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"A non-perturbative approach to the instability
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A non-perturbative approach to the instability of the vacuum in the presence of fermions.

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Abstract

We discuss in detail the possibility that states of fermion number one, consisting of the scalar field in its ground state and the occupancy of the zero-momentum plane-wave one for the fermionic quantum, may be unstable to the formation of a kink-antikink pair each carrying a fermion number of one-half. Although such phenomenon can occur in other dimensional contexts we restrict ourselves to a bidimensional model which considers the Φ^4 theory coupled to fermions by means of a Yukawa term. In particular we take advantage of the well-known bosonization procedure so that the scalar-fermion system becomes in the sequel a pure scalar model. Eventually we consider non-perturbative methods on the lattice, thus going beyond the classical treatments used by other authors.

When studying quantum field models with scalar-fermion interaction couplings, it is often the case that the fermions remain massless unless the Higgs field acquires a non-zero expectation value in its ground state (VEV for short). Once the spontaneous symmetry breaking (SSB) occurs, one could just as well devote his effort to answer the following question: What is the lowest-energy state with fermion number one? At first glance it would seem admissible that the naive fermion, i. e., the scalar field in its VEV and the occupancy of the zero-momentum plane-wave fermionic state, provides us with such a stable configuration. It may be interesting at this point to remind that, however, a very different picture emerges when the fractionization of the fermionic charge comes into play. As certain theories exhibit soliton-like configurations with fractional fermionic number, it could happen that whenever the Yukawa coupling is large enough, the energy associated with the fermion in a constant scalar field background exceeds twice the mass of the topological soliton. In other words, even though the conservation of the fermionic charge is not violated at all, fermions may become unstable against the formation of a kink-antikink pair each carrying a fermion number of one-half [1].

By considering models based on a simple one-space, one-time dimensional field theory, the well-known bosonization procedure can be employed so that the initial scalar-fermion system is easily mapped into a pure scalar model. In such a case different configurations like naive fermions, kinks (topological solitons) or bags (non-topological ones) can be analyzed in terms of the finite-action solutions of a classical potential, thus obtaining the phase diagram of the model at tree-order [2]. It is also noteworthy that quantum contributions can dramatically modify the results obtained at classical level. In this spirit, let us explore the problem a little further. A consistent approach should include the three following steps: i) as the fermions can acquire mass through the Yukawa coupling the computation of the VEV for the scalar field provides us with the fermion mass E_f ; ii) in addition we have soliton-like configurations each carrying fermion number one-half and energy E_k ; iii) at this point, the comparison between $2 E_k$ and E_f to find the lowest-energy state with fermion number unity is feasible.

Going to the VEV for the scalar field one could employ perturbative methods so that the quantum corrections up to leading order in the loop expansion together with the classical potential should enable to study the vacuum structure of the model. With regards to the kink mass it is a well-

known fact that topological solitons are analyzed semiclassically. In accordance with such approaches the first term in the expansion represents just the solution to a non-linear field equation. Almost from the very beginning of those treatments the one-loop bosonic contribution to the kink energy have been taken into account [3]. As far as we know Campbell and Liao were the first to consider the one-loop fermionic effect although their computations restrict to a special set of coupling constants which in fact renders the model supersymmetric [4]. In addition a new method of computing exactly one-fermion-loop contributions have been recently introduced, regardless the values of coupling constants [5]. For the sake of shortness we simply point out that in this paper the VEV for the scalar field as well as the kink mass are calculated by means of non-perturbative methods on the lattice. The advantage we get in such a framework is twofold: i) we get rid of the embarrassing ultraviolet divergences and the subtleties associated with the renormalization procedures; ii) we go far beyond the perturbative studies, classical or one-loop order at best, which can be found in preceding works.

In order to illustrate these ideas in a simple context, we consider in the following the (1+1) dimensional model governed by the lagrangian density

$$L = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{4}(\Phi^2 - v)^2 + i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - g\bar{\Psi}\Psi\Phi, \quad (1)$$

where Φ represents as usual a real scalar field while Ψ corresponds to a fermionic excitation. By neglecting the fermion source terms, it is the case that the scalar sector of the model has a soliton-like configuration (the kink) which is given by

$$\Phi_k(x) = v \tanh\left(\frac{x}{\sqrt{2}}\right), \quad (2)$$

so that the Dirac equation in the presence of the kink exhibits just a single self-conjugate zero-energy solution [6]. In such a case the occupancy of this state has no effect on the energy of the kink mass but originates a twofold vacuum degeneracy so that the kink itself has a fractional fermionic number F . (In absence of nonzero energy excitations F is either $-1/2$ or $1/2$ depending on the occupancy of the zero-mode).

