$N = 1$ and 2 Superstrings as Supertopological Models on Higher-Genus Super Riemann Surfaces

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ABSTRACT

On arbitrary higher-genus super Riemann surfaces we show that the $N = 1$ and 2 superstrings realize supertopological algebras which are superfield extensions of the twisted $N = 2$ superconformal algebra. Some restrictions on their dimensions are found for both $N = 1$ and 2 superstrings. Superaffine and superprojective connections are automatically introduced into the algebras, so that they guarantee coordinate-independence of the algebras.

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1. Introduction

Topological field theories^{1),2)} are reparametrization invariant theories which either do not depend on the metric, or can be consistently restricted to a metric independent sector. The latter are known as cohomological topological field theories. The characteristic feature of them is the existence of a nilpotent BRST charge Q_B such that the stress-energy tensor $T_{\alpha\beta}(x)$ is the BRST trivial

$$T_{\alpha\beta}(x) = \{Q_B, G_{\alpha\beta}(x)\}, \quad (1.1)$$

where $G_{\alpha\beta}(x)$ is some function of basic fields. In a physical sector defined by the cohomology of Q_B the theories are metric independent. Here one can see a remarkable example: Let $W_0(x)$ be an operator which is gauge-invariant, BRST-invariant, but not BRST-trivial. Then, a correlation function

$$\langle W_0(x_1)W_0(x_2) \cdots W_0(x_n) \rangle \quad (1.2)$$

is independent of $x_1, x_2, \dots, x_n$.¹⁾

In two-dimensional conformal theories, topological models were first considered by Eguchi and Yang.³⁾ They pointed out that by twisting the $N = 2$ superconformal algebra one obtains a topological conformal algebra (TCA) which involves the relation (1.1) and has a vanishing Virasoro central charge. Dijkgraaf, Verlinde and Verlinde⁴⁾ then found that this algebra is in fact the most general one which can be written with four chiral fields, i.e., the stress-energy tensor $T(z)$, the BRST current $Q(z)$, and their BRST partners $G(z)$ and $J(z)$. By the BRST partners we mean they have relations such that

$$T(z) = \{Q_B, G(z)\} \quad (1.3)$$

$$Q(z) = -[Q_B, J(z)], \quad (1.4)$$

where $Q_B = \frac{1}{2\pi i} \oint Q(z)dz$ is the BRST charge. Realizations of this $N = 2$ twisted superconformal algebra and its extension were extensively studied in Ref.5).

In previous works⁽⁶⁾ we considered TCA on an arbitrary higher-genus Riemann surface M . The TCA on M is realized by various models and takes the following form

$$T(z_1)T(z_2) = \frac{2T(z_2)}{z_{12}^2} + \frac{\partial T(z_2)}{z_{12}} + \dots \quad (1.5a)$$

$$T(z_1)Q(z_2) = \frac{Q(z_2)}{z_{12}^2} + \frac{\partial Q(z_2)}{z_{12}} + \dots \quad (1.5b)$$

$$T(z_1)G(z_2) = \frac{2G(z_2)}{z_{12}^2} + \frac{G(z_2)}{z_{12}} + \dots \quad (1.5c)$$

$$T(z_1)J(z_2) = -d \left[\frac{1}{z_{12}^3} - \frac{\Gamma(z_2)}{2z_{12}^2} - \frac{\partial \Gamma(z_2)}{2z_{12}} \right] + \frac{J(z_2)}{z_{12}^2} + \frac{\partial J(z_2)}{z_{12}} + \dots \quad (1.5d)$$

$$Q(z_1)G(z_2) = d \left[\frac{1}{z_{12}^3} + \frac{\Gamma(z_2)}{2z_{12}^2} \right] + \frac{J(z_2)}{z_{12}^2} + \frac{T(z_2)}{z_{12}} + \dots \quad (1.5e)$$

$$Q(z_1)J(z_2) = -\frac{Q(z_2)}{z_{12}} + \dots \quad (1.5f)$$

$$J(z_1)G(z_2) = -\frac{G(z_2)}{z_{12}} + \dots \quad (1.5g)$$

$$J(z_1)J(z_2) = \frac{d}{z_{12}^2} + \dots \quad (1.5h)$$

$$Q(z_1)Q(z_2) \cong 0, \quad (1.5i)$$

where $z_{12} = z_1 - z_2$, d is a parameter and $\Gamma(z)$ is an affine connection which guarantees coordinate-independence of the algebra. From (1.5e) and (1.5f) we have the relations (1.3) and (1.4), respectively. When $\Gamma = 0$, this algebra coincides with the usual TCA of Eguchi and Yang.

In this paper we would like to consider $N = 1$ and 2 superfield extensions of the above TCA (1.5) on arbitrary higher-genus super Riemann surfaces. In the following these extended algebras will be called the $N = 1$ and 2 supertopological algebras, respectively. We will use the $N = 1$ and 2 superstrings to realize the supertopological algebras. It was discussed by several authors that topological superstring theories may have $N = 3$ twisted supersymmetry,^{(7),(8),(9)} and the superfield extension of the twisted $N = 2$ superconformal algebra is identical to the twisted $N = 3$ superconformal algebra. Their results will be compared with ours in the last section. In §2 we consider the $N = 1$ superstring and in §3 the $N = 2$ superstring. The last section is devoted to concluding remarks.

2. The $N = 1$ Superstring

We follow the superspace formulation of the super Riemann surface proposed in Refs.10,11). The chiral $N = 1$ superstring field $X^\mu(Z)$ is a function of the superspace variables $Z = (z, \theta)$. In terms of component fields we get the expansion

$$X^\mu(Z) = x^\mu(z) + \theta \psi^\mu(z). \quad (\mu = 0, 1, 2, \dots, d-1) \quad (2.1)$$

We also introduce a superdilaton field

$$\Phi(Z) = \phi(z) + \theta \psi(z). \quad (2.2)$$

The chiral stress-energy tensor for these free matter fields X^μ and Φ is given by

$$T_m = T_X + T_\Phi, \quad (2.3)$$

$$T_X = -\frac{1}{2}DX^\mu \partial X_\mu, \quad (\eta^{00} = -1), \quad (2.4)$$

$$T_\Phi = -\frac{1}{2}D\Phi \partial \Phi + \alpha(D + 2\Gamma)(D + \Gamma)D\Phi, \quad (2.5)$$

where the covariant derivative D is defined by

$$D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial z}, \quad \partial = \frac{\partial}{\partial z}, \quad (2.6)$$

$\Gamma(Z)$ a superaffine connection and α a real parameter.

