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POSSIBLE DYNAMICAL ORIGINS OF INTERMITTENCY IN HIGH-ENERGY COLLISIONS

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Abstract

This is a brief review of the dynamical origins of intermittency. For elementary processes involving leptons and hadrons, we discuss toy models, perturbative QCD, Monte Carlo and other models. For complex processes involving nuclear collisions we review theoretical ideas on hadronic fluctuations as consequences of phase transition.

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1. Introduction

After nearly thirty years of study the subject of multiparticle production at high energy is still not solved and the problems continue to be interesting. There has been spectacular progress in understanding hard processes, for which perturbative QCD is applicable. In soft processes, the advances made have been less dramatic, but not less persistent in the gradual clarification of the underlying dynamics of a very complex many-body strong-interaction problem. For hard processes the problem is basically deductive, since there is a reliable theoretical framework for detailed calculations. For soft processes it is essentially inductive with a great deal of emphasis on phenomenology. Many models have been constructed to account for the data. Thus clever ideas in the analysis of data hold the key to progress. Intermittency is one such idea that has in recent years advanced the subject to a new frontier.

The conventional observables in multiparticle production are such quantities as $\langle n \rangle$, dn/dy , d^2n/dy_1dy_2 , P_n , $d\sigma/dp_T^2$, etc. They are global averages: global, in the sense of all phase space, and average, in the usual sense of averaging over all events. The new frontier is on observables that quantify local fluctuations: local, in the sense of restricted regions of phase space, and fluctuations, representing deviations from the average. There is nothing interesting, if the fluctuations are statistical. It is the nonstatistical fluctuation of dynamical origin that is important to extract; furthermore, it is the universal features of such fluctuations that are phenomenologically manageable and theoretically appealing. Intermittency refers to the self-similar behavior of the fluctuation that is independent of the scale of the bin sizes in which the local fluctuations are measured [1].

Before intermittency analysis became the focal point of the experimental effort in multiparticle production, there were many phenomenological models that all claim success in being able to reproduce the data. One need only examine the review volume on hadronic multiparticle production [2] to recall the following models: Lund model, Dual Parton Model, Quantum Statistical Model, Geometrical Model, Branching Model, Supercluster Model, Fireball Model and Statistical Field Theoretical Model. Only a few of them are still being discussed. Many of the models lack Monte Carlo codes to generate the fluctuations that can be compared to experiment. Some have no specific dynamical origin for fluctuations, so no event generator will be developed. Those models that do are now motivated to develop MC codes in order to check intermittency. Not many can meet the test, as we shall discuss in the following. Thus we see here the strong role that intermittency has played in recent development of the subject of particle production.

There are several reviews of intermittency already in the literature [3]-[6] and two more will soon appear in print [7, 8]. Most reviews place heavy emphases on the experimental data.

I shall therefore take the complementary approach here and concentrate on the theoretical issues, especially on dynamics. That means that I shall have to leave out many topics, including the newly proposed measures of intermittency and correlations, since they can meaningfully be discussed only in the context of real data, not in abstraction. Instead of phenomenology my focus will be on the dynamical origins of intermittency.

2. An Overview

It is useful to give a bird's eye view of how the various topics in the subject are related to one another. First of all, intermittency refers to the power-law behavior [1]

$$F_q \propto \delta^{-\varphi_q} \quad (1)$$

of the normalized factorial moments as a function of the bin size δ , as δ becomes small. The justification and verification of such a behavior have become the main concern of many who have attempted to understand multiparticle production in recent years. The effort should be recognized as a confrontation between theory and experiment in an arena which can broadly be labeled as intermittency, but more specifically includes various ramifications that go beyond the particular behavior in (1) of the moments F_q .

In that arena many topics can be mentioned as efforts either to improve the sensitivity of the dynamical effect or to broaden the scope of the scaling behavior. The primary example of the former is the work of the Arizona group in their advocacy of the correlation integrals [9], which have been shown to improve dramatically the statistics of the power-law behavior. In the latter effort attempts have been made to explore more fully the multifractal structure of the multiplicity distribution [10, 11, 12].