Being a bidimensional model the Ψ field, to be more specific its bilinear expressions, can be written in terms of a new scalar field σ . By using the well-known transformations [7]

$$\begin{aligned} \bar{\Psi}\Psi &\rightarrow \rho \cos(2\sqrt{\pi}\sigma) \\ i\bar{\Psi}\gamma^\mu \partial_\mu \Psi &\rightarrow \frac{1}{2}(\partial_\mu \sigma)^2 \\ j^\mu &= \bar{\Psi}\gamma^\mu \Psi \rightarrow \frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu \sigma, \end{aligned} \quad (3)$$

where ρ represents in principle a parameter to fix subsequently, one obtains a pure scalar model depending on the fields Φ and σ . As the question we wish to address concerns the lowest-energy state with $F = 1$, it seems interesting to remind that in the spirit of the bosonization procedures the fermion number F of a configuration is measured by

$$F = \int dx j^0(x) = \frac{1}{\sqrt{\pi}}(\sigma(\infty) - \sigma(-\infty)), \quad (4)$$

and results proportional to the change in the σ field when going from $-\infty$ to $+\infty$. The resulting system can be analyzed classically in terms of the finite-energy field configurations of the potential $V(\Phi, \sigma)$ [2]. The different classical solutions which appear have been classified according to the two infinite sets of minima of $V(\Phi, \sigma)$, namely:

$$\begin{aligned} \Phi &= -\Phi_0, \quad 2\sqrt{\pi}\sigma = 2n\pi \\ \Phi &= \Phi_0, \quad 2\sqrt{\pi}\sigma = (2n+1)\pi, \end{aligned} \quad (5)$$

being Φ_0 the positive solution to the equation

$$\Phi(\Phi^2 - v^2) - g\rho = 0. \quad (6)$$

Thus we can find two kinds of classical solutions: i) when Φ has the same sign at $x = \pm\infty$ only states with $\Delta\sigma = n\sqrt{\pi}$ (integer fermion number) are feasible; ii) on the other hand, whenever Φ changes sign we have $\Delta\sigma = (n + 1/2)\sqrt{\pi}$ (half-integral fermion number). At least as far as this description is reliable, the question of fermion stability can be analyzed by means of an approach which takes into account numerical solutions as parametrized by the coupling constants of the model. We will return on this later on.

Now we set $\rho = \pi g v^2/16\Phi_0$ according to the following consideration: once the model is bosonized, the *naïve* fermion is found solving the lagrangian (1)

for σ in the case where $\Phi = \Phi_0$; as we obtain the sine-Gordon equation, the fermion mass is that of the sine-Gordon soliton. With the previous choice of ρ , we make this mass equal to that of the *naive* fermion before bosonization ($E_f = gv$). We scale the magnitudes we work with as: $\Phi \rightarrow \hat{\Phi} = \Phi/v$, $\sigma \rightarrow \hat{\sigma} = 2\sqrt{\pi}\sigma$, $x \rightarrow \hat{x} = vx$. In the following we omit the carets and use the parameters $G = g\rho/v^3$ and $\beta = v^2$.

Let us now explain in outline the non-perturbative method on the lattice that we have used to study the hypothetical instability of fermions. By taking scalar variables Φ and σ on each site of a bidimensional symmetric lattice, we obtain the discretized euclidean action of the bosonized version of (1) which depends on two independent parameters (β, G) [2], i.e.

$$S_{lat} = \beta \left(d \sum_n \Phi_n^2 - \sum_{n,\mu} \Phi_n \Phi_{n+\mu} + \frac{1}{4} \sum_n (\Phi_n^2 - 1)^2 + G \sum_n \Phi_n \cos(\sigma_n) \right) + \frac{1}{4\pi} \left[d \sum_n \sigma_n^2 - \sum_{n,\mu} \sigma_n \sigma_{n+\mu} \right], \quad (7)$$

where $d = 2$ is the dimension of the lattice, n runs over the lattice sites while μ runs along the space/time directions.