The stress-energy tensor for the superghosts $B(Z)$ and $C(Z)$ is given by

$$T_{gh} = -C\partial B + \frac{1}{2}DC \cdot DB - \frac{3}{2}\partial C \cdot B, \quad (2.7)$$

where $C(Z) = c(z) + \theta \gamma(z)$ is a fermionic ghost with a weight $h = -1$ and $B(Z) = \beta(z) + \theta b(z)$ is a bosonic ghost with a weight $h = 3/2$. The BRST current $Q_0(Z)$ and the superghost number current $J_0(Z)$ are also known to be

$$Q_0 = C(T_m + \frac{1}{2}T_{gh}), \quad (2.8)$$

$$J_0 = CB. \quad (2.9)$$

In the following we modify the above Q_0 and J_0 by total derivative terms. It will be shown that the modified Q and J together with T and B realize the $N = 1$

supertopological algebra on an arbitrary higher-genus super Riemann surface M_1 . The new four operators are defined by

$$T(Z) = T_m + T_{gh} - \Omega, \quad (2.10)$$

$$Q(Z) = C(T_m + \frac{1}{2}T_{gh} - \Omega) - \frac{1}{4}D(CDC \cdot B) + \gamma D(C\partial\Phi) - \frac{\gamma}{2}D(DC \cdot D\Phi)$$

$$+ \frac{\gamma^2}{2}D(D - \Gamma)(D - 2\Gamma)C + D(\frac{1}{2}\Lambda DC - D\Lambda \cdot C), \quad (2.11)$$

$$G(Z) = B, \quad (2.12)$$

$$J(Z) = CB + \beta D\Phi + \Lambda, \quad (2.13)$$

where $\Omega(Z)$ and $\Lambda(Z)$ are c-number functions with weights $h = 3/2$ and $h = 1/2$, respectively, determined later, and γ , β are real parameters. The total derivative forms here are suggested from the work of Bershadsky et al.,⁹⁾ though their approach is not a full superspace formulation.

As for the quantization of fields on M_1 we follow the global operator method of Krichever-Novikov,¹²⁾ and only refer to the results of our previous works.^{6),13)} The contractions of fields are given by

$$< X^\mu(Z_1) X^\nu(Z_2) > = \eta^{\mu\nu} [-\ln Z_{12} + f(Z_1, Z_2)] \quad (2.14)$$

$$< \Phi(Z_1) \Phi(Z_2) > = -\ln Z_{12} + f(Z_1, Z_2), \quad (2.15)$$

$$< B(Z_1) C(Z_2) > = \frac{\theta_{12}}{Z_{12}} + l(Z_1, Z_2), \quad (2.16)$$

where

$$Z_{12} = z_1 - z_2 - \theta_1 \theta_2, \quad (2.17)$$

$$\theta_{12} = \theta_1 - \theta_2, \quad (2.18)$$

f and l are holomorphic except for P_+ and P_- which are arbitrary fixed points on M_1 . When genus $g = 0$, one may set $f = l = \Gamma = \Lambda = \Omega = 0$.

By using the contractions (2.14)~(2.16) we can derive the $N = 1$ supertopological algebra on M_1 in the operator product expansion form

$$T_1 T_2 = \frac{3\theta_{12}}{2Z_{12}^2} T_2 + \frac{\theta_{12}}{Z_{12}} \partial T_2 + \frac{1}{2Z_{12}} D T_2 + \dots \quad (2.19a)$$

$$T_1 Q_2 = \frac{\theta_{12}}{2Z_{12}^2} Q_2 + \frac{\theta_{12}}{Z_{12}} \partial Q_2 + \frac{1}{2Z_{12}} D Q_2 + \dots \quad (2.19b)$$

$$T_1 G_2 = \frac{3\theta_{12}}{2Z_{12}^2} G_2 + \frac{\theta_{12}}{Z_{12}} \partial G_2 + \frac{1}{2Z_{12}} D G_2 + \dots \quad (2.19c)$$

$$T_1 J_2 = \frac{\theta_{12}}{2Z_{12}^2} J_2 + \frac{\theta_{12}}{Z_{12}} \partial J_2 + \frac{1}{2Z_{12}} D J_2 + \frac{\gamma^2}{2} \left[-\frac{1}{Z_{12}^2} + \frac{\theta_{12}}{Z_{12}^2} \Gamma_2 + \frac{2\theta_{12}}{Z_{12}} \partial \Gamma_2 + \frac{1}{Z_{12}} D \Gamma_2 \right] + \dots \quad (2.19d)$$

$$Q_1 G_2 = -\frac{\theta_{12}}{2Z_{12}^2} J_2 - \frac{1}{2Z_{12}} D J_2 + \frac{\theta_{12}}{Z_{12}} T_2 - \frac{\gamma^2}{2} \left[\frac{1}{Z_{12}^2} \Gamma_2 + \frac{\theta_{12}}{Z_{12}} \Gamma_2 + \frac{1}{Z_{12}} D \Gamma_2 \right] + \dots \quad (2.19e)$$

$$Q_1 J_2 = -\frac{\theta_{12}}{Z_{12}} Q_2 + \dots \quad (2.19f)$$

$$J_1 G_2 = \frac{\theta_{12}}{Z_{12}} G_2 + \dots \quad (2.19g)$$

$$J_1 J_2 = -\frac{\gamma^2}{Z_{12}} + \dots \quad (2.19h)$$

$$Q_1 Q_2 \cong 0, \quad (2.19i)$$

$$G_1 G_2 \cong 0, \quad (2.19j)$$

where we need constraints

$$\dot{\epsilon}_{1\alpha} = \dot{\epsilon} - 10 + 8\alpha^2 = 0, \quad (2.20)$$

$$\beta = -\gamma, \quad (2.21)$$

$$\frac{1}{2} + \alpha\beta = -\frac{\gamma^2}{2}, \quad (2.22)$$

$\dot{\epsilon}_{1\alpha}$ being the total central charge of the matter-ghost system ($\dot{\epsilon} = d + 1$). From (2.21) and (2.22) we have

$$\alpha = \frac{1}{2} \left(\gamma + \frac{1}{\gamma} \right). \quad (2.23)$$

Since γ is a real parameter, the minimum value of α^2 is 1. Hence, from (2.20) we