Space does not permit even giving the definitions of the notions mentioned above. It suffices to remark here that besides correlation integrals and multifractal analysis there are numerous other proposals to investigate the resolution-dependent properties of the multiplicity fluctuations. They include factorial correlators [1], higher-dimensional analyses [13]-[15], normalized factorial cumulants [16], double trace moments [17], Bose-Einstein correlation [18]-[20], star integrals [21], wavelets [22], etc. All of them are different methods to analyze the data, and are therefore topics that belong to the arena lying between theory and experiment. They facilitate the confrontation.

What theoretical predictions are there that can be put to test in the face of all those sophisticated measures of fluctuations and correlations mentioned above? Woefully few. In this field experiments are ahead of theory, which is a very challenging situation for theorists, although it seems that there is never enough data to give us clear guidance to the heart of the problem of soft interaction. To be discussed in the following sections is a selection of

theoretical topics representing work that have attempted to find the dynamical origin of the type of fluctuations observed. They are summarized first in the remainder of this overview.

Before so doing it should be remarked that recent data suggest the dominance of Bose-Einstein correlations between like-sign charged hadrons in the intermittency phenomena [23, 24]. It has led some people to regard the BE correlation as the origin of intermittency. That is hardly a satisfactory view, since it leaves open the question of what gives rise to the specific (power-law) behavior observed in the correlation function. Besides, the self-similar behavior in unlike-charge moments remains to be explained. In the following we shall leave out the topic of Bose-Einstein correlations, since the origin of their power-law dependences on Q^2 is not understood well enough for a review.

One may characterize the present approaches to the question of dynamical origin as having three levels. At the top includes toy models that are naive and simple, but may contain the essence or what is needed in a realistic model to generate intermittency. In the middle is the approach using Monte Carlo simulations based on some phenomenological models, where quantitative comparison with data can be made. At the most fundamental level is the work based on perturbative QCD, which can make precise predictions, albeit only for hard processes at very high energy.

The above three approaches are for elementary processes involving leptons and hadrons, either hard or soft. There are also nuclear collisions being studied, for which the emphasis is on the possibility of using intermittency as a means of detecting collective behavior. In that realm there are also toy models, like the Ising models, and phenomenological models, like the Ginzburg-Landau model. Computer simulations have also been tried. But there are no first-principle calculations of the very difficult problem of quark-hadron phase transition, as far as fluctuations are concerned. Despite the complications there are recently some interesting results that are worthy of mention.

3. Elementary Processes

3.1. Toy Models

In the same paper where intermittency was first suggested [1], the random cascade model was shown to give rise to the power-law behavior of (1). The simplest example of such models is the α -model, in which particles are successively partitioned into left and right subbins, where at each branching the particle density in the original bin is modified by a random factor W_- for those particle partitioned into the left half, and the density of particles on the right is modified by a factor W_+ , with a probability distribution $r(W)$ such that $\langle W \rangle = 1$.

It can be shown that the intermittency index φ_q , defined in (1), is then

$$\varphi_q = \log \langle W^q \rangle / \log 2 \quad . \quad (2)$$

This is a simple model with a simple result, but it contains the essence of a mechanism for intermittency. Branching models to be discussed below all have the same basic ingredients: successive random subprocesses.

At a level closer to reality but still in the category of toy models are some branching models based on field theory: ϕ^3 theory in 6 dimensions and pure gluon model, both of which are asymptotically free. The splitting functions at each branching vertex are known. Evolution of the branching processes through the degradation of virtualities has been carried out numerically [25]. It is found that the F_q moments possess power-law behaviors as in (1) for both models, and that the intermittency exponent for ϕ^3 theory is much stronger than that for pure gluons, by more than an order of magnitude [25].