Now we can classify physical configurations according to topological arguments:

i. Configurations describing $F = 1$ states (*naive* fermions), characterized by

$$\sigma_{l+L} - \sigma_l = 2\pi, \quad \Phi_{l+L} = \Phi_l, \quad (8)$$

where σ_l, Φ_l run over the left boundary of the lattice, whose length is L .

ii. Configurations with $F = \frac{1}{2}$ (kinks), characterized by

$$\sigma_{l+L} - \sigma_l = \pi, \quad \Phi_{l+L} = -\Phi_l. \quad (9)$$

In order to calculate the masses of the configurations given by (8) and (9) we define a *disorder operator*, after Kadanoff and Ceva [8]. We place ourselves in the dual lattice and consider the function (μ_{R_1}, μ_{R_2}) , which is the dual transform of the correlation function (Φ_1, Φ_2) . The *disorder operator* is defined in

the following way: Let us consider a general action $S = \sum_{n,\mu} J_{n,\mu} \Phi_n \Phi_{n+\mu} + \sum_n O(\Phi_n)$; we trace a path in the dual space, connect the dual points R_1 and R_2 and change the signs of the coupling constant in every link crossed by such a path. We now define

$$\langle \mu_{R_1}, \mu_{R_2} \rangle = \frac{1}{Z} \sum_{\{\Phi\}} \exp \left\{ - \sum_{n,\mu} J_{n,\mu} \Phi_n \Phi_{n+\mu} + \sum_{n,\mu}' J_{n,\mu} \Phi_n \Phi_{n+\mu} \right\}, \quad (10)$$

where $\sum_{\{\Phi\}}$ runs over all the configurations of the field, $\sum_{n,\mu}'$ takes into account the links with their signs changed and $\sum_{n,\mu}'$ refers to the rest of the links. In other words

$$\langle \mu_{R_1}, \mu_{R_2} \rangle = \frac{1}{Z} \sum_{\{\Phi\}} \exp \left\{ S - 2 \sum_{n,\mu} J_{n,\mu} \Phi_n \Phi_{n+\mu} \right\} = \langle \exp \left\{ -2 \sum_{n,\mu} J_{n,\mu} \Phi_n \Phi_{n+\mu} \right\} \rangle, \quad (11)$$

We can otherwise express the correlation function in terms of a "twisted action" $S_t = S - 2 \sum_{n,\mu} J_{n,\mu} \Phi_n \Phi_{n+\mu}$ and its partition function $Z_t = \sum_{\{\Phi\}} \exp \{-S_t\}$, i. e.

$$\langle \mu_{R_1}, \mu_{R_2} \rangle = \frac{Z_t}{Z}. \quad (12)$$

If we consider a vertical line between the dual points R_1 and R_2 , a topological excitation lasting a time $\tau = R_2 - R_1$ is induced. As (μ_{R_1}, μ_{R_2}) decays exponentially with the distance we obtain masses like

$$m = \log \frac{\langle \mu(\vec{n}, t) \mu(\vec{n}, 0) \rangle}{\langle \mu(\vec{n}, t + a) \mu(\vec{n}, 0) \rangle}, \quad (13)$$

where a is the lattice spacing. Now we realize that configurations (8) and (9) are generated by imposing adequate boundary conditions:

i. Configurations (8) are equivalent to consider periodic boundary conditions for Φ and antiperiodic ones for σ . We can simulate this situation by placing the system at the minimum $\Phi = \Phi_0$, $\sigma = -\pi$ and imposing twist conditions for the field σ along the t-axis. Consequently

$$\exp(-E_f L) = \left\langle \exp \left(-\frac{1}{2\pi} \sum_{n,\mu} \sigma_n \sigma_{n+\mu} \right) \right\rangle, \quad (14)$$

where the summatory is taken along the t-boundary.

ii. We can obtain configurations like (9) by placing again the system in the minimum $\Phi = \Phi_0$, $\sigma = -\pi$ with twist conditions for the field Φ while the periodic conditions for σ are broken by decoupling the left and right boundaries. Then we get

$$\exp[-E_k L] = \left(\exp \left(-2\beta \Phi_n \Phi_{n,\mu} - \frac{1}{4\pi} \sum_{n,\mu} \sigma_n \sigma_{n,\mu} \right) \right). \quad (15)$$

Expressions (14) and (15) handle strongly non-local operators. In consequence, we may fear finite size effects which can spoil our results. On the other hand, the operator has an exponential form so that the errors may dramatically be amplified. After Groeneveld, Jurkiewicz and Korthals-Altes [9] we can find an alternative way of calculating the masses by means of local well-behaved quantities. In fact, from (14) and (15) we write

$$m = -\frac{1}{L} \log(\mu(\vec{n}, L) | \mu(\vec{n}, 0)) = -\frac{1}{L} \log \frac{Z_t(\tau_0, \lambda_0)}{Z(\tau_0, \lambda_0)} = \int_{\beta_0}^{\beta_1} \Omega(\beta_0), \quad (16)$$

