Relative Size of SU(3) Symmetry-Breaking Effects and Penguin Diagrams

B. F. L. Ward

Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996-1200,
Stanford Linear Accelerator Center, P.O. Box 4349, Stanford, California 94309
and TH Division, CERN, Geneva 23, Switzerland

(Received 11 June 1992)

We use the data on the decays $D \rightarrow \pi\pi$ and $D \rightarrow KK$ as a test vehicle for evaluating the effects of penguin diagrams in exclusive heavy meson decays. Evaluating $\Gamma(D \rightarrow \pi^+\pi^-)$ and $\Gamma(D \rightarrow K^+K^-)$, we find good agreement between our theoretical $SU(2L) \times U(1) \times SU(3)^c$ -based predictions and observation. Contributions from penguin diagrams are small and SU(3) flavor breaking effects account for the large value of $\Gamma(D \rightarrow K^+K^-)/\Gamma(D \rightarrow \pi^+\pi^-)$. Implications for B decays are discussed briefly.

PACS numbers: 13.25.+m, 12.38.Bx

An outstanding issue [1] in the theory of exclusive non-leptonic two-body weak decays of heavy mesons is the ratio $R_{+-} = \Gamma(D \rightarrow K^+K^-)/\Gamma(D \rightarrow \pi^+\pi^-)$: Naive tree diagram considerations and phase space arguments would give

$$R_{+-} = (f_K^2/f_\pi^2)(m_D^2/4 - m_K^2)^{1/2}/(m_D^2/4 - m_\pi^2)^{1/2} \cong 1.24,$$

whereas the average of the recent available data [2] is $R_{+-} = 2.21 \pm 0.41$. [The 1992 Particle Data Group (PDG) [2] value for R_{+-} is 2.51 ± 0.38 .] The original quark-diagram-based analysis of Chau and Cheng [3] gives $R_{+-} = 1.29$ unless the familiar penguin diagram contributions play a significant role. In this Letter, we analyze the decays $D \rightarrow \pi\pi$, $D \rightarrow KK$ in the context of the standard $SU(2L) \times U(1) \times SU(3)^c$ model with an eye toward isolating calculational techniques which can reliably describe the effects of penguin transitions in such decays. We find that penguin diagram contributions are small and that SU(3) breaking effects are the dominant source of the large value of R_{+-} . In this way, we resolve this problem with the size of R_{+-} in the theory of exclusive, hadronic heavy meson weak decays and arrive at a quantitative view of such two-body heavy meson weak decays. Possible implications for B decays are noted.

Specifically, the quark-level transitions of interest to us here are shown in Fig. 1, where (a) is the usual tree-level transition and (b) is the penguin transition. To assess the effects of (b) in $D \rightarrow K^+K^-$, we shall use the fact that QCD is perturbative at scales $\sim m_D$ to follow the methods of Lepage and Brodsky [4] to realize these quark-level transitions in Fig. 1 as the hadron transitions

FIG. 1. Quark-level standard model transitions: (a) usual charge current reaction; (b) penguin transition.

in Fig. 2. In this approach, each process in Fig. 2 corresponds to a convolution of the respective meson distribution amplitudes $\phi_M(x_1, x_2)$ with a corresponding QCD hard interaction amplitude [4]. In our case, the relevant distribution amplitudes [4] are $\phi_D(x_1, x_2) = a_D \delta(x_1 - x_2)$, $\phi_M(y_1, y_2) = \sqrt{N_c} f_M y_1(1 - y_2)$, $M = \pi, K$, where $a_D = f_D/\sqrt{12}$ and $x_2 = \bar{m}_u/m_D$ with $\bar{m}_u = \hat{m}_u - \hat{m}_c - \hat{m}_c(\hat{m}_c + \hat{m}_u - m_D)/(\hat{m}_c + \hat{m}_u) \cong 0.234$ GeV for $\hat{m}_u = 0.33$ GeV and $\hat{m}_c = 1.65$ GeV, standard constituent quark values. Thus, $\bar{m}_u$ reflects the effects of binding on $\hat{m}_u$ in the D meson. Since $\hat{m}_c \gg \hat{m}_u$, most of the four-momentum of the D resides on the c quark so that our form for ϕ_D is a good approximation. Here, we work with the leading terms in the Lepage-Brodsky expansion so that certain transverse momentum effects are thereby neglected; as explained in Ref. [4], these neglected transverse momentum effects should be small so long as the scale of the hard interac-

