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The Dynamics of Luminous Galaxies in Isothermal Halos

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ABSTRACT

We examine the dynamics of luminous galaxies in singular isothermal mass distributions by fitting the line-of-sight velocity dispersion profiles of 37 elliptical galaxies assuming that the galaxies are spherical, non-rotating and have a constant velocity anisotropy. We find that the observed central velocity dispersion, σ_c , is a good estimate for the velocity dispersion of the halo σ_{DM} . If the σ_{DM} are fitted with a power-law of the luminosity, $\sigma_{DM} = \sigma_{DM*}(L/L_*)^{1/\gamma}$, we find $\sigma_{DM*} = 223 \pm 10 \text{ km s}^{-1}$ and $1/\gamma = 0.24 \pm 0.05$, compared to $\sigma_{c*} = 228 \pm 14 \text{ km s}^{-1}$ and $1/\gamma = 0.29 \pm 0.07$ for the central velocity dispersions. The scatter about the power laws is correlated with the central surface brightness and the effective radius (the “fundamental plane”) and is uncorrelated with ellipticity and rotation. The existence of a fundamental plane that causes intrinsic scatter about the power law relations significantly modifies the number counts of galaxies as a function of velocity dispersion.

Subject headings: galaxies: kinematics and dynamics – galaxies: structure – galaxies: elliptical – gravitational lensing

1 INTRODUCTION

The dynamics of the central regions of elliptical galaxies provide little evidence for the presence of a dark matter component (see the review by de Zeeuw & Franx 1991). Constant mass to light ratio models (M/L) successfully model the inner regions, and the inferred values for M/L are consistent with the stellar populations. Some ellipticals have dynamical tracers like X-ray halos (see the review by Fabbiano 1989) or gaseous disks and rings (eg Kent 1990, de Zeeuw 1991) outside the inner regions of the galaxies that imply much larger M/L s, but it is not clear these systems are typical ellipticals. While the stellar dynamical data are consistent with constant M/L , it does not rule out the presence of dark matter inside R_e . For example, Saglia, Bertin & Stiavelli (1992) could fit the dynamical data on 10 elliptical galaxies with self-consistent dark matter models.

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Recently two important new pieces of information from gravitational lensing bear on the structure of elliptical galaxies. Maoz & Rix (1993) demonstrated that galaxies with de Vaucouleurs (1948) mass distributions and the best dynamical estimates for their masses and effective radii are inconsistent with the statistics of gravitational lenses, while Kochanek (1993b) demonstrated that galaxies with singular isothermal mass distributions are consistent with the statistics of gravitational lenses. The problem is significant because the dynamically estimated M/L for an L_* galaxy is $(M/L)_B \simeq (10 \pm 2)h$ in the B band (van der Marel 1991, $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$) while $(M/L)_B \simeq (22 \pm 3)h$ is needed to fit the lens surveys. Fits to the radio ring MG1654+134 (Langston et al 1989) by Kochanek (1994) show that a nearly singular isothermal mass profile is consistent with the data and that more centrally concentrated profiles like the de Vaucouleurs model are not. Unlike the X-ray or ring/disk systems, we have no reason to think that the lens galaxy in MG1654+134 is dynamically special.

The statistical studies of lenses do not show, however, that the dark matter models are consistent with both gravitational lenses and elliptical dynamics. Maoz & Rix (1993) added a softened isothermal halo to the de Vaucouleurs models to fit the data, but their halo models are dynamically inconsistent (Breimer 1993). Kochanek (1993b) fit the halo velocity dispersion scale using the lens data, and addressed the issue of dynamical consistency only qualitatively. Thus while lens statistics are clearly inconsistent with constant M/L models, it is not clear if any of the dark matter models are consistent with both parts of the problem.

The dark matter model commonly used in lens statistics is the singular isothermal sphere (SIS), and the more complicated dark matter models of Maoz & Rix (1993) and Breimer (1993) are qualitatively similar. Gott (1977) argued that galaxy luminosity densities are approximately r^{-3} power laws, and for a r^{-3} luminosity density in an r^{-2} SIS mass density profile the velocity dispersion of the dark matter, σ_{DM} , is related to the velocity dispersion of the luminous matter, σ , by $\sigma_{DM}^2 = (3/2)\sigma^2$ independent of radius and orbital isotropy. Turner, Ostriker, & Gott (1984) advocated using this correction for estimating the statistics of gravitational lenses, and most studies have included it (eg Turner 1990, Fukugita & Turner 1991, Krauss & White 1991, Wallington & Naryan 1992, Kassiola & Kovner 1993). Only Kochanek (1991, 1992, 1993b) either did calculations both with and without the correction, or fitted the dark matter dispersion using the lens data. The correction is critical in lens statistics because the expected number of lenses depends on σ_{DM}^4 , and the average separation depends on σ_{DM}^2 , so the $(3/2)^{1/2}$ correction more than doubles the expected number of lenses and increases their average separations by 50%.