where β_c is the value of β from which spontaneous symmetry breaking occurs. As $Z_t(\tau_0, \lambda_0)$ becomes different from $Z(\tau_0, \lambda_0)$ we have

$$\Omega(\beta_0) = -\frac{1}{L} \frac{\partial}{\partial \beta_0} \log \frac{Z_t}{Z} = \frac{1}{L} \{ \langle S_{\beta_0} \rangle_t - \langle S_{\beta_0} \rangle \}. \quad (17)$$

and

$$S_{\beta} = \sum_n \Phi_n^2 - \sum_{n,\mu} \Phi_n \Phi_{n+\mu} + \frac{1}{4} \sum_n (\Phi_n^2 - 1)^2 + G \sum_n \Phi_n \cos(\sigma_n). \quad (18)$$

The system has been simulated in an especially-designed transputer based parallel machine, RTN (Reconfigurable Transputer Network), with 64 T-805 transputers, distributed in 8 boards with 8 transputers each. Its peak power is about 100 Mflops. We have used and adaptive Monte Carlo (MC) algorithm, keeping the acceptance rate between 45 % and 55 %.

We have simulated two lattice sizes ($16^2, 32^2$), with (3000, 5000) iterations of thermalization and take (2000, 4000) measurements in each transputer. Between two consecutive measurements we took (10, 5) decorrelation MC iterations. All the transputers belonging to the same board studied the same

point in the parameter space (identical values for β and G), thus we have a total of (16000, 32000) measurements for every point. The errors have been estimated using the jack-knife method. In addition a big size of the lattice is needed when measuring non-local quantities such as correlations of the fields.

If one analyzes classically the model represented by (7), both naive fermion and kink coexist no matter what the values of coupling constants are. When considering quantum corrections, for small values of β and G , the fluctuations can make the system jump easily over the potential barriers. Only when β reaches a certain critical value β_c the SSB phenomenon comes into play and the minima of the model become reliable. Once this happens we compare the masses of both kinds of configurations. Always with $G = 1$ we have the following situations:

- a. We start from fields $\Phi = \Phi_0$, $\sigma = -\pi$ with periodic boundary conditions. We study the evolution of $\langle \Phi \rangle$, $\langle \Phi^2 \rangle$, $\langle \sigma \rangle$, $\langle \sigma^2 \rangle$ and measure the quantities $\langle \exp[-\frac{1}{2\pi} \sum_{n,\mu} \sigma_n \sigma_{n,\mu}] \rangle$ and $\langle \exp[-2\beta \Phi_n \Phi_{n,\mu} - \frac{1}{4\pi} \sum_{n,\mu} \sigma_n \sigma_{n,\mu}] \rangle$, where the summatories are taken along the t-boundary. From the physical point of view we calculate from them the masses of the $F = 1$ and $F = \frac{1}{2}$ configurations using formulae (14) and (15).
- b. We start from fields $\Phi = 1$ and $\sigma_l = -\pi$, where σ_l refers to the value along the left boundary of the lattice. Then we impose the boundary conditions $\Phi_n = \Phi_{n+L}$, $\sigma_{n+L} = \sigma_n + 2\pi$ and force the appearance of the $F = 1$ configuration.
- c. We start from fields $\Phi = 0$ and $\sigma_l = -\pi$, with boundary conditions like $\Phi_{n+L} = -\Phi_n$, $\sigma_{n+L} = \sigma_n + \pi$. In this way we get nicely the kink configuration.

In the mean-field approximation, we obtain $\langle \Phi \rangle = \Phi_0$, $\langle \Phi^2 \rangle = \Phi_0^2$, $\langle \sigma \rangle = 0$, $\langle \sigma^2 \rangle = \pi^2/2$ in case b, and $\langle \Phi \rangle = 0$, $\langle \Phi^2 \rangle = \Phi_0^2$, $\langle \sigma \rangle = -\pi/2$, $\langle \sigma^2 \rangle = \pi^2/2$ in case c.

As we have said earlier, these three situations are indistinguishable for small β values since the energy barriers are easily jumped over. Now we must find the critical value β_c from which the results from a, b and c become different.

In fig. 1 we can see the evolution of the expected values of the fields as we increase β in cases b and c. We see that for $\beta > 0.6$ the minima seem to stabilize. We also compare the measured values with the mean-field predictions. For large β the agreement is very good as expected.