FIG. 2. Hadron-level standard model weak transitions: (a) charged current reaction; (b) penguin transition. Here, $M_{u\bar{q}}$ is, for example, the meson composed of u and $\bar{q}$, where $u = u' = u$ and $q = u, d, s$; G is a QCD gluon. ($D, D' = d, s$.) In the text, E_A is P_A^0 , where P_A is the four-momentum of A .

tion amplitude is large compared to Λ_{QCD} . Our hard interaction scale, m_D , satisfies this requirement. Accordingly, using the standard manipulations as formulated in Ref. [4] and the penguin diagram expression from Ref. [5] we get the following representation of the amplitudes for $D \rightarrow \pi^+ \pi^-, K^+ K^-$:

$$\begin{aligned} \mathcal{M}(D \rightarrow K^+ K^-) = & \left[i(G_F/\sqrt{2})V_{cs}^*V_{us}a_1(-g_s^2C_F)\sqrt{2}f_K(\sqrt{3}f_K/2)a_DP_{K^+a} \frac{\int_0^1 dy_1 y_1}{[2E_K+2E_K-2m_D(2\pi)^9]^{1/2}} \right. \\ & \times \text{tr} \left\{ \frac{1}{(4x_2^2 E_K^2 - m_D^2)} [\gamma_\eta \gamma_5 (\not{P}_K - + m_K) \gamma^\eta (\not{P}_K - x_2 \not{P}_D + m_s) \gamma^a (1 - \gamma_5) \gamma_5 (\not{P}_D - m_D)] \right. \\ & + \frac{\gamma_\eta \gamma_5 (\not{P}_K - + m_K) \gamma^a (1 - \gamma_5) (\not{P}_D - y_2 \not{P}_K - + m_c) \gamma^\eta \gamma_5 (\not{P}_D - m_D)}{(-2x_2 E_K - m_D)(y_1 m_D^2 - m_c^2 + i\epsilon)} \left. \right\} \\ & - i \frac{G_F}{\sqrt{2}} \alpha_p f_K \frac{\sqrt{3}f_K}{(\sqrt{2})^3} C_F^2 g_s^2 a_D \frac{\int_0^1 dy_1 y_1}{[2E_K - 2E_K + 2m_D(2\pi)^9]^{1/2}} \\ & \times \text{tr} \left\{ \frac{1}{(4x_2^2 E_K^2 - m_D^2)} \gamma_\rho \gamma_5 (\not{P}_K - + m_K) \gamma^\rho (\not{P}_K - x_2 \not{P}_D + m_s) \right. \\ & \times \gamma_\eta \gamma_5 (\not{P}_K + + m_K) \gamma^\eta (1 - \gamma_5) \gamma_5 (\not{P}_D - m_D) \\ & + \frac{\gamma_5 (\not{P}_K - + m_K) \gamma_\eta \gamma_5 (\not{P}_K + + m_K) \gamma^\eta (1 - \gamma_5) (\not{P}_D - y_2 \not{P}_K - + m_c) \gamma^\rho \gamma_5 (\not{P}_D - m_D) \gamma_\rho}{(-2x_2 E_K - m_D)(y_1 m_D^2 - m_c^2 + i\epsilon)} \left. \right\} \\ & \times (2\pi)^4 \delta^4(P_D - P_{K^+} - P_{K^-}), \end{aligned} \quad (1)$$

$$\mathcal{M}(D \rightarrow \pi^+ \pi^-) = \mathcal{M}(D \rightarrow K^+ K^-)|_{f_K \rightarrow f_\pi, m_K \rightarrow m_\pi, E_K \rightarrow E_\pi, s \rightarrow d, P_K \rightarrow P_\pi}, \quad (2)$$

where the penguin coefficient α_p is well known from Ref. [5], for example, and is given by

$$\alpha_p = \frac{\alpha_s(q^2)}{2\pi} \sum_D V_{cd}^* V_{ud} \left[\frac{2}{3} \ln(M_W^2/m_D^2) \right]. \quad (3)$$