The validity of the Gott (1977) argument depends on how well the luminosity profile is approximated by a r^{-3} power law. Kochanek (1993b) showed that the $(3/2)^{1/2}$ correction applies to the average of the line-of-sight velocity dispersion over the entire galaxy for any luminosity profile in an SIS potential, but for more realistic models (such as the Hernquist (1990) or Jaffe (1983) models) the central velocity dispersion is comparable to the dark matter dispersion. The only way to resolve the issue is to study the dynamics of ellipticals in an isothermal halo using the observed photometric and velocity dispersion profiles. We study the sample of 37 elliptical galaxies fitted by van der Marel (1991) with axisymmetric models, using spherical, non-rotating dynamical models. We can control for the effects of ellipticity

and rotation by examining how the results depend on the axis (major, minor, intermediate) used for the luminosity profile, how the results vary if highly elliptical or rapidly rotating galaxies are excluded, and whether the residuals correlate with ellipticity or rotation.

In §2 we outline the model and the statistical procedures used to analyze the data. In §3 we fit a “Faber-Jackson” (1976) power law relation between the galaxy luminosity and the velocity dispersion of the isothermal halo. In §4 we examine the scatter about the “Faber-Jackson” law, fit a fundamental plane to reduce the residuals, and examine how the fundamental plane affects the number counts of galaxies as a function of velocity dispersion. In §5 we consider the implications of these results.

2 DYNAMICAL MODELS

We use the same sample of 37 elliptical galaxies examined by van der Marel (1991). The photometry for the sample is from Djorgovski (1985), Jedrezejewski (1987), Franx, Illingworth, & Heckman (1989a), and Peletier et al (1990), and the kinematic data is from Davies & Birkenshaw (1989), Jedrezejewski & Schechter (1989), Franx, Illingworth, & Heckman (1989b), and Binney, Davies, & Illingworth (1990). The distances to the galaxies are from Faber et al (1987) and the effective radii and magnitudes are from Burstein et al (1987). A table summarizing the galaxies and their properties can be found in van der Marel (1991), and is not reproduced here. All parameters are scaled with a Hubble constant of $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ (van der Marel (1991) used $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$).

We invert the photometric profile $\Sigma(R)$ using a linear regularization technique (Press et al 1992) to determine the luminosity density, $\nu(r)$, where R is the polar radius on the sky, z is the coordinate along the line of sight, and $r = (R^2 + z^2)^{1/2}$. We only use the data outside $4''$ where seeing corrections are minimal. The results do not change significantly if we use $8''$, or if we include seeing corrections for the galaxies with complete photometric profiles. The luminosity density is extrapolated as r^{-4} beyond the edge of the observed profile. Since the kinematic data is usually only available well inside the edge of the photometric profile, the extrapolation has no effect on the dynamics. For lens statistics we are primarily interested in normalizing the mass/dispersion scale near the effective radius R_e , so excluding the core is unimportant. We normalize the scale of all the profiles to be the intermediate axis of the elliptical galaxies.

Next we solve the spherical Jeans equation to determine the radial velocity dispersion of the luminous matter, σ_r , given the mass distribution, M , and the orbital anisotropy of the luminous matter, $\beta = 1 - \sigma_\theta^2/\sigma_r^2$,

$$\frac{d}{dr} (\nu \sigma_r^2) + \frac{2\beta}{r} \nu \sigma_r^2 = -\frac{GM\nu}{r^2}. \quad (1)$$

For a general solution, the anisotropy β is a function of radius with $\beta \leq 1$ (see Binney & Mamon 1982, or Tonry 1983). The dispersion profiles are too noisy to invert accurately, so we are restricted to fitting models with predetermined isotropy functions and for simplicity we used constant anisotropy. For β constant, the Jeans equation (1) is easily inverted to

determine the radial velocity dispersion

$$\sigma_r^2 = r^{-2\beta} \int_r^\infty \frac{GM\nu}{r^2} r^{2\beta} dr \quad (2)$$

and the line-of-sight velocity dispersion

$$\Sigma v_{los}^2 = \int_0^\infty dz \left(1 - \beta \frac{R^2}{r^2} \right) \nu \sigma_r^2. \quad (3)$$

The density profile of the singular isothermal sphere (SIS) is $\rho(r) = \sigma_{DM}^2/2\pi Gr^2$, and the mass distribution is $M(r) = 2\sigma_{DM}^2 r/G$ where σ_{DM}^2 is the velocity dispersion of the dark matter. For an SIS mass distribution, a power law luminosity density, $\nu \propto r^{-n}$, and constant anisotropy, the solution to equation (2) is $\sigma_{DM}^2 = (n - 2\beta)\sigma_r^2$, and the line-of-sight dispersion is

$$v_{los}^2 = 2\sigma_{DM}^2 \frac{n - (n - 1)\beta}{n(n - 2\beta)}. \quad (4)$$

For the special case of $n = 3$, $v_{los}^2 = (2/3)\sigma_{DM}^2$ independent of the value of β (Gott 1977, White 1981). Kochanek (1993) proved the general result that the luminosity weighted average of v_{los}^2 over the entire luminous galaxy always satisfies $\langle v_{los}^2 \rangle = (2/3)\sigma_{DM}^2$ if the background mass distribution is the SIS model and the total luminosity is finite, independent of the anisotropy function. However, the velocity dispersions of the inner regions of realistic galaxies are higher than this average value, and the velocity dispersions of the outer regions are lower. Combining the real luminosity profile and the observational aperture is critical to correctly estimating the dispersion of the dark matter.