see

$$\hat{c} = d + 1 \leq 2, \quad (2.24)$$

which means that the dimensions of X^μ should be $d \leq 1$. The unknown functions Ω and Λ are fixed to be

$$\Omega(Z) = \frac{1}{4} [\hat{c}(R_m - R) - 10(R_{gh} - R)], \quad (2.25)$$

$$\Lambda(Z) = l(Z, Z) + \Gamma(Z), \quad (2.26)$$

where the R 's are superprojective connections defined by

$$R(Z) = \partial\Gamma + \Gamma D\Gamma, \quad (2.27)$$

$$R_m(Z) = 2[\partial_1 D_2 f(Z_1, Z_2)]_{Z_1=Z_2=Z}, \quad (2.28)$$

$$R_{gh}(Z) = -\frac{1}{5}[2\partial_1 l(Z_1, Z_2) + 3\partial_2 l(Z_1, Z_2) - D_1 D_2 l(Z_1, Z_2)]_{Z_1=Z_2=Z}, \quad (2.29)$$

and $-l(Z, Z)$ is a superaffine connection. The c -number functions Ω and Λ are tensors with weights $3/2$ and $1/2$, respectively. We shall prove that these R 's and $-l(Z, Z)$ are actually subject to transformation rules of superprojective and superaffine connections.

Before we prove the algebra (2.19) with constraints (2.20)~(2.22), we would like to mention a reason why this algebra is topological, and also make a remark about a role of the superaffine connections in the algebra. By using formulas*

$$\frac{1}{2\pi i} \oint_{C_2} dZ_1 Z_1^{-n-1} = 0, \quad \frac{1}{2\pi i} \oint_{C_2} dZ_1 \partial_{12} Z_1^{-n-1} = \delta_{n,0}, \quad (2.30)$$

and from (2.19e) we find the relation

$$T(Z) = [Q_B, G(Z)], \quad (2.31)$$

* The integration $\oint_{C_2} dZ_1$ means $\oint_{C_2} dz_1 d\theta_1 d\bar{\theta}_1$, where the integration over z_1 is carried out along a contour C_2 winding around the point z_2 .¹⁰⁾ The integration without any assignment of paths below means that the integration over z is carried out along a contour winding around P_+ on M_1 .

where Q_B is the super BRST charge defined by

$$Q_B = \frac{1}{2\pi i} \oint dZ Q(Z). \quad (2.32)$$

The relation (2.31) means that the stress-energy tensor $T(Z)$ is BRST-trivial. We also see from (2.19a) or (2.19i) that the total central charge $\hat{c}_{tot}$ is zero. The existence of the nilpotent BRST charge Q_B and the BRST-trivial stress-energy tensor $T(Z)$ is the characteristic feature of topological theories.

A role of the affine connection Γ is as follows: The affine connection guarantees coordinate-independence of the algebra. In order to see this more explicitly, we define the super Virasoro operator

$$L_V = \frac{1}{2\pi i} \oint dZ V(Z) T(Z) \quad (2.33)$$

and

$$J_A = \frac{1}{2\pi i} \oint dZ A(Z) J(Z), \quad (2.34)$$

where $V(Z)$ is a supervector field with weight $h = -1$ and $A(Z)$ is a scalar field. From (2.19d) we calculate their commutator to give

$$[L_V, J_A] = J_{(V,A)} - \frac{\gamma^2}{2} \chi(V, A), \quad (2.35)$$

where

$$J_{(V,A)} = \frac{1}{2\pi i} \oint dZ (-\partial A \cdot V + \frac{1}{2} DA \cdot DV) J, \quad (2.36)$$

$$\chi(V, A) = \frac{1}{2\pi i} \oint dZ AD(D - \Gamma)(D - 2\Gamma)V. \quad (2.37)$$

Comparing (2.19d) with (2.35) we see a correspondence

$$H_{12}^{(13)} \equiv \left[\frac{1}{Z_{12}^2} \Gamma_2 - 2 \frac{\theta_{12}}{Z_{12}} \partial \Gamma_2 - \frac{1}{Z_{12}} D \Gamma_2 \right] \longleftrightarrow D(D - \Gamma)(D - 2\Gamma). \quad (2.38)$$

Namely, the affine connection Γ appears in the super-covariant derivative $(D + 2h\Gamma)$ in the anomaly $\chi(V, A)$. Since, for ϕ_h with weight h , $(D + 2h\Gamma)\phi_h$ has a weight

$h + (1/2)$, we see that $D(D - \Gamma)(D - 2\Gamma)V$ has a weight $1/2$. Hence, $dZAD(D - \Gamma)(D - 2\Gamma)V$ behaves like a scalar, so that $\chi(V, A)$ is coordinate-independent. Note that more directly $H_{12}^{(13)}$ behaves like a tensor with weight $1/2$ with respect to Z_1 and that with weight $3/2$ with respect to Z_2 if one ignores finite terms. From this fact we see that J is not anomalous but a tensor with weight $1/2$. T is also not anomalous but a tensor with weight $3/2$. Hence, L_V , J_A and $J_{(VA)}$ are coordinate-independent (one can see $-\partial A \cdot V + \frac{1}{2}DA \cdot DV$ to be a scalar). We also see a similar correspondence

$$\frac{1}{Z_{12}^2} + \frac{\theta_{12}}{Z_{12}^2}\Gamma_2 + \frac{1}{Z_{12}}D\Gamma_2 \longleftrightarrow (D + 2\Gamma)(D + \Gamma)D. \quad (2.39)$$

Derivation of (2.19a)