3.2. Hard Processes

To be realistic enough so that model predictions can be compared with data, it is necessary to use Monte Carlo simulations with full treatment of the hadronization process. The two most commonly considered codes are JETSET [26] and HERWIG [27], which represent two ways of implementing QCD branching. They have been used to calculate the F_q moments for e^+e^- annihilation. Data from LEP experiments have all been analyzed for intermittency studies, and the results are on the whole in good agreement with the simulations [5]. What small discrepancies that exist are not serious enough to cast any doubt on the conclusion that the essential elements of the dynamics of hadron production in e^+e^- annihilation are contained in those MC codes that are QCD inspired. One may further state that the origin of intermittency is rooted in the branching dynamics; it is the degree of intermittency that is tested by those specific codes. Although one may adopt the view that not much about e^+e^- annihilation has been learned from intermittency studies beyond what is contained in those MC codes, not all predictions from those codes have so far been checked [28]. Further experimental studies on the subject are still needed.

It would be unsatisfactory if intermittency can only be found in MC simulations without theoretical proof for its existence based on fundamental theory. In recent years a number of groups have derived analytical results in perturbative QCD regarding intermittency [29]-[32]. Despite some differences in details, they all agree in the basic conclusion. First of all, the intermittency pattern should be measured in accordance to solid angles defined relative to the jet axis. If θ is the half angle of a cone positioned at θ_0 from the jet axis, then angular

intermittency is the power-law behavior

$$F_q \propto (\theta_0/\theta)^{\varphi_q} \quad . \quad (3)$$

The intermittency indices φ_q can be related in general to the fractal dimension D_q by

$$\varphi_q = (q-1)(d-D_q) \quad , \quad (4)$$

where d is the usual (topological) dimension. In pQCD if the coupling constant is held fixed, then

$$D_q = \frac{q+1}{q}\gamma_0, \quad \gamma_0 = \sqrt{\frac{6\alpha_S}{\pi}} \quad , \quad (5)$$

where γ_0 is the QCD anomalous dimension. If α_S runs, then γ_0 in (5) is replaced by $\bar{\gamma}$ which, for fixed momentum k , is an average of $\gamma_0(\alpha_S(\theta))$ over $d\theta/\theta$ from θ to θ_0 . In so doing F_q loses the strict power-law behavior, since $\bar{\gamma}$ depends on θ and θ_0 separately. The behavior becomes even more complicated when the energy dependence is taken into account. The proper variable to study is $x = (\ln\theta_0/\theta) / (\ln E\theta_0/\Lambda)$. At small x , the constant α_S approximation is roughly valid and (3)-(5) are recovered. At larger x (i.e. small θ), α_S gets larger and $\log F_q$ starts to saturate as x increases until it bends over, at which point α_S is too large to justify pQCD.

Experimentally, neither (5) nor the running-coupling version of the predictions of pQCD have been verified. Even for fixed total rapidity interval, data [33] are only just beginning to show the oscillating nature of the cumulant moments K_q , as q increases, a property that has been predicted in pQCD [34, 35]. It can be shown that K_q is far more sensitive to dynamical details than F_q . For example, the negative binomial distribution (NBD) has been known to be an appropriate description in the double-log approximation; specifically, $k=5$ for the NBD parameter gives an adequate approximation of F_q for the pQCD result. But K_q does not oscillate for NBD, while the exact solution of the QCD equations in the fixed-coupling case yields oscillating K_q [35]. Thus it will be of interest to investigate how intermittency behavior exhibits itself in terms of the K_q moments.

3.3. Soft Processes

While multiparticle production in hard processes is now reasonably well understood, just the opposite is the case for soft interaction. Perturbative QCD is not applicable, and nonperturbative QCD is not feasible. Many models considered in the '80s are not even brought up to confront the intermittency data. The dual parton model (DPM) has been shown to disagree badly with the data on intermittency indices [36]. The recent review article

on DPM [37] makes no effort to address that issue. The Fritiof model [38] does better, but still misses the NA22 data [36]. The deficiency is apparently being amended by the insertion of extra sources of pion production [39]. The role of intermittency in the screening and improvement of models is evidently important.

