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"A non-perturbative approach to the instability of the vacuum in the presence of fermions"
Vacuum structure of the model. With regards to the Higgs mass, it is a very important consideration together with the classical potential problem that is studied in the paper. Understanding the relationship between the VEV of the scalar field and the coupling at a fixed vacuum structure is of great importance. This study shows that the Higgs mass is strongly coupled to the Higgs quartic coupling of the model. However, even though the quartic coupling can be measured from the Higgs field, the quartic coupling of the Higgs field can only be measured from the Higgs field. This is because the quartic coupling of the Higgs field is not measured from the Higgs field. Therefore, the Higgs quartic coupling is strongly coupled to the Higgs quartic coupling of the model. However, even though the quartic coupling of the Higgs field is not measured from the Higgs field, this study shows that the Higgs mass is strongly coupled to the Higgs quartic coupling of the model. However, even though the quartic coupling can be measured from the Higgs field, the quartic coupling of the Higgs field can only be measured from the Higgs field. Therefore, the Higgs quartic coupling is strongly coupled to the Higgs quartic coupling of the model. However, even though the quartic coupling of the Higgs field is not measured from the Higgs field, this study shows that the Higgs mass is strongly coupled to the Higgs quartic coupling of the model.
known fact that topological solitons are analyzed semiclassically. In accordance with such approaches the first term in the expansion represents just the solution to a non-linear field equation. Almost from the very beginning of those treatments the one-loop bosonic contribution to the kink energy have been taken into account [3]. As far as we know Campbell and Liao were the first to consider the one-loop fermionic effect although their computations restrict to a special set of coupling constants which in fact renders the model supersymmetric [4]. In addition a new method of computing exactly one-fermion-loop contributions have been recently introduced, regardless the values of coupling constants [5]. For the sake of shortness we simply point out that in this paper the VEV for the scalar field as well as the kink mass are calculated by means of non-perturbative methods on the lattice. The advantage we get in such a framework is twofold: i) we get rid of the embarassing ultraviolet divergences and the subtleties associated with the renormalization procedures; ii) we go far beyond the perturbative studies, classical or one-loop order at best, which can be found in preceding works.

In order to illustrate these ideas in a simple context, we consider in the following the (1+1) dimensional model governed by the lagrangian density

$$L = \frac{1}{2}(\partial_{\mu}\Phi)^2 - \frac{1}{4}(\Phi^2 - v^2)^2 + i \overline{\Psi} \gamma^{\mu} \partial_{\mu} \Psi - m^{2} \overline{\Psi} \Psi,$$

(1)

where \(\Phi\) represents as usual a real scalar field while \(\Psi\) corresponds to a fermionic excitation. By neglecting the fermion source terms, it is the case that the scalar sector of the model has a soliton-like configuration (the kink) which is given by

$$\Phi_{s}(x) = v \tanh \left(\frac{x}{\sqrt{2}}\right),$$

(2)

so that the Dirac equation in the presence of the kink exhibits just a single self-conjugate zero-energy solution [6]. In such a case the occupancy of this state has no effect on the energy of the kink mass but originates a twofold vacuum degeneracy so that the kink itself has a fractional fermionic number \(F\). In absence of nonzero energy excitations \(F\) is either -1/2 or 1/2 depending on the occupancy of the zero-mode.

Being a bidimensional model the \(\Psi\) field, to be more specific its bilinear expressions, can be written in terms of a new scalar field \(\sigma\). By using the well-known transformations [7]

$$\overline{\Psi} \Psi \rightarrow \rho \cos (2\sqrt{\pi} \sigma),$$

$$i \overline{\Psi} \gamma^{\mu} \partial_{\mu} \Psi \rightarrow \frac{1}{2} (\partial_{\mu} \sigma)^{2},$$

$$j^{\nu} = \overline{\Psi} \gamma^{\nu} \Psi \rightarrow \frac{1}{\sqrt{\pi}} \epsilon^{\mu \nu \rho \sigma} \partial_{\rho} \sigma,$$

