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AN APPROACH TO FINITE-SIZE PARTICLES WITH SPIN

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Abstract

A short review of the theory of relativistic spin particles is presented. The diverse ways of approach to the equations of motion of classical spin particles are described. The properties of classical spin particles are discussed and the similarities to quantum-mechanical properties of Dirac's electron are stressed. Then the attempts of the quantization of the theory of spin particles are sketched.

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1 Introduction

Quantum mechanics, quantum electrodynamics and quantum chromodynamics deal with point particles. In quantum theory of fields the point particles are causes of grave difficulties. Quantum field theory based on special theory of relativity should satisfy three principles. It should :

I be Lorentz-invariant

II be gauge-invariant

III give finite results, which should agree with experimental data.

These three principles cannot be simultaneously satisfied. There were and there are made attempts to improve quantum field theories. In most cases of improvement of the theory two principles were not changed: Lorentz invariance and gauge-invariance. There are two main ways to improve the theory in the third point:

1. the renormalization without changing the point-like character of the particles,

2. the passage to extended particles by introducing form factors.

The procedure 2 violates the principles I or II and therefore it was given up.

There were also made another attempts of introducing particles of finite sizes to the physics of elementary particles, between them:

1. theory of relativistic spin particles,

2. theory of strings and membranes.

Here we shall be concerned with the classical spin particles and their quantum analogues.

2 Classical equations of motion of dipole particles

2.1 Weyssenhoff's derivation

The equations of motion of dipole particles can be derived from different points of view. We begin with the most simple derivation (Weyssenhoff 1947c, 1958), resulting from the simple generalization of the mechanics of material points, by assuming that the energy-momentum vector G^μ of the particle is not necessarily parallel to its four-velocity u^μ . Then the angular momentum is not conserved ($\dot{\cdot} = \frac{d}{ds}$)

$$\frac{d}{ds}(x_\mu G_\nu - x_\nu G_\mu) = x_\mu \dot{G}_\nu - x_\nu \dot{G}_\mu + u_\mu G_\nu - u_\nu G_\mu. \quad (1)$$

We shall restrict ourselves to the case of free particle. Then

$$\dot{G}_\mu = 0 \quad (2)$$

and the equation (1) reduces to

$$\frac{d}{ds}(x_\mu G_\nu - x_\nu G_\mu) = u_\mu G_\nu - u_\nu G_\mu. \quad (3)$$

¹ $\mu, \nu, \dots = 0, 1, 2, 3, \quad i, j, \dots = 1, 2, 3.$

We assume the pseudoeuclidean metrics + - - -. Then $u_\mu u^\mu = 1$.

In order to save the law of the conservation of angular momentum we introduce the bivector $S_{\mu\nu}$, satisfying

$$\dot{S}_{\mu\nu} = u_\mu G_\nu - u_\nu G_\mu. \quad (4)$$

Then

$$\frac{d}{ds}(x_\mu G_\nu - x_\nu G_\mu) + \dot{S}_{\mu\nu} = 0. \quad (5)$$

Multiplying (4) by u^ν we get

$$G_\mu = mu_\mu + \frac{1}{c^2} S_{\mu\nu} \dot{u}^\nu. \quad (6)$$

where

$$m = \frac{1}{c^2} G_\mu u^\mu. \quad (7)$$

It can be seen that every bivector $S^{\mu\nu}$ may be written in the form

$$S^{\mu\nu} = s^{\mu\nu} + n^\mu u^\nu - n^\nu u^\mu, \quad (8)$$

where $s^{\mu\nu}$ is the bivector and

$$s^{\mu\nu} u_\nu = 0, \quad n^\mu u_\mu = 0. \quad (9)$$

We call $s^{\mu\nu}$ the spin, n^μ the dipole moment of the particle. The relation (9) gives in the proper system of the particle

$$s^{\mu\nu} = (s, q) \quad \text{and} \quad q = \frac{1}{c} s \times u.$$

The equations (2), (4), (6), (7), (8), (9) form the system of classical equations of motion of the dipole particles. Two cases are important:

$s_{\mu\nu} \neq 0, n_\mu = 0$, spin particle (Frenkel 1926), (Mathisson 1936), (Weyssenhoff 1947 a,c),
 $n_\mu \neq 0$, pole-dipole particle (Hönl 1939).