Using the fundamental contractions (2.14)~(2.16) and Wick's theorem we calculate the contraction

$$\begin{aligned} < T(Z_1)T(Z_2) > = < T_1^X T_2^X > + < T_1^{gh} T_2^{gh} > + \frac{1}{4} < : D\Phi_1 \partial\Phi_1 :: D\Phi_2 \partial\Phi_2 : > \\ & + \alpha(D_1 + 2\Gamma_1)(D_1 + \Gamma_1) < D\Phi_1, -\frac{1}{2}D\Phi_2 \partial\Phi_2 > \\ & + \alpha(D_2 + 2\Gamma_2)(D_2 + \Gamma_2) < -\frac{1}{2}D\Phi_1 \partial\Phi_1, D\Phi_2 > \\ & + \alpha^2(D_1 + 2\Gamma_1)(D_1 + \Gamma_1)(D_2 + 2\Gamma_2)(D_2 + \Gamma_2) < D\Phi_1, D\Phi_2 > \\ & = \frac{3\theta_{12}}{2Z_{12}^2}(T_1^m + T_2^h) + \frac{\theta_{12}}{2Z_{12}^2}\partial(T_1^m + T_2^h) + \frac{1}{2Z_{12}}D(T_1^m + T_2^h) \\ & + \hat{c}\left(\frac{1}{4Z_{12}^3} - \frac{3\theta_{12}}{8Z_{12}^2}R_2^m - \frac{\theta_{12}}{4Z_{12}}\partial R_2^m - \frac{1}{8Z_{12}}DR_2^m\right) \\ & - 10\left(\frac{1}{4Z_{12}^3} - \frac{3\theta_{12}}{8Z_{12}^2}R_2^{gh} - \frac{\theta_{12}}{4Z_{12}}\partial R_2^{gh} - \frac{1}{8Z_{12}}DR_2^{gh}\right) \\ & + \alpha^2\left(\frac{2}{Z_{12}^3} - \frac{3\theta_{12}}{Z_{12}^2}R_2 - \frac{2\theta_{12}}{Z_{12}}\partial R_2 - \frac{1}{Z_{12}}DR_2\right) + \dots, \end{aligned} \quad (2.40)$$

where $\hat{c}$ and the R 's were defined before. The most singular terms proportional to Z_{12}^{-3} in (2.40) vanish when the following relation holds:

$$\hat{c} - 10 + 8\alpha^2 = 0. \quad (2.41)$$

This is nothing but the condition that the total central charge $\hat{c}_{tot}$ given by (2.20)

should be zero. Collecting R terms proportional to θ_{12}/Z_{12}^2 , we have

$$\begin{aligned} -\frac{3}{8}\frac{\theta_{12}}{Z_{12}^2}(\hat{c}R_2^m - 10R_2^{gh} + \alpha^2 R_2) &= -\frac{3}{8}\frac{\theta_{12}}{Z_{12}^2}[\hat{c}(R_2^m - R_2) - 10(R_2^{gh} - R_2)] \\ &\equiv -\frac{3}{2}\frac{\theta_{12}}{Z_{12}^2}\Omega_2, \end{aligned} \quad (2.42)$$

where we have used the relation (2.41) and Ω_2 is defined by (2.25). Similar calculations for R terms proportional to (θ_{12}/Z_{12}) and $(1/Z_{12})$ yield $-(\theta_{12}/Z_{12})\partial\Omega$ and $-(1/2Z_{12})D\Omega$, respectively. Putting all these terms together we finally obtain (2.19a).

Derivation of (2.19f)

In a similar manner as we did in the derivation of (2.19a) we calculate the contraction

$$\begin{aligned} < Q(Z_1)J(Z_2) > = < C_1(T_1^m - \Omega_1) - \frac{1}{4}DC_1DC_1B_1 + \frac{1}{2}C_1DC_1DB_1 \\ & - \frac{1}{2}C_1\partial C_1B_1 + \frac{\gamma^2}{2}D\partial C_1 - \partial(\gamma^2\Gamma_1 + \Lambda_1)C_1 \\ & - \frac{1}{2}(\gamma^2\Gamma_1 + \Lambda_1)\partial C_1 - \frac{1}{2}D(\gamma^2\Gamma_1 + \Lambda_1)DC_1 + \frac{\gamma}{2}DC_1\partial\Phi_1 \\ & - \gamma C_1D\partial\Phi_1 - \frac{\gamma}{2}\partial C_1D\Phi_1|C_2B_2 > \\ & + \beta < C_1T_1^m D\Phi_2 > + \frac{\beta\gamma}{2} < DC_1\partial\Phi_1 D\Phi_2 > \\ & - \beta\gamma < C_1D\partial\Phi_1 D\Phi_2 > - \frac{\beta\gamma}{2} < \partial C_1D\Phi_1 D\Phi_2 > \\ & = -\frac{\theta_{12}}{Z_{12}}\left[C(T_m - \Omega) - \frac{1}{4}DCDCB + \frac{1}{2}CCDCDB - \frac{1}{2}C\partial CB \right. \\ & \left. - \left(\frac{1}{2} + \alpha\beta\right)D\partial C - \frac{\beta}{2}DC\partial\Phi + \beta C D\partial\Phi + \frac{\beta}{2}\partial C D\Phi\right]_2 \\ & + \frac{1}{Z_{12}}\left[\left(\frac{1}{2} + \alpha\beta - \frac{1}{2}\beta\gamma\right)\partial C_2 + \frac{1}{2}(\beta + \gamma)C_2\partial\Phi_2\right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{Z_{12}^2} \left(\frac{1}{2} - \frac{\gamma^2}{2} + \alpha\beta - \beta\gamma \right) C_2 \\
& + \frac{\theta_{12}}{Z_{12}^2} \left[\left(\frac{1}{2} + \alpha\beta - \frac{1}{2}\beta\gamma \right) DC_2 - \frac{1}{2}(\beta + \gamma)C_2 D\Phi_2 \right] \\
& + \frac{\theta_{12}}{2Z_{12}^2} [l(2, 2) - \gamma^2\Gamma_2 - \Lambda_2 - 2\alpha\beta\Gamma_2] C_2 \\
& + \frac{1}{2Z_{12}} D_1 [l(1, 2) + l(2, 1) - \gamma^2\Gamma_1 - \Lambda_1 - 2\alpha\beta\Gamma_1]_{z_1=z_2} C_2 \\
& + \frac{\theta_{12}}{Z_{12}} \left\{ \partial_1 [l(1, 2) + l(2, 1) - 2\alpha\beta\Gamma_1]_{z_1=z_2} C_2 \right. \\
& + \left[\frac{1}{2}l(2, 2) - \alpha\beta\Gamma_2 \right] \partial C_2 \\
& + \left. \frac{1}{2} D_1 [l(1, 2) + l(2, 1) - 2\alpha\beta\Gamma_1]_{z_1=z_2} DC_2 \right\} + \dots
\end{aligned} \tag{2.43}$$