(3)

where \(\rho\) represents in principle a parameter to fix subsequently, one obtains a pure scalar model depending on the fields \(\Phi\) and \(\sigma\). As the question we wish to address concerns the lowest-energy state with \(F = 1\), it seems interesting to remind that in the spirit of the bosonization procedures the fermion number \(F\) of a configuration is measured by

$$F = \int d\sigma \delta(x) = \frac{1}{\sqrt{\pi}} (\sigma(+\infty) - \sigma(-\infty)),$$

(4)

and results proportional to the change in the \(\sigma\) field when going from \(-\infty\) to \(+\infty\). The resulting system can be analyzed classically in terms of the finite-energy field configurations of the potential \(V(\Phi, \sigma)\) [2]. The different classical solutions which appear have been classified according to the two infinite sets of minima of \(V(\Phi, \sigma)\), namely:

$$\Phi = -\Phi_{0},\ 2\sqrt{\pi} \sigma = 2n\pi$$

$$\Phi = \Phi_{0},\ 2\sqrt{\pi} \sigma = (2n + 1)\pi,$$

(5)

being \(\Phi_{0}\) the positive solution to the equation

$$\Phi(\Phi^2 - v^2) - \rho \Phi = 0.$$

(6)

Thus we can find two kinds of classical solutions: i) when \(\Phi\) has the same sign at \(x = \pm \infty\) only states with \(\Delta \sigma = n \sqrt{\pi}\) (integer fermion number) are feasible; ii) on the other hand, whenever \(\Phi\) changes sign we have \(\Delta \sigma = (n + 1/2) \sqrt{\pi}\) (half-integral fermion number). At least as far as this description is reliable, the question of fermion stability can be analyzed by means of an approach which takes into account numerical solutions as parametrized by the coupling constants of the model. We will return on this later on.

Now we set \(\rho = x g u^2/16 \Phi_{0}\) according to the following consideration: once the model is bosonized, the naive fermion is found solving the lagrangian (1)
where the summation is taken along the boundary.

\[
\int \left( \nabla \cdot \mathbf{v} \right) d\mathbf{x} = (\gamma / \gamma - 1) \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

Next, consider the periodic boundary conditions.

The periodic boundary conditions are equivalent to considering periodic boundary conditions on the lattice span. We apply these boundary conditions to the corresponding boundary conditions in the lattice space. We can then write the condition as:

\[
\mathbf{Z} = \mathbf{Z}(1 / 2, 1 / 2, 1 / 2)
\]

The action and the partition function are given by:

\[
\mathcal{Z} = \frac{1}{(2\pi)^3} \int d\mathbf{x} \mathcal{L} = \frac{1}{(2\pi)^3} \int d\mathbf{x} \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

The partition function is given by:

\[
\mathcal{Z} = \frac{1}{(2\pi)^3} \int d\mathbf{x} \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

where \( \mathcal{L} \) is the lattice action.

We can apply the classical approximation to the partition function in terms of \( \sum_{i} \).

The classical approximation is given by:

\[
\mathcal{Z} \approx \frac{1}{(2\pi)^3} \int d\mathbf{x} \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

The action is:

\[
\mathcal{L} = \int \left( \frac{\partial \mathbf{u}}{\partial x} \right) d\mathbf{x}
\]

The partition function is given by:

\[
\mathcal{Z} = \frac{1}{(2\pi)^3} \int d\mathbf{x} \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

where \( \mathbf{u} \) is the action density.

The partition function is then given by:

\[
\mathcal{Z} = \frac{1}{(2\pi)^3} \int d\mathbf{x} \sum_{i} \int \left( \frac{\partial \mathbf{u}^i}{\partial x_i} \right) d\mathbf{x}
\]

where \( \mathcal{L} \) is the lattice action.