The equations of motion were also derived for dipole particles in external field of forces.

2.2 Mathisson's derivation

Mathisson (1936) considered the fundamental problem of general relativity (which was studied since 1919) of deriving the equations of motion of the particle in the gravitational field from the equations of this field as the equations

of the world line of the singularity in this gravitational field. Mathisson's predecessors studied the case, when this singularity had (in a properly chosen coordinate frame) spherical symmetry and they got the equations of the geodesics. Mathisson was the first, who considered non-spherically singularities of gravitational field.

He introduced into his variational principle, based on the equations of gravitational field, the multipole moments of this singularity. Assuming the presence of only positive masses he obtained in the lowest approximation the equations of the geodesics. In the next approximation he got for positive masses the equations of motion of the spin particle, which for the gravitational field with metrics close to Minkowski's one, reduced to the equations (2) - (9). (See also (Średniawa 1983)).

2.3 Lubański's derivation

By expanding gravitational retarded potentials of gravitational multipoles (Lubański 1937) and using for the momentum-energy tensor $T^{\mu\nu}$ the relation

$$\nabla_\nu T^{\mu\nu} = 0, \quad (10)$$

Lubański obtained (for positive masses) the equations of motion of the spin particle.

2.4 Weyssenhoff's and Raabe's derivation

Weyssenhoff and Raabe (Weyssenhoff and Raabe 1947 a) considered the incoherent fluid, for which they assumed the momentum-energy tensor of the form

$$T^{\mu\nu} = g^{\mu\nu} u^\nu, \quad (11)$$

where g^μ was the density of momentum and assumed the existence of the intrinsic angular momentum density $s^{\mu\nu}$. They formulated the equations of motion of such a fluid and integrated them for its very small volume. They obtained again the equations of motion of the spin particle.

2.5 Hönl's and Papapetrou's derivation

Hönl and Papapetrou (1939) applied Lubański's method to a particle characterized by the mass m , dipole moment n^λ , assuming $s^{\mu\nu} = 0$. They obtained the equations of motion of the pole-dipole particles.

3 Solutions of the equations of motion of the spin particle and the pole-dipole particle

We assume that G^μ is a time-like vector.

3.1 Spin particle

The solution (Weyssenhoff and Raabe 1947 a) in the proper frame of reference of G^μ (where $G^i = 0$) is in the uniform circular motion of radius

$$r = \frac{s_0 v}{m_0(c^2 - v^2)} \quad (12)$$

and frequency

$$\omega = \frac{m_0(c^2 - v^2)}{s_0}, \quad (13)$$

where

$$s_0^2 = s_{\mu\nu}s^{\mu\nu} = \text{const.}, \quad m_0 \text{ is constant, } v^2 = (v^i)^2.$$

3.2 Pole-dipole particle

In order to obtain here the definite solution, supplementary condition must be assumed. Having put $n^\mu n_\mu = \text{const.}$, (Šredniawa 1948) we get for the pole-dipole particle in proper frame of G^μ also circular motion.

These solutions suggest that the whole circle should be regarded as "macroparticle", in contradistinction to the "microparticle", which circulates along the circle.

4 Passage to the velocity of light for the spin particle and analogy to Dirac's electron

Up to now we considered dipole particles moving slower than light. Then the "microparticle" has still one arbitrary parameter, namely the velocity v , which can make the radius of the circle arbitrary large. Therefore Weyssenhoff and Raabe (1947 b) made the passage $v \rightarrow c$. Since then $ds \rightarrow 0$, and proper time cannot be used, they introduced before the passage to the limit $v \rightarrow c$ the new parameter π , such that $\frac{ds}{d\pi} > 0$. Having made the passage they choose the time t as π , and they got the equations of motion, whose

solution in the proper frame C of G^μ was the circle of the definite radius

$$r_c = \frac{s_c}{cM_c} \quad (14)$$

and definite frequency

$$\omega_c = \frac{M_c c^2}{s_c}, \quad (15)$$

where

$$s_c = (s_c^i)^2, \quad M_c = G^\mu u_\mu.$$

The spin particle moving with the velocity of light has following properties similar to those of Dirac's electron : (Weyssenhoff 1947 d):