We fit the line-of-sight dispersion to the observed profiles using a χ^2 statistic

$$\chi_0^2 = \sum_i \frac{(\sigma_i^2 - v_{los}^2(R_i))^2}{e_i^2} \quad (5)$$

where the σ_i are the measured line of sight dispersions, and the e_i are the estimated errors in the square of the dispersion. For a fixed anisotropy β the only parameter of the model is the dark matter dispersion scale σ_{DM} . After minimizing χ_0^2 with respect to σ_{DM} (for $\beta \simeq 0$) we typically find that half the galaxies are statistically consistent with the model (less than two standard deviations from the expected value of χ_0^2), a quarter are between 2 and 4 standard deviations away, and a quarter have larger errors. The inconsistent values of χ_0^2 are largely caused by underestimating the magnitude of the error e_i that normalizes χ_0^2 . This underestimate is partially a systematic problem from using a spherical model and superimposing major and minor axes dispersion profiles, but there are also many galaxies with outlying data points that are statistically inconsistent with any smoothly varying velocity dispersion profile given their formal errors.

Underestimating the dispersion errors does not affect the parameters of the best fit model, but both the magnitude of the errors in the parameters and the goodness of fit will be underestimated. One plausible way to control these two problems is to increase the estimated errors and define a new χ^2 estimator where the best fit model has a χ^2 equal to the

number of degrees of freedom. If the minimum χ_0^2 for the current galaxy is χ_{0min}^2 , and the number of degrees of freedom is N_{dof} , we define the renormalized statistic $\chi_1^2 \equiv N_{dof} \chi_0^2 / \chi_{0min}^2$ if $\chi_{0min}^2 > N_{dof}$, and $\chi_1^2 = \chi_0^2$ if $\chi_{0min}^2 < N_{dof}$. This increases the errors e_i so that the best fit model is statistically consistent with the data. For most galaxies the renormalization corresponds to increasing the estimated errors in the velocity dispersion measurements by 50%. We use the renormalized χ_1^2 to estimate the 90% confidence intervals on the model parameters.

Given the estimated velocity dispersion σ_{DM} and its error for each galaxy, we next study how the dispersions depend on the galaxy luminosity, L , effective radius, R_e , and central surface brightness, μ_B . The Faber-Jackson (1976) law $L = L_*(\sigma_c/\sigma_{c*})^\gamma$ is a power law relation between the central velocity dispersion σ_c and the luminosity L , and we call the equivalent relation between the dark matter dispersion and the luminosity the σ_{DM} - L relation

$$L = L_* \left(\frac{\sigma_{DM}}{\sigma_{DM*}} \right)^\gamma \quad (6)$$

where L_* corresponds to $B_{T*} = -19.9 + 5 \log h$ mags (Efsthathiou, Ellis, & Peterson 1988) and σ_{DM*} and γ are parameters. We also examine the dark matter versions of the fundamental plane (Djorgovski & Davis 1987, Dressler et al 1987) by including either the central surface brightness μ or the effective radius of the galaxies R_e as a second variable in the model. The σ_{DM} - L - μ relation has the form $\sigma/\sigma_* = (L/L_*)^a (I/I_*)^b$ where $-2.5 \log(I/I_*) = \mu_B - \mu_{B*}$, and the σ_{DM} - L - R_e relation has the form $\sigma/\sigma_* = (L/L_*)^a (R_e/R_{e*})^b$. where μ_* and R_{e*} are the average surface brightness and effective radius of an L_* galaxy.

We can fit the various power law relations to either σ_{DM} or to its logarithm $\log(\sigma_{DM})$, where the two approaches differ in how they weight the errors. In general we will fit the logarithm because this allows us to use linear rather than nonlinear fitting techniques, but the parameters for the two approaches differ by approximately one standard deviation. The value for σ_{DM*} found fitting σ_{DM} is usually 6-7 km s⁻¹ smaller than the value found for fitting $\log(\sigma_{DM})$. To estimate the errors on the parameters of the power law relations, we again scale the χ^2 for the model to equal the number of degrees of freedom, and give the 90% confidence interval assuming Gaussian errors.