Here, we use the short-distance quark masses $m_u = 5.1$ MeV, $m_c = 1.3$ GeV, $m_d = 8.9$ MeV, and $m_s = 0.175$ GeV. On substituting the results of Gilman and Kleinknecht [2] for the central values of the Cabibbo-Kobayashi-Maskawa matrix elements V_{ud} (the QCD improvement coefficient a_1 is set [6] at the typical value 1.3), we get the results

$$\begin{aligned} \mathcal{M}(D \rightarrow K^+ K^-) &= \frac{4iG_F}{\sqrt{2}} \frac{f_K^2 \sqrt{3} C_F^2 g_s^2 a_D (2\pi)^4 \delta^4(P_D - P_{K^+} - P_{K^-})}{2\sqrt{2}[2E_K - 2E_K + 2m_D(2\pi)^9]^{1/2}} [-10.2 + 8.05i + \{-1.08 + 0.41i\}], \\ \mathcal{M}(D \rightarrow \pi^+ \pi^-) &= \frac{4iG_F}{\sqrt{2}} \frac{f_\pi^2 \sqrt{3} C_F^2 g_s^2 a_D (2\pi)^4 \delta^4(P_D - P_{\pi^+} - P_{\pi^-})}{2\sqrt{2}[2E_\pi + 2E_\pi - 2m_D(2\pi)^9]^{1/2}} [9.98 - 9.83i + \{-0.48 + 0.43i\}], \end{aligned} \quad (4)$$

where the terms in curly brackets are the respective penguin diagram contributions [1], g_s is the QCD coupling constant with $\Lambda_{\text{QCD}}^{(5)} \approx 100$ MeV, and G_F is Fermi's constant. We see that penguin contributions are small in (4). The details of the manipulations used to arrive at (4) from Fig. 1 are thus entirely analogous to those used in Ref. [7] in our $B \rightarrow \Psi/JK^*$ analysis and more examples of their use will appear elsewhere [8]. Here, $C_F = \frac{4}{3}$. Further, we use the values $f_\pi = 0.093$ GeV, $f_K = 0.112$ GeV, and $a_D = 0.056$ GeV [the value of a_D drops out of our analysis of R_{+-} , for a_D cancels in the ratio of $\Gamma(D \rightarrow K^+ K^-)$ to $\Gamma(D \rightarrow \pi^+ \pi^-)$]. The coupling constant $g_s(Q)$ also cancels out of R_{+-} . It is evaluated at $Q^2 = m_D^2$ for calculation of the amplitudes in (1); this implies a threshold adjustment to $g_s(Q)$ as Q passes below the scale $\mu = m_b = 4.25$ GeV if one starts with a formula

for $g_s(Q)$ which is valid for $Q \geq m_b$ based on $\Lambda_{\text{QCD}}^{(5)}$, for example. We use $\alpha_s(m_D) = g_s^2(m_D)/4\pi = 0.23$. In our graphs in Figs. 1 and 2, the short-distance quark masses are as given above; these masses are taken from Gaisser and Leutwyler in Ref. [9]. We note also that the main dependence of our amplitudes on x_2 will drop out of R_{+-} ; thus, our results for R_{+-} will be relatively insensitive to our model and QCD parameters over a reasonably wide range of respective variation. This gives us added confidence in our findings.