Going to the masses computation we have followed two methods: the first uses a cumbersome, non-local function while the second one requires the integration of a local quantity S_β . Although in previous works [10] it has been found that this second method is more efficient, the situation in our case is quite different. When we study the mass of the $F = 1$ configuration, this method has a serious problem: in the mean-field approximation $S_\beta \rightarrow 0$ as we increase β . In our simulations, it is different from zero only in a small region around β_c , as can be seen in fig. 2. That means that the relative errors of the integrand in (16) are very high and it affects seriously the precision of the result. In fig. 2 we compare ($F = 1$) the quantity $O(\beta)$ with the decay of (μ_R, μ_{R_2}) . This latest quantity results in fact to be more useful in order to measure the mass.

Now we focus our attention in the region of the parameter space where cases a, b and c become distinguishable. We stress that, if the expected values $\langle \Phi \rangle$, $\langle \Phi^2 \rangle$, $\langle \sigma \rangle$ and $\langle \sigma^2 \rangle$ are null, the values of the masses obtained either by (14) and (15) or by (16) are zero, and only become nonzero when the three cases become distinct. In fig. 3 we compare the values of the masses obtained via both methods and confirm the value $\beta_c = 0.65 \pm 0.05$.

Finally, in fig. 4 we compare twice the kink mass with that of the *naive* fermion. We see that, once the minima of the potential have stabilized ($\beta > \beta_c = 0.65 \pm 0.05$, for $G = 1$) there is a short range of β where the decay of the fermion into a kink-antikink pair is feasible. This situation persists until $\beta = 1.3 \pm 0.1$ since for greater values of β the *naive* fermion is energetically favoured.

To sum up, we have discussed the possibility that fermions could be unstable to the formation of a kink-antikink pair each carrying a fermion number of one-half. Two independent non-perturbative methods on the lattice, using either local or nonlocal quantities, provide us with compatible results. Once the constant G is fixed at value 1, the evolution of β traces the phases of the model. Three different regions nicely appear:

i) A nonzero VEV for Φ field comes into play when we reach the critical point β_c ($\beta_c = 0.65 \pm 0.05$). Below such a value we have no spontaneous symmetry breaking, so that the kinklike configurations are absent.

ii) Whenever $\beta_c < \beta < 1.3 \pm 0.1$, the mass of the fermion E_f exceeds twice the kink mass E_k and decay processes are feasible.

iii) For β values greater than 1.3 ± 0.1 the lowest-energy fermion number one configuration corresponds to the *naive* fermion.

In conclusion, the stable state with fermion number one is very different depending upon the parameters. Our numerical results are in principle consistent with the previous findings of Mackenzie and Palmer [2]. However, two main differences can be remarked:

i) As they employ a classical framework the nonzero VEV for Φ field, SSB phenomenon and kinklike configurations emerge from the very beginning ($\beta = 0$).

ii) We have made no effort to include in our treatment nontopological solitons (bags). Although it is not an easy task to handle such states on the lattice, a careful analysis of them would provide us with a more accurate phase diagram. It is our hope to present new results on this challenging problem in the next future.

** The numerical simulations have been possible thanks to the RTN machine, for which we are grateful to the RTN group. We also acknowledge the Spanish CICYT for financial support.

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Figure captions

Fig. 1 $\langle \Phi \rangle$, $\langle \Phi^2 \rangle$, $\langle \sigma \rangle$ and $\langle \sigma^2 \rangle$ for a 16^2 lattice for configurations $F = 1$ (left) and $F = \frac{1}{2}$ (right) ($G = 1$). We find that, for $\beta > 0.6$, the expected values become different in both cases and tend to the mean-field predictions (continuous line).

Fig. 2 In a 32^2 lattice, we draw $\Omega(\beta)$ for $F = 1$ (left) and $F = \frac{1}{2}$ (right). In the $F = 1$ case $\Omega(\beta)$ is non-zero only in a small range of β ; thus the relative errors become very high and the method using the decay of the correlation function $\langle \mu\mu \rangle$ becomes more efficient. In the top window, we draw $\log \langle \mu\mu \rangle$ for $\beta = 0.7, 0.8, 0.9$.

Fig. 3 In a 32^2 lattice, we compare the masses of the configurations with fermion number $F = 1$ (left) and $F = \frac{1}{2}$ (right). The agreement is satisfactory, even in the $F = 1$ case, where the relative errors associated to the integration method are considerable; in this case we have omitted the error bars.

Fig. 4 In a 32^2 lattice, we draw the mass of the naive fermion and twice the mass of the kink. We draw the fermion mass calculated using both methods and the kink mass calculated from the decay of $\langle \mu\mu \rangle$.

Fig. 2

Fig. 1

Fig. 3

Fig. 4