3 THE "FABER-JACKSON" LAW FOR ISOTHERMAL HALOS

We fit all 37 galaxies using the SIS mass model and the same constant anisotropy parameter β , computing the best fit dark matter dispersion for each galaxy. Figure 1 shows the variation in the total χ_0^2 for all 37 galaxies as a function of β . The best fitting model has slightly radial orbits ($\beta = 0.13$) and $\chi_0^2/N_{dof} = 3.01$ for $N_{dof} = 864$ degrees of freedom. Statistically this is a terrible model given the number of degrees of freedom because we underestimate the velocity dispersion errors in χ_0^2 . If we renormalize the models so that the best fit model has $\chi_1^2 = N_{dof}$ then the models with $-0.4 \lesssim \beta \lesssim 0.4$ have total likelihoods of fitting the data within two standard deviations of the best model. (This is not a parameter estimate for the anisotropy. If we really want an error estimate on the anisotropy parameter β , the $\Delta\chi^2$ from the best fit model is distributed as a χ^2 distribution with $N_{dof} = 1$ degree of

FIG. 1.—Total χ_0^2 per degree of freedom for the fits to all 37 galaxies using the SIS mass model as a function of the common, fixed anisotropy parameter β . The total number of degrees of freedom is $N_{dof} = 864$. The expected standard deviation of χ_0^2/N_{dof} from unity is $(2/N_{dof})^{1/2} = 0.048$ and the horizontal line schematically shows a two standard deviation increase in χ_1^2/N_{dof} .

freedom, so the two standard deviation limit on the parameter β is formally $\beta = 0.13 \pm 0.05$.) For comparison, if we fit the data with Hernquist (1991) models roughly scaled to match the photometry (Hernquist scale length $a = 0.45R_e$ effective radii) the best model has $\beta \simeq 0.3$ and $\chi_0^2/N_{dof} = 3.6$, and the best model for any scale length that is proportional to the effective radius has $a \simeq R_e$, $\beta \simeq 0.7$, and $\chi_0^2/N_{dof} = 3.3$. The inferred mass-to-light ratios for $a = 0.45R_e$ are close to the van der Marel (1991) values.

Figure 2 shows the best and worst fits to the individual galaxies for an isotropic dispersion tensor ($\beta = 0$). The SIS model tracks the observed profile in both cases. The reason one model has $\chi_0^2/N_{dof} = 0.4$ ($N_{dof} = 23$) and the other has $\chi_0^2/N_{dof} = 8.2$ ($N_{dof} = 59$) is the difference in the intrinsic scatter of the dispersions measured at the same radii from the galaxy but at different position angles. In NGC4486, measurements at the same radius but different position angles differ by more than is consistent with their error bars, while in NGC4494 they do not. The difference between the two cases is not attributable to problems with fitting spherical models to elliptical rotating galaxies: NGC4494 is the more elliptical ($\epsilon = 0.16$ versus $\epsilon = 0.04$) and faster rotating galaxy ($v_{rot}/\sigma \simeq 0.42$ versus $v_{rot}/\sigma = 0.026$).

For each value of the anisotropy, we fit the $\sigma_{DM}-L$ relation to the dispersions. Figure 3 shows the fitted values for σ_{DM*} and γ as a function of the anisotropy β and the data for the isotropic case ($\beta = 0$) are shown in Figure 4. As with the fits to the individual galaxies, the scatter of the points relative to a power-law of the luminosity is much larger than the errors for the individual galaxies. For models, however, we want the range for the parameters

FIG. 2.—The best and worst fit galaxies for an isotropic dispersion tensor ($\beta = 0$). No smoothly varying curve can fit NGC4486 with $\chi_0^2 = N_{dof}$ because of the enormous scatter in the data points compared to the formal errors. The dashed lines show the shifts in the model at the 90% confidence limit variations from the χ_1^2 statistic on the dark matter dispersion.

that is consistent with the data given the assumption of a power-law model, and this means rescaling the χ^2 of the best model at each value of the anisotropy so that $\chi^2/N_{dof} = 1$ and then estimating the 90% confidence interval errors on σ_{DM*} and γ given the rescaled model.

There is no significant difference between the isotropic model and the best fitting (slightly radial) model, so we simply give results for the isotropic model. When we fit the power law to the logarithm of the dispersions we find $\sigma_{DM*} = 223 \pm 10 \text{ km s}^{-1}$ and $\gamma = 4.1 \pm 0.9$ for the isotropic model using our estimated 90% confidence intervals. The rms scatter of the points about this power law is 40 km s^{-1} . Fitting the dispersions directly gives $\sigma_{DM*} = 216 \pm 10 \text{ km s}^{-1}$ and $\gamma = 3.4 \pm 0.7$. Figure 3 shows the difference between the two fitting methods for all values of β . The origin of the differences is the change in the contribution to the χ^2 from outlying points like NGC1379 ($M_B = -19.1 + 5 \log h$, $\sigma_{DM} = 106 \pm 7 \text{ km s}^{-1}$, $\chi_0^2/N_{dof} = 2.3$, $N_{dof} = 14$). Fitting the major axis instead of the intermediate axis photometry or using the data outside $8''$ instead of $4''$ only changes the dispersion scale by about 1 km s^{-1} . Radial models have lower dispersion scales ($\sigma_{DM*} = 216 \pm 11 \text{ km s}^{-1}$, $\gamma = 4.0 \pm 1.0$ for $\beta = 0.4$), and tangential models have higher dispersion scales ($\sigma_{DM*} = 229 \pm 9 \text{ km s}^{-1}$, $\gamma = 4.1 \pm 0.8$ for $\beta = -0.4$). If we allow each galaxy to have a different, constant anisotropy, we find $\sigma_{DM*} = 219 \pm 10 \text{ km s}^{-1}$ and $\gamma = 4.1 \pm 0.9$. This model is almost identical to the mildly isotropic model, although the total $\chi_0^2/N_{dof} = 2.8$ ($N_{dof} = 828$) for fitting the dispersion profiles is significantly better.