It is straightforward to compute the rates associated with (4). We get

$$\begin{aligned} \Gamma(D \rightarrow K^+ K^-) &\cong 9.51 \times 10^{-15} \text{ GeV}, \\ \Gamma(D \rightarrow \pi^+ \pi^-) &\cong \Gamma(D \rightarrow K^+ K^-)/2.00, \end{aligned} \quad (5)$$

so that

$$R_{+-} \cong 2.00. \quad (6)$$

These results should be compared to the recent measurements [2] (as we noted, the earlier measurements [10] gave a somewhat larger value of R_{+-}):

$$\Gamma(D \rightarrow K^+ K^-) = (7.85 \pm 0.92) \times 10^{-15} \text{ GeV}, \quad (7)$$

and

$$\Gamma(D \rightarrow \pi^+ \pi^-) = (3.56 \pm 0.52) \times 10^{-15} \text{ GeV}. \quad (8)$$

The agreement between the theory and the experiments is quite reasonable, in view of the 25% errors on the theoretical methods which we have used [7]. Thus, we see from (5) and (6) that penguin contributions are indeed small in $D \rightarrow \pi\pi$ and $D \rightarrow KK$, but that their $\sim 10\%$ effects, taken together with the SU(3) flavor breakings associated with f_K/f_π and m_K/m_π [in (4)], account for the deviation of R_{+-} from the naive expectations.

One expects our methods to apply to $D \rightarrow \pi^0 \pi^0$, since its amplitude follows from that in (2) by straightforward substitutions and group theoretical interchanges. This is not the case for $D \rightarrow K^0 \bar{K}^0$, where the gluon analogous to that in Fig. 2 is not necessarily hard enough to justify the use of the methods of Ref. [4] so that, as is shown in Refs. [9,10], nonperturbative, final state interactions are important. We get the results

$$\Gamma(D \rightarrow \pi^0 \pi^0) \cong 0.173 \Gamma(D \rightarrow \pi^+ \pi^-), \quad (9)$$

$$\Gamma(D \rightarrow \pi^0 \pi^0) \cong 0.823 \times 10^{-15} \text{ GeV},$$

to be compared with the experimental result

$$\Gamma(D \rightarrow \pi^0 \pi^0) \cong (1.53 \pm 0.52) \times 10^{-15} \text{ GeV}. \quad (10)$$

We see that there is again reasonable agreement between the theory and the experiment (when the 25% accuracy of our methods is taken into account, the theory is within 2σ of the data).

Our resolution of the R_{+-} paradox should be compared with those in Refs. [11,12]. Chau and Cheng also find that substantial SU(3) breaking effects are required to explain R_{+-} using an entirely different approach—their quark diagrammatic approach. An equivalent conclusion was reached by Czarnecki, Kamal, and Xu [12] using the BSW model with final state interactions as well as an inelastic coupled channel model. What we have done in this Letter is to show how the results in Refs. [11,12] for R_{+-} can be obtained in the context of perturbative QCD corrections to the spectator and W -exchange graphs in D decay, including the effects of penguin diagrams. Like Chau and Cheng, and Czarnecki, Kamal, and Xu, we also find that penguin contributions are small and that SU(3) breaking and strong interactions are primarily responsible for the deviation of R_{+-} from the naive expectations (our analysis of $D \rightarrow \pi^0 \pi^0$ supports this finding [8]). We may therefore

now turn to the application of our methods to $B \rightarrow \pi\pi$ transitions, for example, with the expectation that we can obtain a $\sim 25\%$ accurate prediction of the expected branching ratio; this will allow a quantitative assessment of the usability of certain isospin considerations [13] in removing would-be penguin contamination from these transitions in their exploitation for CP -violation studies for the SLAC-LBL-LNL asymmetric B -factory project. Such modes provide the avenue to the CP -violating angle α in the famous unitarity triangle—their importance cannot be overemphasized and we shall report our results for them elsewhere [14].