Although intermittency started out as a characterization of the dependence of F_q on rapidity bin size, generalization to higher dimensional analysis reveal stronger dependence on the cell size [13, 36]. Deviation from simple power-law behavior motivates the search for other scaling forms. It has been found both in models and in data that $\log F_q$ exhibits linear dependence on $\log F_2$ [13, 15], whether or not F_q itself behaves as (1). Thus there is a different scaling law

$$F_q \propto F_2^{\beta_q} \quad (6)$$

that seems to be more prevalent than (1). However, the dynamical origin of that behavior is not known.

Theoretically, there is another possible scaling law based on linked pair approximation (LPA) [40]. In that approximation the (unintegrated) cumulant $k_q(1, 2, \dots, q)$, which is the normalized q -body connected correlation function, can be expressed in terms of the products of two-body cumulants; for example, in the case of k_3 , LPA implies

$$k_3(1, 2, 3) = \frac{A_3}{3} \sum_{\text{perm}} k_2(ij)k_2(jk) \quad (7)$$

where the sum is over all permutations of $\{i, j, k\}$ over $\{1, 2, 3\}$, and A_3 is some constant. To facilitate comparison with data, one would consider the integrated cumulant moments K_q , which is the integral of $k_3(y_1, y_2, y_3)$ over the rapidity interval δy of all three variables. This integration can be made much simpler if one assumes translational invariance in y , whereupon one obtains the trivial scaling behavior

$$K_q \approx A_q K_2^{q-1} \quad , \quad (8)$$

where A_q is unknown, but independent of δy . This behavior is not well supported by data at lower energy (NA22) where translational invariance is not justified; at higher energies the agreement with data is better, but the presence of minijets casts doubt on the relevance of LPA. Since LPA is presumably an approximation on the nature of soft interaction, the question remains, even after (8) is found to be good, on what one would have learned about the soft interaction dynamics. For example, how does one write a Monte Carlo code based on LPA to simulate particle production?

For soft production the geometrical branching model (GBM) [41] has some success in reproducing both the global average observables mentioned in Sec. 1 and the local fluctuations

from averages. Since it has branching, GBM contains the germ for intermittency, and has been demonstrated by its MC code, called ECCO [42], to reproduce the intermittency data in 1, 2, and 3 dimensions [24]. The quantity that evolves is not virtuality, since the model is for soft interaction, but the mass of a cluster of partons. The basic idea is that, as the quark and gluon system expands, an initially dense cluster of partons undergoes successive fission processes into color-singlet subclusters in a scale invariant way. The complex branching tree allows crisscrossing so that a left cluster can give rise to a daughter cluster far on the right. In the mass distribution of branching

$$D(m, m_1, m_2) = \left(\frac{m_1 + m_2}{m} \right)^\gamma, \quad (9)$$

where m is the mass of the mother cluster and m_1 and m_2 are those of the daughters, there is a parameter γ , which is the only parameter in ECCO that plays a dynamical role. Since it controls the length of evolution chain, it determines the energy dependence of all observables, starting with $\langle n \rangle$, and ending with intermittency. It turns out that essentially all data on soft production can be fitted by the choice $\gamma = 1.0$ [42]. Although there is no understanding of the origin of (9), there is now at least a MC code that works well for soft processes.

4. Complex Processes

4.1. Nuclear Collisions

Having been effective in deepening our understanding about particle production in elementary processes, hard or soft, intermittency can also be used as a tool to help us learn more about complex processes in dense quark-gluon systems. Such systems are created in high-energy nuclear collisions. Instead of asking the phenomenological question “Is there intermittency in nuclear collisions?”, one can ask the more probing question “Can intermittency help to determine when collective behavior becomes important or when phase transition occurs?”