For comparison, the Faber-Jackson (1976) relation for the central velocity dispersions σ_c (from Whitmore, McElroy, & Tonry 1985, or the average dispersion inside $3''$) of this sample has $\sigma_{c*} = 228 \pm 14 \text{ km s}^{-1}$ and $\gamma = 3.4 \pm 0.8$ with a rms scatter about the power law of 40 km s^{-1} . This is consistent with the estimates by de Vaucouleurs & Olson (1982) of

FIG. 3.—The variations in the best fit dispersion scale and exponent for a power law model of the variation between luminosity and dark matter velocity dispersion, $L/L_* = (\sigma_{DM}/\sigma_{DM*})^\gamma$. The solid lines with squares and error bars show the result of fitting $\log \sigma_{DM}$, and the dashed lines with triangles show the result of fitting σ_{DM} . The results are different because of the change in how the errors are weighted. The error bars and the outer dashed lines show the 90% confidence limits on the parameters after renormalizing the χ^2 so that the best fit model for each β has a $\chi^2/N_{dof} = 1$.

$\sigma_{c*} = 227 \pm 10 \text{ km s}^{-1}$ and $\gamma = 3.6 \pm 0.5$ from a larger number of ellipticals. This confirms the suggestion of Kochanek (1993) that the central velocity dispersion is usually a good estimate of the dark matter dispersion. If the Gott (1977) model for a galaxy in an isothermal halo were correct, we would expect to find that $\sigma_{DM*} = (3/2)^{1/2} \sigma_{c*} = 278 \text{ km s}^{-1}$, but this is more than 3 standard deviations from the best fitting model. Figure 4 shows the best fit isotropic σ_{DM} - L model, the best fit Faber-Jackson (1976) law for the central dispersions, and the Gott (1977) model for the dark matter dispersions.

4 THE FUNDAMENTAL PLANE

Djorgovski & Davis (1987) and Dressler et al (1987) pointed out that the residuals from the Faber-Jackson (1976) law are correlated with the central surface brightness and the effective radius. Figure 5 shows the residuals after fitting the σ_{DM} - L model as a function of the galaxy ellipticity, rotation measure, central surface brightness, and effective radius. While we see no dependence on either the ellipticity or the rotation measure, there is a clear dependence on the central surface brightness and the effective radius.

The effective radii of the van der Marel (1991) sample scale with luminosity as $R_e/R_{e*} = (L/L_*)^c$ with $c = 0.6 \pm 0.1$ and $R_{e*} = (3.5 \pm 0.3)h^{-1} \text{ kpc}$ compared to an exponent of $c = 1.2 \pm 0.3$ and $R_{e*} = (4 \pm 1)h^{-1} \text{ kpc}$ found in larger samples (eg Kormendy & Djorgovski (1989), Rix (1991)). These exponents are not consistent with each other, and the problem may be differences in the values of R_e van der Marel (1991) assigned to the galaxies from

FIG. 4.—The best fit power law relationship for $\beta = 0$ is superimposed on the individual estimates of σ_{DM*} for the galaxies. The solid lines show the best fit σ_{DM-L} model and the models with σ_{DM*} increased or decreased to its 90% confidence limits. The lower dashed line shows the best fit Faber-Jackson (1976) law for the central velocity dispersions, and the upper dashed line shows the Gott (1977) model for the dark matter dispersions.

Burstein et al (1987) and those assigned by Djorgovski & Davis (1987). If we fit the dark matter dispersions by the σ_{DM-L-R_e} model with $\sigma_{DM}/\sigma_{DM*} = (L/L_*)^a (R_e/R_{e*})^b$, we find that $\sigma_{DM*} = 220 \pm 8 \text{ km s}^{-1}$, $a = 0.42 \pm 0.10$, and $b = -0.29 \pm 0.14$. The central surface brightnesses of the sample scale with luminosity as $\mu_B = \mu_{B*} - c(M_B - M_{B*})$ we find that $\mu_{B*} = 21.25 \pm 0.18 + 5 \log h$ mag per square arcsecond, and $c = 0.27 \pm 0.21$. If we fit the dispersions by the $\sigma_{DM-L-\mu}$ model with $\sigma_{DM}/\sigma_{DM*} = (L/L_*)^a (I/I_*)^b$ we find that $\sigma_{DM*} = 219 \pm 6 \text{ km s}^{-1}$, $a = 0.33 \pm 0.04$, and $b = 0.24 \pm 0.05$. These exponents are consistent with Djorgovski & Davis (1987) who found $a \simeq 0.29$ and $b \simeq 0.25$. In both ways of expressing the fundamental plane, the rms residuals are approximately 25 km s^{-1} , compared to 40 km s^{-1} for the simpler σ_{DM-L} model. The change in the residuals implies that the intrinsic scatter about the σ_{DM-L} model from projecting the fundamental plane is 30 km s^{-1} .