Notes added.—(1) H. Simma and D. Wyler (Ref. [15]) have analyzed *charged* B decays from the standpoint of penguin effects using the methods of Ref. [4] as realized by A. Szczepaniak, E. M. Henley, and S. J. Brodsky (Ref. [16]). The latter authors analyzed neutral B decays without considering penguin effects. Thus, what we will present in Ref. [14] will be the quantitative assessment of penguin effects in neutral B decays. (2) One may question why our result for R_{+-} and the conclusion that penguin contributions are small in $D \rightarrow K^+ K^-$, $\pi^+ \pi^-$ can be trusted, since our main Glashow-Iliopoulos-Maiani (GIM) cancellation involves d and s quarks in the W loop in Fig. 1(b) and the masses of these quarks are small compared with m_c . We emphasize that what matters is the scale of the typical four-momentum in the W loop. If this scale is large compared to Λ_{QCD} , then QCD will be perturbative during the respective GIM cancellation and our results for the penguin are reliable. Here, we note that from gauge invariance it follows that our effective penguin diagram G^a - u - c vertex operator has the form $-ig_s \{F_1(q^2)(q^2 \gamma_\mu - \not{q} q_\mu) + i\sigma_{\mu\nu} q^\nu [(1 + \gamma_5)m_c + (1 - \gamma_5)m_u]F_2(q^2)\} \lambda^a$, where $q^2 = (P_u - P_c)^2$ and where the form factors $F_{1,2}$ are given in Ref. [5] and are entirely finite even without GIM cancellation. Hence, both are dominated by virtual W four-momenta $\sim M_W$ and so are their GIM cancellations. This shows that QCD is indeed perturbative in the regime of W -loop four-momenta which determine our penguin amplitude in (1).

The author is grateful to Professor J. Dorfan of SLAC and Professor D. Hitlin of Caltech for giving him the opportunity to participate in the SLAC B -factory physics and simulation working group, wherein the material in the manuscript was conceived and implemented. The author benefited from discussion with Dr. V. Luth, Dr. H. Quinn, Dr. A. Snyder, and Dr. S. Wagner.

This work was supported in part by DOE Contracts No. DE-FG05-91ER40627 and No. DE-AS03-76SF-00515 and by TNRLC Grant No. RCFY9101.

[1] We thank Professor D. Hitlin for calling this problem to our attention.

[2] J. Alexander *et al.*, Phys. Rev. Lett. **65**, 1184 (1990); M.

- Danilov, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, 1991*, edited by S. Hegarty *et al.* (World Scientific, Singapore, 1992), Vol. 2, p. 331; H. Albrecht *et al.*, *Z. Phys. C* **46**, 9 (1990); Wei-Ming Yao, in *Proceedings of the 1991 Annual Meeting of the Division of Particles and Fields of the American Physical Society, Vancouver, 1991*, edited by D. Axen *et al.* (World Scientific, River Edge, NJ, 1992); B. Gittelman, *ibid.*; J. C. Anjos *et al.*, Report No. FERMI-LAB-Pub-91/147, 1991 (to be published). For a summary of earlier data, see the Particle Data Group, M. Aguilar-Benitez *et al.*, *Phys. Rev. D* **45**, Pt. II (1992).
- [3] Ling-Lie Chau and Hai-Yang Cheng, *Phys. Rev. D* **36**, 137 (1987); *Phys. Rev. Lett.* **56**, 1655 (1986); *Phys. Rev. D* **39**, 2788 (1989).
- [4] G. P. Lepage and S. J. Brodsky, *Phys. Rev. D* **22**, 2157 (1980).
- [5] T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981).
- [6] M. Bauer, B. Stech, and M. Wirbel, *Z. Phys. C* **34**, 103 (1987).
- [7] B. F. L. Ward, *Phys. Rev. D* **45**, 1544 (1992), and references therein.
- [8] B. F. L. Ward (to be published).
- [9] J. Gaiser and H. Leutwyler, *Phys. Rep.* **87**, 77 (1982).
- [10] See the PDG in Ref. [2], for example.
- [11] Ling-Li Chau and Hai-Yang Cheng, Report No. UCD-PHYS-PUB-91-26, 1991 (to be published).
- [12] A. Czarnecki, A. N. Kamal, and Q. Xu, Alberta Report No. Thy-42-91, 1991 (to be published).
- [13] See, for example, M. Gronau and D. London, *Phys. Rev. Lett.* **65**, 3381 (1990); Y. Nir and H. Quinn, *Phys. Rev. Lett.* **67**, 541 (1991).
- [14] B. F. L. Ward (to be published).
- [15] H. Simma and D. Wyler, *Phys. Lett. B* **272**, 395 (1991).
- [16] A. Szczepaniak, E. M. Henley, and S. J. Brodsky, *Phys. Lett. B* **243**, 287 (1990).