Assuming that one has a reliable model for pp collisions, one can first consider the possibility that nuclear collision involves nothing more than the superposition of many independent pp collisions, i.e., no collective flow, with the proviso that care must be exercised to account for the change of the nucleon states for subsequent collisions downstream after suffering a collision upstream. The observables to be studied should start with average quantities such as $\langle n \rangle$, which can pin down properly the nuclear physics part of the problem, but should move on to fluctuations such as intermittency, which are far more sensitive indicators of the dynamics. If the predictions on the properties of F_q differ significantly from the data, then one may reasonably regard the discrepancy as being due to new physics not taken into con-

sideration, notably collective behavior and possibly (but not necessarily) quark-hadron phase transition. Undoubtedly, new physics in complex systems have new behaviors of fluctuations.

Assuming no new physics, ECCO has been used to calculate F_q in nuclear collisions and it is found that intermittency, if it exists at all, is very weak for $q \leq 6$ [43] in qualitative agreement with experimental data that have serious statistics problem [44]. However, what has not been tested is the prediction that at high q ($q > 8$ for S-Au) the intermittency index φ_q will become as large as for pp collisions at lower q . This is understandable because F_q receives contribution only from events in which there are at least q particles in some bin. In pp collisions, that can cause a severe cut on the number of contributing events when q is large and the bin size is small, thereby allowing F_q to select rare events with large spikes. In AA collisions since so many particles are produced, nearly all bins of every event would contribute to F_q for $q < 6$ even when bin sizes are small, so fluctuations tend to cancel resulting in no measurable intermittency. It is only when q is greater than the average multiplicity in a bin can interesting fluctuations be detected. F_q for such high q have not been determined so far in experiments on nuclear collisions; when they become available, we shall be able to know whether independent pp collisions form sufficient basis for nuclear collisions at the energies being studied.

4.2. Second-Order Phase Transition

If there is “new” physics, it may be just collective hydrodynamical flow, in which case fluctuations are suppressed. But really new physics would be quark-hadron phase transition, for which fluctuations can be large. Intermittency would be a possible manifestation of such a process.

It is not feasible at present to derive observable consequences of phase transition (PT) from first principles. A simple toy model is the Ising model in 2d that can provide some hint on the nature of a second-order PT. Its intermittency behavior has been studied both analytically and numerically [45, 46]. It is found that $\varphi_q = (q - 1)/8$, where the anomalous fractal dimension $d_q = \varphi_q/(q - 1) = 1/8$ can be related to the critical exponent η of the Ising model. Since d_q is a constant independent of q , one may say that the Ising model exhibits a monofractal pattern of intermittency. Based on that finding, it has been conjectured that in nuclear collisions the intermittency behavior of hadronic fluctuations following a PT may also be monofractal [47].

To do better than toy models and conjectures, it is necessary to consider a phenomenological model that is physically more relevant. Since the confinement of quarks by vacuum condensate in a second-order PT may be thought of as being analogous to the confinement

of magnetic fields into fluxoids by superconducting condensate in a type II superconductor, it should be sensible to consider the problem of intermittency in the framework of the Ginzburg-Landau (GL) theory[48]-[51]. The intermittency properties in that problem was first studied in [52] in the case of a homogeneous system, i.e., in the GL free-energy density

$$\mathcal{F}[\phi] = a |\phi|^2 + b |\phi|^4 + c |\partial\phi/\partial\vec{r}|^2 \quad (10)$$

the last term is neglected. When the order parameter ϕ is varied, the observed multiplicity distribution P_n is obtained from the Poissonian distribution $\hat{P}_n[\phi]$ according to

$$P_n = \frac{1}{Z} \int \mathcal{D}\phi \hat{P}_n[\phi] e^{-\int dz \mathcal{F}[\phi]} \quad , \quad (11)$$

where $Z = \int \mathcal{D}\phi e^{-\int dz \mathcal{F}[\phi]}$ and $|\phi|^2$ is the mean particle density in $\hat{P}_n[\phi]$. It is found that with this P_n there is no intermittency in the limit of vanishing bin size. However, the system possesses the scaling behavior (6) extremely well. Furthermore, the indices β_q satisfy the relation [52]

