EE

TRI-PP-94-6
Jan 1994The Electromagnetic Force for "Vortical" Optics and Traps
of ParticlesV. E. Shapiro
TRIUMF, 4004 Westbrook Mall, Vancouver B.C. Canada V6T 2A3

Abstract

Particles in resonance electromagnetic fields experience a sharp increase of the recoil forces. The quasi-elastic and cooling action of the recoil has found application in traps and optics of particles. This paper aims at drawing attention to the vortical recoil action; it can dominate and has peculiar features important for the application. Basic notions that specify the vortical force are addressed.

(submitted to Hyperfine Interactions, in press)

TRI PP 94-6
SU 94-11

The ideas of electromagnetic "vortical" trap and "vortical" optics of particles arise in association with phenomena like vortical jet propulsion in sputs and generation of large scale vortices due to frequent small scale oscillations in turbulent media. The basis for such ideas is that an electromagnetic force similar to the vortical force, responsible for the phenomena mentioned, does exist. Attention to such forces and their general properties was drawn for the first time in [1].

The vortical force differs essentially from the gyro forces, e.g. the Lorentz force, $F_{gyr} = q \mathbf{V} \times \mathbf{B}$, exerted on a particle of charge q and velocity $\mathbf{V}$ in magnetic field $\mathbf{B}$. The gyro forces are always directed perpendicular to $\mathbf{V}$ and change sign as $\mathbf{V}$ changes sign, while the vortical forces do not. Their simplest functional structure is

$$\mathbf{F}_{vor} = K \mathbf{r}, \quad (1)$$

where K is an antisymmetric matrix, with components $K_{\mu\nu} = -K_{\nu\mu}$, characterizing vorticity of the vortical force field. The same form [1] characterizes the forces of quasi-elastic (or gradient) type but with a symmetric matrix K representing coefficients of elasticity. The work done by the action of the vortical force over a closed trajectory of the particle motion $W = \oint \mathbf{F}_{vor} \cdot d\mathbf{r}$ is not equal to zero, and depends on the trajectory and direction of motion along it. The vortical force is an even function of $\mathbf{V}$, and in this respect it is different from the ordinary friction forces and their particular hysteresis type.

Time independent electromagnetic fields cannot produce the vortical force; the energy conservation law forbids that. The origin of the vortical force is in the recoil process and it can be, for example, a part of the force acting on a dipole particle

$$\mathbf{F} = \frac{\partial}{\partial \mathbf{r}} (d(t) \mathbf{E}(\mathbf{r}, t)), \quad \mathbf{E} \sim e^{i\omega t}. \quad (2)$$

The response of d to the field $\mathbf{E}$ is delayed, $d(t) = d(t, |\mathbf{E}(\mathbf{r}(\tau), \tau)|)$, $\tau \leq t$, and this results in the appearance of the mean friction force (positive or negative) and, in rather general conditions, of the mean vortical force. Both forces sharply increase at frequency $\omega \sim \omega_0$, where ω_0 's are resonance frequencies of the response of d to $\mathbf{E}$, but depend quite differently on the resonance detunings. This and other features were elucidated in [1-4].

Figure 1 shows a "motor" exploiting the vortical force effect. Movable circuit III moves along circular trajectories due to resonant excitation of the immovable circuits I and II. The presence of a number of excited modes is important for the vortical forces. The model was described in [1] and then realized at the Kirensky Institute of Physics. The excitation frequency was 5 KHz, while the rotation was at 1 Hz. The threshold of build-up of the vibration was much smaller than with circuit I or II disconnected (then only the negative friction force builds up the vibration).