Taking account of the constraints (2.21) and (2.22), we find

$$\frac{1}{2} + \alpha\beta - \frac{1}{2}\beta\gamma = 0, \tag{2.44}$$

$$\frac{1}{2} - \frac{\gamma^2}{2} + \alpha\beta - \beta\gamma = 0. \tag{2.45}$$

As a result we see that the second, third and forth terms on the r.h.s. of (2.43) identically vanish, i.e.,

$$\frac{1}{Z_{12}} \left[\left(\frac{1}{2} + \alpha\beta - \frac{1}{2}\beta\gamma \right) \partial C_2 + \frac{1}{2}(\beta + \gamma)C_2 \partial\Phi_2 \right] = 0, \tag{2.46}$$

$$\frac{1}{Z_{12}^2} \left(\frac{1}{2} - \frac{\gamma^2}{2} + \alpha\beta - \beta\gamma \right) C_2 = 0, \tag{2.47}$$

$$\frac{\theta_{12}}{Z_{12}^2} \left[\left(\frac{1}{2} + \alpha\beta - \frac{1}{2}\beta\gamma \right) DC_2 - \frac{1}{2}(\beta + \gamma)C_2 D\Phi_2 \right] = 0. \tag{2.48}$$

Substituting from (2.22) into the last three terms on the r.h.s. of (2.43) and noting

$$D_1 [l(1, 2) + l(2, 1)]_{z_1=z_2} = D_2 l(2, 2), \tag{2.49}$$

$$\partial_1 [l(1, 2) + l(2, 1)]_{z_1=z_2} = \partial_2 l(2, 2), \tag{2.50}$$

we have

$$\frac{\theta_{12}}{2Z_{12}^2} [l(2, 2) - \gamma^2\Gamma_2 - \Lambda_2 - 2\alpha\beta\Gamma_2] C_2 = \frac{\theta_{12}}{2Z_{12}^2} [l(2, 2) + \Gamma_2 - \Lambda_2] C_2, \tag{2.51}$$

$$\begin{aligned}
& \frac{1}{2Z_{12}} D_1 [l(1, 2) + l(2, 1) - \gamma^2\Gamma_1 - \Lambda_1 - 2\alpha\beta\Gamma_1]_{z_1=z_2} C_2 \\
& = \frac{1}{2Z_{12}} D_2 [l(2, 2) + \Gamma_2 - \Lambda_2] C_2,
\end{aligned} \tag{2.52}$$

$$\begin{aligned}
& \frac{\theta_{12}}{Z_{12}} \left\{ \partial_1 [l(1, 2) + l(2, 1) - 2\alpha\beta\Gamma_1]_{z_1=z_2} C_2 \right. \\
& + \left[\frac{1}{2}l(2, 2) - \alpha\beta\Gamma_2 \right] \partial C_2 \\
& + \left. \frac{1}{2} D_1 [l(1, 2) + l(2, 1) - 2\alpha\beta\Gamma_1]_{z_1=z_2} DC_2 \right\} \\
& = \frac{\theta_{12}}{Z_{12}} \left\{ [\partial_2 (l(2, 2) + \Gamma_2) + \gamma^2 \partial\Gamma_2] C_2 \right. \\
& + \left[\frac{1}{2}l(2, 2) + \Gamma_2 \right] + \frac{\gamma^2}{2} \Gamma_2 \partial C_2 \\
& + \left. \left[\frac{1}{2} D_2 (l(2, 2) + \Gamma_2) + \frac{\gamma^2}{2} D\Gamma_2 \right] DC_2 \right\}.
\end{aligned} \tag{2.53}$$

Let us fix the unknown function Λ by (2.26). Then Eqs. (2.51) and (2.52) vanish, and Eq.(2.53) becomes

$$\frac{\theta_{12}}{Z_{12}} \{ \} = \frac{\theta_{12}}{Z_{12}} \left\{ \partial(\gamma^2\Gamma + \Lambda)C + \frac{1}{2}(\gamma^2\Gamma + \Lambda)\partial C + \frac{1}{2}D(\gamma^2\Gamma + \Lambda)DC \right\}_2. \tag{2.54}$$

Collecting terms proportional to θ_{12}/Z_{12} one finally finds Eq.(2.43) to be

$$-\frac{\theta_{12}}{Z_{12}} Q_2 + \dots \tag{2.55}$$

This completes the proof of (2.19f).

Other remaining equations in (2.19) can be derived in a similar way by carrying out straightforward but long calculations.

A transformation rule for $-l(Z, Z)$

We would like to prove that $-l(Z, Z)$ is subject to the transformation rule of superaffine connection. First we note that under a superconformal transformation, $Z = (z, \theta) \longrightarrow \tilde{Z} = (\tilde{z}, \tilde{\theta})$, the contraction $< B(Z_1)C(Z_2) >$ behaves like

$$< B(Z_1)C(Z_2) > = (D\tilde{\theta}_1)^3(D\tilde{\theta}_2)^{-2} < \tilde{B}(\tilde{Z}_1)\tilde{C}(\tilde{Z}_2) >. \quad (2.56)$$

Substituting (2.16) into both sides of the above equation, we have

$$\frac{\theta_{12}}{Z_{12}} + l(Z_1, Z_2) = (D\tilde{\theta}_1)^3(D\tilde{\theta}_2)^{-2} \left[\frac{\tilde{\theta}_{12}}{\tilde{Z}_{12}} + \tilde{l}(\tilde{Z}_1, \tilde{Z}_2) \right] \quad (2.57)$$

or

$$l(Z_1, Z_2) - \frac{(D\tilde{\theta}_1)^3 \tilde{\theta}_{12}}{(D\tilde{\theta}_2)^2 \tilde{Z}_{12}} = \frac{(D\tilde{\theta}_1)^3 \tilde{\theta}_{12}}{(D\tilde{\theta}_2)^2 \tilde{Z}_{12}} - \frac{\theta_{12}}{Z_{12}} = \frac{D^2 \tilde{\theta}_2}{D\tilde{\theta}_2} + O(1, 2). \quad (2.58)$$

In (2.58) we take a limit $Z_1 \longrightarrow Z_2 = Z$ to give

$$-l(Z, Z) = (D\tilde{\theta})(-\tilde{l}(\tilde{Z}, \tilde{Z})) - \frac{D^2 \tilde{\theta}}{D\tilde{\theta}}. \quad (2.59)$$

This transformation rule shows that $-l(Z, Z)$ is one of superaffine connections.