Lens statistics depend on the number distribution of galaxies as a function of dynamical variables like σ_{DM*} or mass, rather than the number distribution in luminosity calculated by surveys such as Efstathiou, Ellis, & Peterson (1988). Usually the conversion is treated as a change of variables using the Faber-Jackson (1976) relation or the equivalent relation for the variation of M/L with luminosity. The scatter about the Faber-Jackson (1976) relation produced by the fundamental plane biases the conversion of number counts in luminosity into number counts in velocity dispersion or mass.

FIG. 5.—Residuals from the power law fit for $\beta = 0$ as a function of galaxy ellipticity, rotation measure v_{rot}/σ , surface brightness, and effective radius.

Suppose we have a fundamental plane $\sigma_{DM} = \sigma_{DM*}(L/L_*)^a(f_2/f_{2*})^b$ defined by the luminosity and a second variable f_2 (either R_e or μ), that the second variable has a probability distribution for its value $P(f_2|L)$ for a given luminosity, and that any residual scatter about the fundamental plane is from measurement errors. Let the average value of the second variable vary with luminosity as $\langle f_2 \rangle = f_{2*}(L/L_*)^c$ where the $\langle f_2 \rangle$ is the average of f_2 over the probability distribution at fixed luminosity. The second variable also has a dispersion in its value at fixed luminosity of $\Delta_{rms}\langle f_2 \rangle$ where Δ_{rms} is dimensionless. The σ_{DM} - L (or Faber-Jackson) relation corresponds to averaging the fundamental plane over the second variable to find that

$$\sigma_{DM} = \sigma_{DM*} \left(\frac{L}{L_*} \right)^{a+bc} \left[1 + \frac{b(b-1)}{2} \Delta_{rms}^2 \right]. \quad (7)$$

Thus there is a consistency requirement between the fundamental plane and the σ_{DM} - L relations that $1/\gamma = a + bc$. Substituting the exponents for the σ_{DM} - L - R_e and the σ_{DM} - L - μ models fitted above gives results consistent with the σ_{DM} - L model. There is a small systematic shift in the σ_{DM*} parameter from the width of the distribution of the second variable, but it is a weak effect. The change in the velocity dispersion scale σ_{DM*} between the fundamental plane models and the σ_{DM} - L model sets a limit on the width of the distribution of the second variable, $\Delta_{rms} \lesssim 0.6$. The rms scatter of the dispersions about the σ_{DM} - L model at fixed luminosity is approximately

$$\Delta\sigma_{DM} \equiv \xi_{rms}\sigma_{DM} \simeq |b|\Delta_{rms}\sigma_{DM}. \quad (8)$$

There is little data on the probability distributions for either R_e or μ at fixed luminosity. Sodr & Lahav (1993) modeled the bivariate radius-luminosity function for isophotal diameters and found that $\Delta_{rms} \simeq 0.3$. This gives a predicted width for the σ_{DM} - L relation of $\xi_{rms} \simeq 0.1$, which is roughly consistent with our observed width of $\xi_{rms} \simeq 0.15$.

We know that the galaxy number counts in luminosity are reasonably represented by a Schechter (1976) function, $dn/dL = (n_*/L_*)(L/L_*)^\alpha \exp(-L/L_*)$ with $\alpha \simeq -1.1 \pm 0.1$ (Efsthathiou, Ellis, & Peterson 1988). If $P(\sigma_{DM}|L)$ is the probability distribution for the velocity dispersion at fixed luminosity then the mean dispersion at fixed luminosity defines the σ_{DM} - L relation $\sigma_{DM}(L) = \sigma_{DM*}(L/L_*)^{1/\gamma}$ and the width of the distribution is characterized by ξ_{rms} . The number distribution of galaxies in velocity dispersion is

$$\frac{dn}{d\sigma_{DM}} = \int_0^\infty dL \frac{dn}{dL} P(\sigma_{DM}|L) \quad (9)$$

and if there is no spread in the distribution of dispersions ($\xi_{rms} = 0$) the number distribution is

$$\left(\frac{dn}{d\sigma_{DM}} \right)_0 = \frac{\gamma n_*}{\sigma_{DM*}} \left(\frac{\sigma_{DM}}{\sigma_{DM*}} \right)^{-1+(1+\alpha)\gamma} \exp(-(\sigma_{DM}/\sigma_{DM*})^\gamma). \quad (10)$$