For a slowly moving particle in the weak resonance light of two standing waves $\mathbf{E}(\mathbf{r}, t) = \sum_{j=1,2} \mathbf{E}_j e^{i(\omega_j \pm \mathbf{k}_j \cdot \mathbf{r})}$, when $\mathbf{k}_j \mathbf{V} \ll |\omega - \omega_j| + \gamma_0$ with γ_0 's the resonance linewidths, the vortical force is (4)

$$\mathbf{F}_{vor} = 4 \operatorname{Re}(A_\omega) \operatorname{Im}(E_1 E_2^*) [\mathbf{k}_+ \sin(\mathbf{k}_- \cdot \mathbf{r}) + \mathbf{k}_- \sin(\mathbf{k}_+ \cdot \mathbf{r})]. \quad (3)$$

Here $\mathbf{k}_+ = \mathbf{k}_1 + \mathbf{k}_2$, $\mathbf{k}_- = \mathbf{k}_1 - \mathbf{k}_2$ and

$$A_\omega = \sum_{\alpha} \frac{f_\alpha e^2 / m}{i(\omega_\alpha^2 - \omega^2) + 2\gamma_\alpha \omega},$$

f_a is the strength of dipole oscillator a , e and m are the charge and mass of the electron, and the sum is over all resonances. Figure 2 shows patterns of field lines of the vortical force(3). The force is essentially connected with interference phenomena: it is not at all a sum of partial pressures of individual plane waves, and in a plane wave $F_{\text{vor}} = 0$.

The gradient force of light for the case

$$\mathbf{F}_{\text{grad}} = 2\text{Im}(A_-) [\text{Re}(\mathbf{E}_1 \mathbf{E}_2^*)(\mathbf{k}_+ \sin(\mathbf{k}_- \mathbf{r}) + \mathbf{k}_- \sin(\mathbf{k}_+ \mathbf{r})) + |\mathbf{E}_1|^2 \mathbf{k}_1 \sin(2\mathbf{k}_1 \mathbf{r}) + |\mathbf{E}_2|^2 \mathbf{k}_2 \sin(2\mathbf{k}_2 \mathbf{r})],$$

is of much smaller magnitude in the vicinity of resonances, provided $\text{Im}(\mathbf{E}_1 \mathbf{E}_2^*) \sim \mathbf{E}_1 \mathbf{E}_2$. The same is true for the friction force of light. At the antinodes, $F_{\text{vor}} = 0$ and the $\mathbf{F}_{\text{grad}}$ forms a potential well if $\omega_a > \omega$. However, the vortical forces make the equilibrium state unstable, unless $\omega_a > \omega + 2\gamma_a$, i.e. in almost all the resonance area. At $\omega_a = \omega$ the increment of instability for the case of a single resonance of unit oscillator strength is

$$\delta = 2[\text{Im}(\mathbf{E}_1 \mathbf{E}_2^*) \omega \gamma_a / M \gamma_a]^{1/2} \sim 10^6 \text{ s}^{-1},$$

where $r_0 = e^2/mc^2$. The numerical estimate corresponds to $M \sim 10^3 \text{ au}$, $2\pi/k = 500 \text{ nm}$, $\gamma_a = 10^{-8} \text{ s}^{-1}$ and $(c/4\pi)\text{Im}(\mathbf{E}_1 \mathbf{E}_2^*) \sim 10^{-3} \text{ W cm}^{-2}$. The estimate shows that the particles can be effectively moved by the vortical force of resonance light.

In addition to having a considerable magnitude, the vortical force is very sensitive and depends quite differently from the gradient and fractional forces on the resonance detunings, polarisation, wavevectors and wave phases. Note also that the "vortical" optics of particles, using the vortical type of forces, is free of the restrictions associated with phase space conservation inherent for electromagnetic lenses of the conventional types.

These features give grounds to conclude that the vortical recoil force of a resonance electromagnetic field represents a new possible tool for traps.

Additional Remarks

1. Recent developments: The material presented above was reported to the Workshop for Antimatter and Radiative Nuclei (TRIUMF, February 1993) and then in print [5]. Later, I came across, thanks to Otto Hauser and Thad Walker, a number of publications [6-10] relevant to the considered topic.