A transformation rule for R, R_m and R_{gh}

Let $\Gamma(Z)$ be a superaffine connection, which is subject to the transformation rule

$$\Gamma(Z) = (D\tilde{\theta})\tilde{\Gamma}(\tilde{Z}) - \frac{D^2 \tilde{\theta}}{D\tilde{\theta}}. \quad (2.60)$$

Then, one obtains the transformation rule

$$R(Z) = (D\tilde{\theta})^3 \tilde{R}(\tilde{Z}) - S(\tilde{Z}, Z) \quad (2.61)$$

from (2.27) and (2.60), where

$$S(\tilde{Z}, Z) = \frac{D^4 \tilde{\theta}}{D\tilde{\theta}} - 2 \frac{D^2 \tilde{\theta} D^3 \tilde{\theta}}{(D\tilde{\theta})^2} \quad (2.62)$$

is the super Schwarzian derivative. Here it should be remarked that the super Miura transformation defined by (2.27) is well known to give the superprojective

connection which obeys the transformation rule (2.61). As for R_m defined by (2.28), it was already proved in our previous work¹³⁾ that R_m is one of superprojective connections subject to the transformation rule (2.61). We would like to prove R_{gh} defined by (2.29) to be actually one of superprojective connections. Let us start with Eq.(2.57) which shows the transformation rule for $l(Z_1, Z_2)$. From this we have in the limit $Z_1 \longrightarrow Z_2$

$$\begin{aligned} R_{gh}(Z_2) &= (D\tilde{\theta}_2)^3 \tilde{R}_{gh}(\tilde{Z}_2) - \frac{\tilde{\theta}_{12}}{5\tilde{Z}_{12}^2} \left[-2 \frac{(D\tilde{\theta}_1)^5}{(D\tilde{\theta}_2)^2} + 3(D\tilde{\theta}_1)^3 - \frac{(D\tilde{\theta}_1)^4}{D\tilde{\theta}_2} \right] \\ &\quad - \frac{6\tilde{\theta}_{12}}{5\tilde{Z}_{12}} \left[\frac{(D\tilde{\theta}_1)^2 \partial D\tilde{\theta}_1}{(D\tilde{\theta}_2)^2} - \frac{(D\tilde{\theta}_1)^3 \partial D\tilde{\theta}_2}{(D\tilde{\theta}_2)^3} + \frac{(D\tilde{\theta}_1)^2 \partial \tilde{\theta}_1 \partial \tilde{\theta}_2}{(D\tilde{\theta}_2)^3} \right] \\ &\quad - \frac{1}{5\tilde{Z}_{12}} \left[2 \frac{(D\tilde{\theta}_1)^3 \partial \tilde{\theta}_1}{(D\tilde{\theta}_2)^2} - 3 \frac{(D\tilde{\theta}_1)^3 \partial \tilde{\theta}_2}{(D\tilde{\theta}_2)^2} - 2 \frac{(D\tilde{\theta}_1)^4 \partial \tilde{\theta}_2}{(D\tilde{\theta}_2)^3} + 3 \frac{(D\tilde{\theta}_1)^2 \partial \tilde{\theta}_1}{D\tilde{\theta}_2} \right]. \end{aligned} \quad (2.63)$$

The terms proportional to $\tilde{\theta}_{12}/\tilde{Z}_{12}^2$ and $\tilde{\theta}_{12}/\tilde{Z}_{12}$ on the r.h.s. of (2.63) obviously vanish in the limit $Z_1 \longrightarrow Z_2$. On the other hand, the last term does not vanish. To see this we first note the expansions

$$\begin{aligned} (D\tilde{\theta}_1)^3 \partial \tilde{\theta}_1 &= (D\tilde{\theta}_2)^3 \partial \tilde{\theta}_2 + \tilde{Z}_{12} \tilde{\partial}_2 [(D\tilde{\theta}_2)^3 \partial \tilde{\theta}_2] + \dots \\ &= (D\tilde{\theta}_2)^3 \partial \tilde{\theta}_2 + \tilde{Z}_{12} [2\partial D\tilde{\theta}_2 \partial \tilde{\theta}_2 + D\tilde{\theta}_2 \partial^2 \tilde{\theta}_2] + \dots, \end{aligned} \quad (2.64)$$

$$(D\tilde{\theta}_1)^3 = (D\tilde{\theta}_2)^3 + 3\tilde{Z}_{12} \partial D\tilde{\theta}_2 + \dots, \quad (2.65)$$

$$(D\tilde{\theta}_1)^4 = (D\tilde{\theta}_2)^4 + 4\tilde{Z}_{12} (D\tilde{\theta}_2) \partial D\tilde{\theta}_2 + \dots, \quad (2.66)$$

$$(D\tilde{\theta}_1)^2 \partial \tilde{\theta}_1 = (D\tilde{\theta}_2)^2 \partial \tilde{\theta}_2 + \tilde{Z}_{12} \left[\frac{\partial D\tilde{\theta}_2 \partial \tilde{\theta}_2}{D\tilde{\theta}_2} + \partial^2 \tilde{\theta}_2 \right] + \dots \quad (2.67)$$

Substituting these expansions into the last term of (2.63), we are finally left with a finite term,

$$-S(\tilde{Z}_2, Z_2) = - \left(\frac{D^4 \tilde{\theta}_2}{D\tilde{\theta}_2} - 2 \frac{D^2 \tilde{\theta}_2 D^3 \tilde{\theta}_2}{(D\tilde{\theta}_2)^2} \right), \quad (2.68)$$

in the limit $Z_1 \longrightarrow Z_2$, where $S(\tilde{Z}_2, Z_2)$ is nothing but the super Schwarzian derivative. This shows that R_{gh} defined by (2.29) to be one of superprojective connections.

