Abstract

The electromagnetic force for a particle is given by

\[ F = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \]

where \( F \) is the force, \( q \) is the charge, \( \mathbf{E} \) is the electric field, and \( \mathbf{v} \) and \( \mathbf{B} \) are the velocity and magnetic field, respectively.

The force is a vector quantity, implying that the direction of the force is perpendicular to both the electric field and the magnetic field. Moreover, the force is always in the direction of acceleration.

This expression is fundamental in electrodynamics and is used extensively in the study of charged particles, such as electrons and protons, in electric and magnetic fields.

The electromagnetic force has a significant impact on the motion of charged particles, allowing for a deeper understanding of the behavior of these particles in various environments.
$f_s$ is the strength of dipole oscillator $a$, $e$ and $m$ are the charge and mass of the electron, and the sum is over all resonances. Figure 2 shows patterns of field lines of the vortical force (3). The force is essentially connected with interference phenomena: it is not at all a sum of partial pressures of individual plane waves, and in a plane wave $F_{\text{vor}} = 0$.

The gradient force of light for the case

$$F_{\text{grad}} = 2\text{Im}(A_e)\left[\text{Re}(E_i E_2^*) (k_z \sin(k_z r) + k_z \sin(k_z r))ight] + |E_i| E_2 |k_z \sin(2k_z r)| + |E_i| k_z |\sin(2k_z r)|$$

is of much smaller magnitude in the vicinity of resonances, provided $\text{Im}(E_i E_2^*) \sim E_i E_2$. The same is true for the friction force of light. At the antinodes, $F_{\text{vor}} > 0$ and the $F_{\text{grad}}$ forms a potential well if $\omega_r > \omega$. However, the vortical forces make the equilibrium state unstable, unless $\omega_r > \omega + 2\tau_a$, i.e. in almost all the resonance area. At $\omega_r = \omega$ the increment of instability for the case of a single resonance of unit oscillator strength is

$$\delta = 2\text{Im}(E_i E_2^*) \omega \tau_a M \gamma_a^{1/2} \sim 10^5 \text{s}^{-1},$$

where $\tau_a = e^2 / m c^2$. The numerical estimate corresponds to $M \sim 10^2$ au, $2\pi / k = 500$ nm, $\gamma_a = 10^{-8}$ s$^{-1}$ and $(e^2 / 4\pi) \text{Im}(E_i E_2^*) \sim 10^{-3}$ W cm$^{-2}$. The estimate shows that the particles can be effectively moved by the vortical force of resonance light.

In addition to having a considerable magnitude, the vortical force is very sensitive and depends quite differently from the gradient and frictional forces on the resonance detunings, polarisation, wavevector and wave phases. Note also that the "vortical" optics of particles, using the vortical type of forces, is free of the restrictions associated with phase space conservation inherent for electromagnetic lenses of the conventional types. These features give grounds to conclude that the vortical recoil force of a resonance electromagnetic field represents a new possible tool for traps.

**Additional Remarks**

1. Recent developments: The material presented above was reported to the Workshop for Antimatter and Radiative Nuclei (TRUMF, February 1993) and then in print [5]. Later, I came across, thanks to Otto Hauser and Thad Walker, a number of publications [6–10] relevant to the considered topic.

Henmerich and Hansch [6] and Hemmerich et al. [7], based on concepts which are in essence the same as in Ref [4], performed experiments showing convincing evidence of the vortical effect on atoms of two standing light waves. Walker et al. [8] have effectively used the vortices in atom traps by arranging two counter-propagating laser beams of Gaussian profile which are not coaxial, i.e., are offset from each other. The offset makes one direction of the vortical force action preferable over large areas compared with the wavelength. It is worthy to note that not only such a particular geometry, but even the ones depicted in Fig. 2 favour large-scale (over the scales of periodicity) vortical motion.

2. Citations: In [6–9] the work [10] was cited as the first which predicted the resonance radiation force of the vortical type. My work [1–4] was not mentioned in [6–10]. The concepts claimed in [10] as a discovery, were developed for a two-level atom model, while in [1–3] one finds general concepts. My work addressed specifically to the vortical light forces exerted on atoms was written in 1988, declined in 1989 by Journal de Physique as lacking novelty, and came to light in [4], later than in [10]. By the way, the word "vortical" was in the titles of [1–4], but not of [10], and introduced to classify the radiation processes and forces with respect to symmetry.

3. Terminology: The vortical radiation force has much in common with the "scattering" force, i.e. the part of the atom light force related to the gradient of phase of the laser light. In terms of the complex light field, of components $E_k = E_k e^{i\varphi_k}$, $k = x, y, z$, and complex $d$ expressed via complex polarizability tensor $a = a + i\beta$ as $d = a E$ the mean of force (2) takes the form

$$\langle F \rangle = a_{kk} \nabla \varphi_k + 2\beta_k \nabla \varphi_k,$$

provided $a$ is a symmetric tensor and the basis is taken so that $a$ is diagonal, $a_{kk} = a_k + i\beta_k$. The second sum (over $k = x, y, z$) in $\langle F \rangle$ is the scattering force. When the dependence of $a$ on $E$ is negligible, the vortical force can only be a part of the scattering force. However, beyond these conditions the trend changes. The scattering force also bears the same radiation pressure so the terms "vortical radiation pressure" and "radiation pressure vortices" are in use. Such an association going from [10] may be misleading since the vortical force is not at all a sum of partial pressures of individual plane waves, as discussed in detail in Ref [4].

**References**

Figure Captions

Figure 1. Patterns of radiated forces, excited near the resonance.

Figure 2. Patterns of radiated forces excited by the mechanical connection.