1. in both theories the momentum G^μ is not parallel to the velocity u^μ ,
2. kinematical velocity $v = c$ corresponds to the fact that the eigenvalues of the velocity operator in Dirac's theory are equal to $\pm c$,
3. in both theories angular momenta are not constants of motion, circular motion is the classical "picture" of the "Zitterbewegung",
4. if one puts $s_c = \frac{1}{2}$ and M_c as the experimental mass of the electron, r_c is of the order of the Compton wave-length and ω_c of the order of "Zitterbewegung",
5. condition of stability of the circle in the electromagnetic field is the same as the condition of non forming electron pairs in this field in Dirac's theory.

These analogies suggest that the spin particle can in some sense be regarded as a classical relativistic model of Dirac's electron.

5 Canonical formalism with higher derivatives and attempts of quantization of spin particles

Up to now we considered classical, non quantized dipole particles. In order to try to quantize their theory by generalising the standard way of quantization, the hamiltonian formalism with higher derivatives should be developed, since classical equations of motion are of the third order. This was done by Ostrogradski (1850) and for homogeneous equations of third

order by Weyssenhoff (1951). They started from the lagrangian as a function of position, velocity and acceleration of the particle

$$L(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) \quad (16)$$

Then the canonical momenta to the variables

$$\mathbf{r}, \quad \mathbf{v} = \dot{\mathbf{r}},$$

$$\text{are} \quad p_i = \frac{\partial L}{\partial v_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{v}_i}, \quad s_i = \frac{\partial L}{\partial \ddot{v}_i}, \quad (17)$$

and the hamiltonian is equal to

$$H = \mathbf{v} \cdot \mathbf{p} - \dot{\mathbf{v}} \cdot \mathbf{s} - L(\mathbf{r}, \mathbf{v}, \dot{\mathbf{v}}) \quad (18)$$

after elimination of $\mathbf{v}$.

Bopp (1946) and Weyssenhoff (1951) found the form of the lagrangian of the spin particle for $v = c$, ($' = \frac{d}{dt}$):

$$L = \sqrt{l_0} \sqrt{\frac{w_\mu w^{\mu'}}{w_\mu w^\mu}}, \quad (19)$$

where $w^\mu = \frac{dx^\mu}{dt}$, l_0 is a constant of the dimension of length. From this lagrangian the corresponding hamiltonian could be obtained. Bopp (1948) introduced a complex spinor-like variable ξ on the place of w^μ and the wave function $\Psi(x, \xi, \xi^*)$. Then he extended the Jordan quantization rule to

$$\begin{aligned} \eta &\rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \xi^*}, & \eta^\dagger &\rightarrow \frac{\hbar}{i} \frac{\partial}{\partial \xi} \\ p_\mu &\rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x^\mu}, & p_\mu^\dagger &\rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x^\mu} \end{aligned} \quad (20)$$

and obtained a Dirac-like wave equation, which in simple cases could be reduced to Dirac equation.

Later it turned out (Infeld 1957), (Borelowski 1961), (Borelowski and Średniawa 1962) that from the postulates for L that

1. L should be a scalar not depending on x^μ ,
2. canonical p^μ and u^μ should not in general be parallel, a variety of models of free dipole particles could be obtained.

Remark: One of the other ways of obtaining classical particles with spin leads through the consideration of the magnetic top (see (Barut et al. 1992)).

6 Conclusion

The general result of our considerations consists in the fact, that starting as well from general relativity as from special relativity and generalizing slightly the principles of mechanics by assuming that in general momentum and velocity are not parallel, one is lead to the flat structures of finite sizes. These classical structures, whose motion is described by differential equations of higher order, show before quantization some of the characteristic features of quantum particles. The standard methods of quantizations of the motions of this particles were, alas, not successful.

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