$$\beta_q = (q-1)^\nu \quad , \quad \nu = 1.304 \quad (12)$$

to a high degree of accuracy. Not only is the existence of a numerical constant ν a surprise, there being no numerical input in the problem, but also the independence of ν on the parameters a , b , and the dimension d is an unexpected bonus, which promotes the scaling exponent ν to a universal quantity that is valid for all systems describable by the GL theory. For the quark-gluon system one has no precise knowledge of the transition temperature T_c , nor even approximate information about $a(T)$ and $b(T)$, and one cannot control the temperature T . However, since ν is independent of all of them it may be used as an effective numerical criterion for quark-hadron PT.

Current data on hadronic collisions [24] can be very well described by (6) and (12), but with $\nu = 1.45 \pm 0.04$ [53]. The nucleus-emulsion experiments [44, 54] give $\nu = 1.55 \pm 0.12$ [52]. Thus none of them satisfies our criterion for PT, which is not a surprise. On the other hand, photon production at the threshold of lasing in quantum optics is known to be describable as a second-order PT. A recent experiment measured F_q from photocount and verified (12) to high precision [55].

The approximation $c = 0$ in (10) was relaxed in later investigations [56]. The value of ν was found to increase a little, but still essentially constant. For all values of c studied, the result can be summarized by $\nu = 1.316 \pm 0.012$. With the verification of the scaling prediction and its universality property by the quantum optics experiment, the proposal that ν be used as a criterion for quark-hadron PT in heavy-ion collisions acquires significant weight.

4.3. First-order Phase Transition

If the PT is first order, then there is a mixed region of quarks and hadrons. But what are those hadrons? In cosmic PT they are hadronic bubbles that can grow in size [57]. In heavy-ion collisions the conditions are different; instead of spherical bubbles there can be hadronic clusters of various shapes and sizes. After these clusters emerge from the plasma, they result in clusters of hadrons, the multiplicities of which are indications of the sizes S of the clusters.

The relevant question to ask is whether the fluctuations of the cluster size from the average can reveal interesting features about the PT. Since the hadronic clusters are subject to collisions by quarks and gluons in the mixed region, they behave like Brownian particles performing random walks around a drift motion toward the boundary. In their paths the clusters can coalesce, breakup, or scatter elastically. Successive coalescence would lead to large clusters, which are rare but possible. Thus the shape of the cluster-size distribution $P(S)$ can reveal the dynamical processes that the clusters go through before emerging from the plasma. Simulation of this growth process has been performed with the result that a scaling behavior

$$P(S) \propto S^{-\sigma} \tag{13}$$

is obtained [58], until finite-size effect cuts off at high S .

This is not intermittency in the usual sense. Nevertheless, it is a study of fluctuations that should be emphasized in future experiments on heavy-ion collisions. The experimental discovery of a scaling distribution such as (13) would undoubtedly stimulate some excitement in the field.

5. Conclusion

In this review I have not covered many advances made in recent years in the phenomenology of measuring fluctuations and correlations. Such efforts are important for quantifying the many features of multiparticle production. What I have covered are the various attempts to discover possible dynamical origins of multiplicity fluctuations. They are also important not only because we need to understand the theoretical basis of what are observed, but also to generate new ideas on what else to measure.

Intermittency has been a tremendous stimulus to the study of the nature of fluctuations in hard and soft processes. Successive branching process is the best candidate for the origin of the power-law behavior. It is proven in perturbative QCD, and is built into the successful model for low- p_T hadronic processes.

The normalized factorial moments have a unique feature characterizing second-order phase transition. It has already been verified in laser physics, and may signal quark-hadron phase transition. Preliminary study indicates that if the phase transition is first order, the size distribution of hadron clusters produced may exhibit scaling behavior. Whether that can be related to fractal properties in the spatial structure of the hadronizing system or to self-organized criticality would be intriguing possibilities.

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