3. The $N = 2$ Superstring

The chiral $N = 2$ superstring field $X^\mu(Z)$ is a function of the superspace variable $Z = (z, \theta, \bar{\theta})$.^{10),11)} In terms of component fields we get the expansion

$$X^\mu(Z) = x^\mu(z) + \theta\bar{\psi}^\mu(z) + \bar{\theta}\psi^\mu(z) + \frac{i}{2}\theta\bar{\theta}g^\mu(z), \quad (\mu = 0, 1, 2, \dots, d-1). \quad (3.1)$$

Let us introduce an infinitesimal superdistance dZ by

$$dZ = dz + \frac{1}{2}(\bar{\theta}d\theta + \theta d\bar{\theta}). \quad (3.2)$$

A super analytic transformation $Z \longrightarrow \tilde{Z}$ transforms dZ according to

$$d\tilde{Z} = (D\bar{\theta})(\bar{D}\bar{\theta})dZ, \quad (3.3)$$

where D and $\bar{D}$ are covariant derivatives defined by

$$D = \frac{\partial}{\partial\theta} + \frac{1}{2}\bar{\theta}\frac{\partial}{\partial z}, \quad \bar{D} = \frac{\partial}{\partial\bar{\theta}} + \frac{1}{2}\theta\frac{\partial}{\partial z} \quad (3.4)$$

with properties

$$D^2 = \bar{D}^2 = 0, \quad (3.5)$$

$$\{D, \bar{D}\} = \frac{\partial}{\partial z}, \quad (3.6)$$

$$D = (D\bar{\theta})\bar{D}, \quad \bar{D} = (\bar{D}\bar{\theta})\tilde{D}. \quad (3.7)$$

We also introduce a super line element squared $ds^2 = |eZ|^2$, where e is a super-einbein. We require that the one-form edZ is invariant under the transformation $Z \longrightarrow \tilde{Z}$. This yields a transformation rule of e

$$e = (D\bar{\theta})(\bar{D}\bar{\theta})\tilde{e}. \quad (3.8)$$

From this we find that superaffine connections $\Gamma = De/e$ and $\bar{\Gamma} = \bar{D}e/e$ are subject to transformation rules

$$\Gamma = (D\bar{\theta})\bar{\Gamma} + \frac{\partial\bar{\theta}}{\partial\theta}, \quad \bar{\Gamma} = (\bar{D}\bar{\theta})\tilde{\Gamma} + \frac{\partial\bar{\theta}}{\partial\bar{\theta}}. \quad (3.9)$$

The primary superconformal field Φ_{pq} is characterized by a pair of weights (p, q) and by its parity σ with respect to interchange $\theta \longleftrightarrow \bar{\theta}$. We denote this

characteristic by $(p, q)^\sigma$. Here we give a list of the assignment of weights and parity of primary superconformal fields with which we will concern in what follows.

	Fields	$(p, q)^\sigma$
Superstring	X^μ	$(0, 0)^+$
Superdilaton	Φ	$(0, 0)^+$
Superantighost	B	$(1, 1)^-$
Superghost	C	$(-1, -1)^+$

As for the σ -parity for B and C , we consider the super BRST charge

$$Q_B = \frac{1}{2\pi i} \oint dz d\theta d\bar{\theta} Q(Z). \quad (3.11)$$

Since the volume element $d\theta d\bar{\theta}$ changes its sign under an interchange $\theta \longleftrightarrow \bar{\theta}$, the BRST current $Q(Z)$, which will be explicitly defined by (3.24) below, should have a negative σ -parity. Based on this observation, it is possible to find proper assignments of the σ -parity as in table (3.10). Namely, we have assigned the σ -parity of B to be negative and that of C to be positive.

The transformation rule of Φ_{pq} is given by

$$\Phi_{pq}(Z) = (\bar{D}\bar{\theta})^p (D\bar{\theta})^q \tilde{\Phi}_{pq}(\tilde{Z}). \quad (3.12)$$

Two kinds of supercovariant derivatives $D - p\Gamma$ and $\bar{D} - q\bar{\Gamma}$ are necessary to give the following transformation rules:

$$(D - p\Gamma)\tilde{\Phi}_{pq} = (\bar{D}\bar{\theta})^p (D\bar{\theta})^{q+1} (\bar{D} - p\bar{\Gamma})\tilde{\tilde{\Phi}}_{pq}, \quad (3.13)$$

$$(\bar{D} - p\bar{\Gamma})\tilde{\Phi}_{pq} = (\bar{D}\bar{\theta})^{p+1} (D\bar{\theta})^q (\bar{D} - q\bar{\Gamma})\tilde{\tilde{\Phi}}_{pq}. \quad (3.14)$$

Fundamental contractions on an arbitrary higher-genus super Riemann surface M_2 are given by

$$< X^\mu(Z_1) X^\nu(Z_2) > = \eta^{\mu\nu} [-2\ln Z_{12} + f(Z_1, Z_2)] \quad (3.15)$$

$$\langle \Phi(Z_1)\Phi(Z_2) \rangle = -2\ln Z_{12} + f(Z_1, Z_2), \quad (3.16)$$

$$\langle C(Z_1)B(Z_2) \rangle = \frac{\bar{\theta}_{1,2}\theta_{1,2}}{Z_{12}} + l(Z_1, Z_2), \quad (3.17)$$

$$\langle B(Z_1)C(Z_2) \rangle = \frac{\bar{\theta}_{1,2}\theta_{1,2}}{Z_{12}} - l(Z_2, Z_1), \quad (3.18)$$

where

$$Z_{12} = z_1 - z_2 - \frac{1}{2}(\theta_1\bar{\theta}_2 + \bar{\theta}_1\theta_2), \quad (3.19)$$

$$\theta_{12} = \theta_1 - \theta_2, \quad \bar{\theta}_{12} = \bar{\theta}_1 - \bar{\theta}_2, \quad (3.20)$$

f and l are holomorphic except for P_+ , P_- which are arbitrary fixed points on M_2 .

We then consider following four generators:

$$G = B, \quad (3.21)$$

$$J = CB + l, \quad (3.22)$$

$$T = T_m + T_{gh} - \Omega, \quad (3.23)$$

$$Q = Q^{(0)} + Q^{(1)}, \quad (3.24)$$

where

$$T_m = -\bar{D}X \cdot DX - \bar{D}\Phi D\Phi + \alpha(D + \Gamma)\bar{D}\Phi - \alpha(\bar{D} + \bar{\Gamma})D\Phi, \quad (3.25)$$

$$T_{gh} = \partial(CB) - DC\bar{D}B - \bar{D}CDB, \quad (3.26)$$

$$Q^{(0)} = C(T_m + \frac{1}{2}T_{gh} - \Omega), \quad (3.27)$$

$$Q^{(1)} = \frac{1}{2}[D(CB\bar{D}C) + \bar{D}(CBDC)] - D[(\bar{D})C] - \bar{D}[(D)C]. \quad (3.28)$$

