\mu} u^\mu.
\]

(7)

It can be seen that every bivector \( S^\mu\nu \) may be written in the form

\[
S^\mu\nu = s^\mu\nu + n^\mu u^\nu - n^\nu u^\mu,
\]

(8)

where \( s^\mu\nu \) is the bivector and

\[
s^\mu\nu u_\nu = 0, \quad n^\mu u_\mu = 0.
\]

(9)

We call \( s^\mu\nu \) the spin, \( n^\mu \) the dipole moment of the particle. The relation (9) gives in the proper system of the particle

\[
s^\mu\nu = (s, q) \quad \text{and} \quad q = \frac{1}{c} s \times u.
\]

The equations (2), (4), (6), (7), (8), (9) form the system of classical equations of motion of the dipole particles. Two cases are important:

\[ s^\mu\nu \neq 0, n_\mu = 0, \] spin particle (Frenkel 1926), (Mathisson 1936), (Weyssenhoff 1947 a, c),

\[ n_\mu \neq 0, \text{pole-dipole particle (Hönl 1939).} \]

The equations of motion were also derived for dipole particles in external field of forces.

2.2 Mathisson's derivation

Mathisson (1936) considered the fundamental problem of general relativity (which was studied since 1919) of deriving the equations of motion of the particle in the gravitational field from the equations of this field as the equations of the world line of the singularity in this gravitational field. Mathisson's predecessors studied the case, when this singularity had (in a properly chosen coordinate frame) spherical symmetry and they got the equations of the geodesics. Mathisson was the first, who considered non-spherically singularities of gravitational field.

He introduced into his variational principle, based on the equations of gravitational field, the multipole moments of this singularity. Assuming the presence of only positive masses he obtained in the lowest approximation the equations of the geodesics. In the next approximation he got for positive masses the equations of motion of the spin particle, which for the gravitational field with metrics close to Minkowski's one, reduced to the equations (2) - (9). (See also (Średniawa 1983)).

2.3 Lubański's derivation

By expanding gravitational retarded potentials of gravitational multipoles (Lubański 1937) and using for the momentum-energy tensor \( T^{\mu\nu} \) the relation

\[
\nabla_\nu T^{\mu\nu} = 0,
\]

Lubański obtained (for positive masses) the equations of motion of the spin particle.

2.4 Weyssenhoff's and Raabe's derivation

Weyssenhoff and Raabe (Weyssenhoff and Raabe 1947 a) considered the incoherent fluid, for which they assumed the momentum-energy tensor of the form

\[
T^{\mu\nu} = g^{\mu\nu} u^{\nu},
\]

(11)

where \( g^{\mu\nu} \) was the density of momentum and assumed the existence of the intrinsic angular momentum density \( s^{\mu\nu} \). They formulated the equations of motion of such a fluid and integrated them for its very small volume. They obtained again the equations of motion of the spin particle.

2.5 Hönl's and Papapetrou's derivation

Hönl and Papapetrou (1939) applied Lubański's method to a particle characterized by the mass \( m \), dipole moment \( n^\mu \), assuming \( s^{\mu\nu} = 0 \). They obtained the equations of motion of the pole-dipole particles.
was done by Oettingenstal (1859) and for homogeneous equations of this
depersed, since classical equations of motion are of the third order. Thus
quantization, the hamiltonian formalism with higher derivatives should be
d to be guaranteed that theory of electromagnetic fields by dividing the.

3. Canonical formalism with higher derivatives and
Spinor electrodynamics

3.2. Dipole Particle

3.2.2. Pole-dipole particle

\[ \frac{dz}{\rho^2} = \frac{\sigma}{e\rho^2} \]

and definite frequency

\[ \frac{\rho}{e\rho^2} = \frac{\sigma}{e^2\rho^2} \]

Soliton in the proper frame $C$ of $\omega$, the circle of the definite radius

Hence, the dipole particle moves slower than light.

...
order by Weyssenhoff (1951). They started from the lagrangian as a function of position, velocity and acceleration of the particle

$$L(r, \dot{r}, \ddot{r})$$

Then the canonical momenta to the variables

$$r, \quad v = \dot{r},$$

are

$$p_i = \frac{\partial L}{\partial \dot{v}_i} - \frac{d}{dt} \frac{\partial L}{\partial v_i}, \quad s_i = \frac{\partial L}{\partial v_i},$$

and the hamiltonian is equal to

$$H = v \cdot p - \dot{v} \cdot s - L(r, v, \dot{v})$$

after elimination of \(v\).

Bopp (1946) and Weyssenhoff (1951) found the form of the lagrangian of the spin particle for \(v = c, \left(\gamma = \frac{\kappa}{c}\right)\):

$$L = \sqrt{\gamma} \sqrt{\frac{w_{\mu} w_{\mu}}{w_{\nu} w_{\nu}}},$$

where \(w_{\mu} = \frac{dx_{\mu}}{dt}\), \(l_0\) is a constant of the dimension of length. From this lagrangian the corresponding hamiltonian could be obtained. Bopp (1948) introduced a complex spinor-like variable \(\xi\) on the place of \(w_{\mu}\) and the wave function \(\Psi(x, \xi, \bar{\xi})\). Then he extended the Jordan quantization rule to

$$\eta \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \xi}, \quad \eta^\dagger \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \bar{\xi}},$$

$$p_{\mu} \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x_{\mu}}, \quad p_{\mu}^\dagger \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \bar{x}_{\mu}}$$

and obtained a Dirac-like wave equation, which in simple cases could be reduced to Dirac equation.

Later it turned out (Infeld 1957), (Borelowski 1961), (Borelowski and Średniawa 1962) that from the postulates for \(L\) that

1. \(L\) should be a scalar not depending on \(x_{\mu}\),

2. canonical \(p_{\mu}\) and \(w_{\mu}\) should not in general be parallel, a variety of models of free dipole particles could obtained.

Remark: One of the other ways of obtaining classical particles with spin leads through the consideration of the magnetic top (see Barut et al. 1992).

6 Conclusion

The general result of our considerations consists in the fact, that starting as well from general relativity as from special relativity and generalizing slightly the principles of mechanics by assuming that in general momentum and velocity are not parallel, one is lead to the flat structures of finite sizes. These classical structures, whose motion is described by differential equations of higher order, show before quantization some of the characteristic features of quantum particles. The standard methods of quantizations of the motions of this particles were, alas, not successful.

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