If there is a spread in the dispersions, however, using $(dn/d\sigma_{DM})_0$ underestimates the number of high velocity dispersion galaxies. The integral (9) cannot be solved analytically for a Gaussian probability distribution, but if we assume that the velocity dispersions at fixed luminosity are distributed uniformly from $(1 - \sqrt{3}\xi_{rms})\sigma_{DM} < \sigma < (1 + \sqrt{3}\xi_{rms})\sigma_{DM}$, we

FIG. 6.—The number counts of galaxies as a function of velocity dispersion. The solid lines show the distribution for the best fit power law, isotropic dark matter model ($\sigma_* = 223 \text{ km s}^{-1}$, $\gamma = 4.0$, $\alpha = -1$, equation (10)) and the effect of changing the dispersion scale by $\pm 10 \text{ km s}^{-1}$. The dashed lines show the effect of adding a fractional rms scatter of the dispersions ξ_{rms} to the best fit model (equation (11)).

find that

$$\frac{dn}{d\sigma_{DM}} = n_* \frac{\Gamma[1 + \alpha - \gamma^{-1}, s_+] - \Gamma[1 + \alpha - \gamma^{-1}, s_-]}{2\sqrt{3}\xi_{rms}\sigma_{DM*}} \quad (11)$$

where $s_{\pm} = \sigma_{DM}/(1 \pm \sqrt{3}\xi_{rms})\sigma_{DM*}$ and $\Gamma[a, b] = \int_b^{\infty} u^{a-1} \exp[-u] du$ is an incomplete gamma function. The distribution $dn/d\sigma_{DM}$ has no power expansion for small values of ξ_{rms} if $\sigma_{DM} \gtrsim \sigma_{DM*}\xi_{rms}^{-2/\gamma}$, because ξ_{rms} controls the number of high velocity dispersion galaxies even if it is small. Figure 6 shows both $(dn/d\sigma)_0$ and $dn/d\sigma$ for various values of ξ_{rms} .

The differences between $(dn/d\sigma_{DM})_0$ and $dn/d\sigma_{DM}$ are significant in lens statistics. For an isothermal lens, the optical depth or the expected number of lenses depends on the average value of $\langle \sigma_{DM}^4 \rangle$, and the expected image separation depends on $\langle \sigma_{DM}^6 \rangle / \langle \sigma_{DM}^4 \rangle$ (Turner, Ostriker, & Gott 1984, see Kochanek (1992a) for the analytic expressions in general cosmologies). If the optical depth, average separation, and rms dispersion in the separations derived using $(dn/d\sigma)_0$ are τ_0 , $\langle \theta \rangle_0$, and $\sigma_{\theta 0}$, then the averages corrected to include the effects of an rms fractional scatter ξ_{rms} are $\tau = \tau_0(1 + 6\xi_{rms}^2)$, $\langle \theta \rangle = \langle \theta \rangle_0(1 + 9\xi_{rms}^2)$, and $\sigma_{\theta} = \sigma_{\theta 0}(1 + 25\xi_{rms}^2)$. The formal uncertainty in the parameters of the σ_{DM} - L relation are 5% (10%) at fixed (varying) anisotropy, leading to uncertainties in the optical depth of 20% (40%), and in the mean or dispersion of the image separations of 10% (20%). If the scatter around the σ_{DM} - L relation is $\xi_{rms} = 0.1$ (0.15), then the corrections increase the optical depth by 6% (11%), the mean separation by 9% (20%), and the dispersion of the separations by 25% (56%). Thus the scatter about the σ_{DM} - L relation introduced by the fundamental

plane produces a systematic bias in lens statistics comparable to the uncertainties from the parameters of the σ_{DM} - L relation. The bias is more important for the distribution of image separations than for the optical depth or expected number of lenses. This systematic bias also explains the increase in the average image separations for the bivariate radius-luminosity models examined by Maoz & Rix (1993).

5 CONCLUSIONS

In this paper we examined the dynamics of luminous galaxies in isothermal mass distributions ($\rho = \sigma_{DM}^2/2\pi Gr^2$) to see how the velocity dispersion of the luminous galaxy is related to the velocity dispersion of the dark matter. Gott (1977) predicted that $\sigma_{DM}^2 = (3/2)\sigma_c^2$, where σ_c is the observed central velocity dispersion of the galaxy if the galaxy luminosity density can be approximated by a power law proportional to r^{-3} . More realistic theoretical profiles like the Hernquist (1991) or Jaffe (1983) models have central dispersions comparable to the dark matter dispersion, although the $(3/2)^{1/2}$ correction does appear in the limit of a global average over the galaxy (Kochanek 1993b).