Here l has been defined by $l = l(Z) = l(Z, Z)$ and is a scalar of the type $(0, 0)^-$. Ω is a c -number tensor of the type $(1, 1)^-$ which is explicitly given by

$$\begin{aligned} \Omega = & -\frac{\hat{c}}{2}[\bar{D}_1 D_2 f(Z_1, Z_2) + \bar{D}_2 D_1 f(Z_2, Z_1) - R]_{Z_1=Z_2=Z} \\ & - [\partial l(Z) + \bar{D}_1 D_2 l(Z_1, Z_2) + D_1 \bar{D}_2 l(Z_1, Z_2) + R]_{Z_1=Z_2=Z}, \end{aligned} \quad (3.29)$$

where $\hat{c} = d + 1$ is the dimensions of matter fields (d for X^μ and 1 for Φ), and R

is an $N = 2$ superprojective connection,

$$R = \bar{D}\Gamma - D\bar{\Gamma} + \bar{\Gamma}\Gamma, \quad (3.30)$$

which is subject to a transformation rule

$$R(Z) = (D\bar{\theta})(\bar{D}\bar{\theta})\bar{R}(\bar{Z}) + S(\bar{Z}, Z) \quad (3.31)$$

with the $N = 2$ super Schwarzian derivative,¹¹⁾

$$S(\bar{Z}, Z) = -\frac{\partial\bar{\theta}\partial\bar{\theta}}{\bar{D}\bar{\theta}\bar{D}\bar{\theta}} - \frac{\partial D\bar{\theta}}{\bar{D}\bar{\theta}} + \frac{\partial D\bar{\theta}}{\bar{D}\bar{\theta}}. \quad (3.32)$$

Note that the above four generators have all negative σ -parity. A similar algebra has been considered by Gomis and Suzuki¹⁴⁾ for genus-zero case, but they did not take account of this σ -parity of T_m ; their superdilatation term has positive σ -parity.

In quite the same way as the $N = 1$ case, by using the fundamental contractions (3.15)~(3.18) and applying Wicks theorem we can show that the above four generators satisfy the $N = 2$ supertopological algebra,

$$T_1 T_2 \sim (\Delta_0 + \Delta_1 + \Delta_2) T_2^2 \quad (3.33a)$$

$$T_1 Q_2 \sim \Delta_1 Q_2 \quad (3.33b)$$

$$T_1 G_2 \sim (\Delta_0 + \Delta_1 + \Delta_2) G_2 \quad (3.33c)$$

$$T_1 J_2 \sim (\Delta_1 + \Delta_2) J_2 \quad (3.33d)$$

$$Q_1 G_2 \sim \Delta_1 J_2 + \delta T_2^2 \quad (3.33e)$$

$$Q_1 J_2 \sim -\delta Q_2 \quad (3.33f)$$

$$J_1 G_2 \sim -\delta G_2 \quad (3.33g)$$

$$J_1 J_2 \sim 0 \quad (3.33h)$$

$$Q_1 Q_2 \sim 0 \quad (3.33i)$$

$$G_1 G_2 \sim 0 \quad (3.33j)$$

with a constraint

$$\hat{c} = 2(1 - \alpha^2). \quad (3.34)$$

Here we have set, for brevity,

$$\delta = \bar{\theta}_{12}\theta_{12}/Z_{12} \quad (3.35)$$

$$\Delta_0 = \bar{\theta}_{12}\theta_{12}/Z_{12}^2 \quad (3.36)$$

$$\Delta_1 = (\bar{\theta}_{12}/Z_{12})\bar{D}_2 - (\theta_{12}/Z_{12})D_2 \quad (3.37)$$

$$\Delta_2 = (\bar{\theta}_{12}\theta_{12}/Z_{12})\partial_2 \quad (3.38)$$

and by $A \sim B$ we mean that A is equal to B except for finite terms.

Contrary to the $N = 1$ algebra, there appear no superdilatation terms in J and $Q^{(1)}$, and, moreover, the parameter α can be chosen to be zero. This means that the $N = 2$ algebra is valid without the superdilatation field. In this case the dimensions of X^μ is just $\hat{c} = d = 2$. This coincides with the result obtained by Gomis and Suzuki¹⁴⁾ in genus-zero case. When we have the superdilatation in T , the constraint (3.34) implies $\hat{c} = d + 1 \leq 2$, namely, $\hat{c} = 1$ or $\hat{c} = 2$. The $\hat{c} = 1$ case is realized by the superdilatation only without the superstring X^μ , and the $\hat{c} = 2$ case is realized by the superstring in dimension one plus the superdilatation.

From (3.33e) we obtain the formula

$$T(Z) = [Q_B, G(Z)], \quad (3.39)$$

where Q_B is the super BRST charge, namely, the stress-energy tensor T is BRST-trivial. We also see from (3.33a) and (3.33i) that the total central charge is zero. These features are characteristic of topological theories.

4. Concluding remarks

On arbitrary higher-genus super Riemann surfaces we have shown that the $N = 1$ and $N = 2$ superstrings realize supertopological algebras which are superfield extensions of the twisted $N = 2$ superconformal algebra. In the $N = 1$ case the superdilatation field is necessary in order to validate the algebra, whereas in the $N = 2$ case the superdilatation is not always necessary for the algebra. The $N = 1$ supertopological algebra is realized by the superstring in dimension $d = 1$ plus the free superdilatation field, or by the superdilatation only without the superstring. This conclusion is different from that of other similar works.^{7),9)} Fujikawa and Suzuki⁷⁾ obtained the result of $d = \pm\infty$, where there is no $D(D - \Gamma)(D - 2\Gamma)C$ term in $Q(Z)$. Bershadsky et al.⁹⁾ discussed that any superstring theory in any dimension has the $N = 3$ twisted supersymmetry. However, their approach is not the full superspace formulation. Their conclusion about dimensionality has been derived by introducing the Coulomb gas model of the matter fields. On the other hand, we have not introduced such a model.

The $N = 2$ supertopological algebra is also realized by three models: (i) the superstring in two dimensions, (ii) the superstring in one dimension plus the superdilatation, (iii) the superdilatation only without the superstring.

The conclusion drawn from (i) is the same as the one obtained by Gomis and Suzuki¹⁴⁾ which corresponds to ours in genus-zero case. On the other hand, the other two models yield different results from ours, since the σ -parity of T_m has not been considered seriously.

From the mathematical point of view we have found some new results: (i) the $N = 2$ superprojective connection (3.30) expressed by the superaffine connection Γ , (ii) the transformation rules (3.13) and (3.14) for supercovariant derivatives $D - p\Gamma$ and $\bar{D} - q\bar{\Gamma}$ in the $N = 2$ case.

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