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\]

where \( \mathcal{L} \) is the lattice action.
ii. We can obtain configurations like (9) by placing again the system in the minimum $\Phi = \Phi_0$, $\sigma = -\pi$ with twist conditions for the field $\Phi$ while the periodic conditions for $\sigma$ are broken by decoupling the left and right boundaries. Then we get

$$\exp[-E_0 L] = \langle \exp(-2\beta\Phi_0 \Phi_{\infty} - \frac{1}{4\pi^2} \sum_{n,\omega} \sigma_n \sigma_{n,\omega}) \rangle.$$  

(15)

Expressions (14) and (15) handle strongly non-local operators. In consequence, we may fear finite size effects which can spoil our results. On the other hand, the operator has an exponential form so that the errors may dramatically be amplified. After Groeneveld, Jurkiewicz and Korthals Altes [9] we can find an alternative way of calculating the masses by means of local well-behaved quantities. In fact, from (14) and (15) we write

$$m = -\frac{1}{L} \log \langle \mu(\bar{\pi}, L) \mu(\bar{\pi}, 0) \rangle = -\frac{1}{L} \log \frac{Z(r_0, \lambda_0)}{Z(r_0, \lambda_0)} = \int_{\beta}^{-\beta} \Omega(\beta).$$

(16)

where $\beta$ is the value of $\beta$ from which spontaneous symmetry breaking occurs. As $Z(r_0, \lambda_0)$ becomes different from $Z(r_0, \lambda_0)$ we have

$$\Omega(\beta) = -\frac{1}{L} \frac{\partial}{\partial \beta} \log \frac{Z}{Z} = \frac{1}{L} \langle [S_{\beta}(r) - (S_{\beta})] \rangle.$$ 

(17)

and

$$S_{\beta} = \sum_n \Phi_n^2 - \sum_n \Phi_n \Phi_{\infty} + \frac{1}{4} \sum_n (\Phi_n^2 - 1)^2 + G \sum_n \Phi_n \cos(\sigma_n).$$ 

(18)

The system has been simulated in an especially-designed transputer based parallel machine, RTN (Reconfigurable Transputer Network), with 64 T-805 transputers, distributed in 8 boards with 8 transputers each. Its peak power is about 100 MFlops. We have used and adaptable Monte Carlo (MC) algorithm, keeping the acceptance rate between 45 % and 55 %. We have simulated two lattice sizes ($16^2, 32^2$), with (3000, 5000) iterations of thermalization and take (2000, 4000) measurements in each transputer. Between two consecutive measurements we took (10, 5) decorrelation MC iterations. All the transputers belonging to the same board studied the same point in the parameter space (identical values for $\beta$ and $G$), thus we have a total of (16000, 32000) measurements for every point. The errors have been estimated using the jack-knife method. In addition a big size of the lattice is needed when measuring non-local quantities such as correlations of the fields.

If one analyzes classically the model represented by (7), both noise fermion and kink coexist no matter what the values of coupling constants are. When considering quantum corrections, for small values of $\beta$ and $G$, the fluctuations can make the system jump easily over the potential barriers. Only when $\beta$ reaches a certain critical value $\beta_c$, the SSB phenomenon comes into play and the minima of the model become reliable. Once this happens we compare the masses of both kinds of configurations. Always with $G = 1$ we have the following situations:

a. We start from fields $\Phi = \Phi_0$, $\sigma = -\pi$ with periodic boundary conditions. We study the evolution of $\langle \Phi \rangle$, $\langle \Phi^2 \rangle$, $\langle \sigma \rangle$, $\langle \sigma^2 \rangle$ and measure the quantities $\langle \exp(-\frac{1}{2} \sum_{n,\omega} \sigma_n \sigma_{n,\omega}) \rangle$ and $\langle \exp(-2\beta\Phi_0 \Phi_{\infty} - \frac{1}{4\pi^2} \sum_{n,\omega} \sigma_n \sigma_{n,\omega}) \rangle$, where the summatories are taken along the t-boundary. From the physical point of view we calculate from them the masses of the $F = 1$ and $F = \frac{1}{2}$ configurations using formulae (14) and (15).

b. We start from fields $\Phi = 1$ and $\sigma = -\pi$, where $\sigma$ refers to the value along the left boundary of the lattice. Then we impose the boundary conditions $\Phi_0 = \Phi_{n+L}$, $\sigma_{n+L} = \sigma_n + 2\pi$ and force the appearance of the $F = 1$ configuration.

c. We start from fields $\Phi = 0$ and $\sigma = -\pi$, with boundary conditions like $\Phi_{n+L} = -\Phi_n$, $\sigma_{n+L} = \sigma_n + \pi$. In this way we get nicely the kink configuration.