Hemmerich and Hansch [6] and Hemmerich *et al.* [7], based on concepts which are in essence the same as in Ref. [4], performed experiments showing convincing evidence of the vortical effect on atoms of two standing light waves. Walker *et al.* [8] have effectively used the vortices in atom traps by arranging two counter-propagating laser beams of Gaussian profile which are not coaxial, i.e. are offset from each other. The offset makes one direction of the vortical force action preferable over large areas compared with the wavelength. It is worthy to note that not only such a particular geometry, but even the ones depicted in Fig. 2 favour large-scale (over the scales of periodicity) vortical motion.

2. Citations: In [6-9] the work [10] was cited as the first which predicted the resonance radiation force of the vortical type. My work [1-4] was not mentioned in

[6-10]. The concepts claimed in [10] as a discovery, were developed for a two-level atom model, while in [1-3] one finds general concepts. My work addressed specifically to the vortical light forces exerted on atoms was written in 1988, declined in 1989 by Journal de Physique as lacking novelty, and came to light in [4], later than in [10]. By the way, the word "vortical" was in the titles of [1-4], but not of [10], and introduced to classify the radiation processes and forces with respect to symmetry.

3. Terminology: The vortical radiation force has much in common with the "scattering" force, i.e. the part of the atom light force related [1] to the gradient of phase of the laser light. In terms of the complex light field, of components $E_k = I_k^{1/2} e^{i\psi_k}$, $k = x, y, z$, and complex d expressed via complex polarizability tensor $a = \alpha + i\beta$ as $d = a(E)E$ the mean of force(2) takes the form

$$\langle \mathbf{F} \rangle = \alpha_k \nabla I_k + 2\beta_k \nabla \psi_k,$$

provided a is a symmetric tensor and the basis is taken so that a is diagonal, $a_{kk} = \alpha_k + i\beta_k$. The second sum (over $k = x, y, z$) in $\langle \mathbf{F} \rangle$ is the scattering force. When the dependence of α on E is negligible, the vortical force can only be a part of the scattering force. However, beyond these conditions the trend changes. The scattering force also bears the name radiation pressure so the terms "vortical radiation pressure" and "radiation pressure vortices" are in use. Such an association going from [10] may be misleading since the vortical force is not at all a sum of partial pressures of individual plane waves, as discussed in detail in Ref. [4].

References

- [1] V.E. Shapiro, *Zh Teor Eksp Fiz (Sov. Phys. JETP)* **89**, 1957 (1985).
- [2] V.E. Shapiro, *Zh Teor Eksp Fiz (Sov. Phys. JETP)* **91**, 1280 (1986).
- [3] V.E. Shapiro, *Phys Lett A* **129**, 62 (1988).
- [4] V.E. Shapiro, *J Phys B At Mol Opt Phys* **13**, 2445s (1990).
- [5] V.E. Shapiro, *Hyperfine Interactions* **81**, 223 (1993).
- [6] A. Hemmerich and T.W. Hansch, *Phys Rev Lett* **68**, 1492 (1992); **70**, 410 (1993).
- [7] A. Hemmerich, M. Weidemüller, T. Esslinger and T.W. Hansch, *Europhys Lett* **21**, 445 (1993).
- [8] T. Walker, D. Hofman, P. Feng and R.S. Williamson III, *Phys Lett A* **163**, 309 (1992); *Phys Rev Lett* **69**, 2168 (1992).
- [9] V.S. Bagnato, L.G. Marcassa, M. Oria, G.I. Surdutovich and S.C. Zilio, *Laser Physics* **2**, 172 (1992).
- [10] A.P. Kazantsev and L.V. Krasnov, *J Opt Soc Am B* **6**, 2141 (1989).
- [11] J.P. Gordon and A. Ashkin, *Phys Rev A* **21**, 1606 (1980).

Figure Captions

1. "Motor" of vortical recoil action formed by three inductively connected high Q resonance circuits, of close frequencies, excited near the resonances.
2. Patterns of field lines of vortical force, exerted on a particle, produced by joint impact of two, (a) travelling, and (b) standing, plane light waves. The broken arrows indicate the wavevectors of the waves.

Fig. 1

(a)

(b)

Fig. 2