In this study we fit the photometric and velocity dispersion profiles of the 37 elliptical galaxies studied by van der Marel (1991), and we find that if the true mass distribution in the galaxies is that of the singular isothermal sphere, then the best fit power-law relationship between galaxy luminosity and the dispersion of the isothermal sphere is $\sigma_{DM*} = 223 \pm 10 \text{ km s}^{-1}$ and $\gamma = 4.1 \pm 0.9$ at 90% confidence, where $L/L_* = (\sigma_{DM}/\sigma_{DM*})^\gamma$. The dispersion scale can be raised 6 km s^{-1} if the average orbital anisotropy is tangential, $\beta \simeq -0.4$, and it can be lowered by 7 km s^{-1} if the orbital anisotropy is radial, $\beta \simeq 0.4$. Models that are more radial or tangential than $\beta = \pm 0.4$ are inconsistent with the observed velocity dispersion profiles, and the tangential models may also be ruled out because of theoretical prejudices. If we allow each galaxy its own best fit value for the anisotropy, we find that $\sigma_{DM*} = 219 \pm 10 \text{ km s}^{-1}$ and $\gamma = 4.1 \pm 0.9$. The Faber-Jackson (1976) law for the central velocity dispersions of the sample has $\sigma_{c*} = 228 \pm 14 \text{ km s}^{-1}$ and $\gamma = 3.4 \pm 0.8$. The Gott (1977) model for luminous galaxies in an isothermal halo is incompatible with the data, and the central dispersion is a good estimator of the dark matter dispersion.

The residuals from the σ_{DM} - L relation are correlated with the effective radius and central surface brightness of the galaxy, and they are not correlated with the ellipticity or rotation velocity of the galaxy. The fundamental plane of luminosity, velocity dispersion, and central surface brightness fit to the sample agrees with other, larger, samples, but the average scaling of the effective radius with luminosity does not. The rms residual about the σ_{DM} - L power law is 40 km s^{-1} , while the rms residual about the fundamental plane (σ_{DM} - L - R_e or σ_{DM} - L - μ) is only 25 km s^{-1} , so there is a real intrinsic scatter in σ_{DM} at fixed luminosity of 30 km s^{-1} . This fractional scatter of $\xi_{rms} \simeq 0.15$ is comparable to the scatter predicted from the distributions of galaxy diameters (Sodré & Lahav 1993) and the fundamental plane. The intrinsic scatter about the Faber-Jackson model (or the σ_{DM} - L relation) makes us undercount the number of high velocity dispersion galaxies if we use the Faber-Jackson model to convert from number counts of galaxies as a function of luminosity into number counts of galaxies as a function of velocity dispersion.

Both the estimate of the dark matter dispersion and the scatter introduced by the fundamental plane lead to important corrections in statistical studies of gravitational lenses. Almost all theoretical studies have included the Gott (1977) $(3/2)^{1/2}$ correction factor (eg Turner, Ostriker, & Gott (1984), Turner 1990, Fukugita & Turner (1991), Krauss & White (1992), Wallington & Narayan (1993), Kassiola & Kovner (1993)), so they have overestimated the expected number of lenses by a factor of 2.25, and overestimated the average separations by a factor of 1.5. Maoz & Rix (1993) make a comparable normalization error in their models of galaxies with dark matter halos (Breimer 1993). Thus the studies including the Gott (1977) correction that derived a limit on the cosmological constant greatly overestimated the limits. Kochanek (1991, 1992, 1993b) avoided this problem either by using models both with and without the Gott (1977) correction or by directly fitting the velocity dispersion to the lens data. Nonetheless, the best fit dispersion of $\sigma_{DM*} = 245 \pm 20 \text{ km s}^{-1}$ from the gravitational lens data (Kochanek 1993b) is only marginally consistent with the dynamical estimate of $\sigma_{DM*} = 223 \pm 10 \text{ km s}^{-1}$.

All the theoretical studies of lensing used a power law $L \propto \sigma_{DM}^7$ (or $M \propto L^\beta$ for Maoz & Rix (1993)) to compute the number of galaxies per unit dynamical variable from the galaxy luminosity function. If there is intrinsic scatter around the power law because of the projection of the fundamental plane, this method intrinsically underestimates the number of high velocity dispersion (or high mass) galaxies leading to a systematic error in the lens statistics. If the fractional scatter about the σ_{DM} - L relation is $\xi_{rms} \simeq 0.15$ then the normal method of computing the distribution of galaxies in velocity dispersion underestimates the number of lenses by $1 + 6\xi_{rms}^2 \simeq 1.14$, underestimates their average separation by $1 + 9\xi_{rms}^2 \simeq 1.20$, and underestimates the dispersion in the separations by $1 + 25\xi_{rms}^2 \simeq 1.56$. The systematic correction to the number of lenses is slightly smaller than the uncertainties from the statistical errors in the parameters of the σ_{DM} - L relation, and the corrections to the average and the dispersion of the separation distribution are larger than the contributions from the statistical errors. The systematic corrections to the distributions are the right magnitude to explain the differences between the Kochanek (1993) estimate for σ_{DM*} from the lenses, and the estimate for σ_{DM*} from the dynamics of elliptical galaxies.

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