In the mean-field approximation, we obtain $\langle \Phi \rangle = \Phi_0$, $\langle \Phi^2 \rangle = \Phi_0^2$, $\langle \sigma \rangle = 0$, $\langle \sigma^2 \rangle = \pi/2$ in case b, and $\langle \Phi \rangle = 0$, $\langle \Phi^2 \rangle = \Phi_0^2$, $\langle \sigma \rangle = -\pi/2$, $\langle \sigma^2 \rangle = \pi^2/2$ in case c.

As we have said earlier, these three situations are indistinguishable for small $\beta$ values since the energy barriers are easily jumped over. Now we must find the critical value $\beta_c$ from which the results from a, b and c become different.
Suppose that $A$ is a finite set. We also denote the intersection $\cap$. The intersection of $A$ with itself is $\emptyset$, and $A$ is the union of all its elements.

Let $A$ be a set and $B$ be a subset of $A$. Then $B$ is called a subset of $A$ if every element of $B$ is also an element of $A$. This can be denoted by $B \subseteq A$. Conversely, if $B \subseteq A$, then every element of $B$ is also an element of $A$.

In particular, the empty set $\emptyset$ is a subset of every set $A$. This is because there are no elements in $\emptyset$, so the condition for being a subset is trivially satisfied.

The union of two sets $A$ and $B$ is the set of all elements which are in $A$, in $B$, or in both. This is denoted by $A \cup B$. Moreover, if $A \subseteq B$, then $A \subseteq A \cup B$, and $B \subseteq A \cup B$. This is because the union includes all elements of $A$ and $B$.

If $A \subseteq B$ and $B \subseteq A$, then $A = B$. This is because every element of $A$ is also an element of $B$, and vice versa.

Given two sets $A$ and $B$, the symmetric difference $A \Delta B$ is the set of elements which are in either $A$ or $B$, but not in both. This is denoted by $A \Delta B = (A \cup B) \setminus (A \cap B)$. If $A \subseteq B$, then $A \Delta B = B \setminus A$.
References


Figure captions

Fig. 1 $\langle \Phi \rangle$, $\langle \Phi^2 \rangle$, $\langle \sigma \rangle$ and $\langle \sigma^2 \rangle$ for a 10$^3$ lattice for configurations $F = 1$ (left) and $F = 1/2$ (right) ($G = 1$). We find that, for $\beta > 0.6$, the expected values become different in both cases and tend to the mean-field predictions (continuous line).

Fig. 2 In a 32$^2$ lattice, we draw $\Omega(\beta)$ for $F = 1$ (left) and $F = 1/2$ (right). In the $F = 1$ case $\Omega(\beta)$ is non-zero only in a small range of $\beta$; thus the relative errors become very high and the method using the decay of the correlation function $\langle \mu \mu \rangle$ becomes more efficient. In the top window, we draw $\log(\mu \mu)$ for $\beta = 0.7, 0.8, 0.9$.

Fig. 3 In a 32$^2$ lattice, we compare the masses of the configurations with fermion number $F = 1$ (left) and $F = 1/2$ (right). The agreement is satisfactory, even in the $F = 1$ case, where the relative errors associated to the integration method are considerable; in this case we have omitted the error bars.

Fig. 4 In a 32$^2$ lattice, we draw the mass of the naive fermion and twice the mass of the kink. We draw the fermion mass calculated using both methods and the kink mass calculated from the decay of $\langle \mu \mu \rangle$. 