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ISOSPIN VIOLATION AND HYPERON SEMILEPTONIC DECAYS

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Abstract Isospin violation induces a little mixing between the Λ and Σ^0 baryons, and this in turn leads to small changes in matrix elements for weak currents involving these particles. The corrections are small but not negligible. The relevant formulae to first order in the mixing are listed in this note. The largest effect is in the ratio of rates of semileptonic decay of Σ^+ and Σ^- (to Λ) which changes by about 6%. A modest improvement in the measurement of the branching ratio of $\Sigma^+ \rightarrow \Lambda e \nu$ would allow the experimental determination of the mixing angle. The SU_3 fitting of axial to vector ratios in the octet is also affected.

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Hyperon semileptonic decay data are slowly becoming rather accurate [1]. There is much interest in extracting baryon axial amplitudes from this data, which are then combined with other information, to extract (assuming SU_3 symmetry) axial amplitudes in the proton [1,2]. The experimental information consists of decay rates (life time + branching ratios) and decay correlations. The data are subjected to a host of corrections, such as radiative corrections, form factors, 'small' couplings (other than vector and axial), Coulomb corrections, etc., to extract the interesting axial amplitudes. Ref. [3] gives a detailed review of the state of the art. Isospin violation in the matrix elements appears to have been ignored in this context. I find that the effects of isospin violation are small but not negligible at the present level of accuracy. A modest improvement in the accuracy of the data will allow the confirmation of the isospin violation and give an independent determination of the relevant isospin violating parameter $(m_d - m_u)/(m_s - \hat{m})$. The SU_3 analysis of amplitudes within the octet is also affected.

(i) $(\Lambda-\Sigma)$ mixing. The mass difference between up and down quarks and electromagnetic interactions are the agents of isospin violation in the standard model. The violation accounts for 'electromagnetic' mass differences between various members of the same isospin multiplet. Aside from these mass differences, isospin violation is also responsible [4,5,6] for the small mixing (ϕ) between the isoscalar state (here denoted by Λ_8) and the neutral component of the isotriplet (Σ^0), to form the physical mass eigenstates Λ , Σ^0 :

$$\Lambda = \Lambda_8 \cos \phi + \Sigma_8^0 \sin \phi \quad (1a)$$

$$\Sigma^0 = -\Lambda_8 \sin \phi + \Sigma_8^0 \cos \phi$$

The mixing angle is determined by quark mass differences and is given by [5,6]

$$\sin \phi \approx \phi = -\frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}} \quad (1b)$$

where $\hat{m}$ is the average mass of the u,d quarks. The same formula holds both in chiral perturbation theory [6] and in the naïve quark model (NQM) [5] with hyperfine interactions [9]. Note that the convention of sign in (1a) differs from reference [6], which accounts for the minus sign in (1b). In SU_3 the relative phase between Λ , Σ^0 is fixed, while in the NQM one can choose this phase, but then one should compute both the mixing and decay amplitudes, so that the choice becomes immaterial. The sign is important here since we shall deal with interferences. The numerical value of ϕ is about -0.015; this value is obtained for example by using the baryon masses of ref [1a] in the formula of ref [4], or values from ref [6] in eqn. (1b).

The analogous effect in pseudoscalar mesons (π^0 η mixing) has the same mixing angle ϕ , and is responsible for decays like $\psi' \rightarrow \psi \pi^0$ which have been observed. Recently it has been noted [7] that the angle ϕ for pseudoscalars, depends on q^2 . The same should be true for $\Lambda - \Sigma^0$ but I shall ignore this complication, since in semileptonic decays the range of q^2 is small.

(ii) Effects on weak amplitudes: Assume that SU_3 symmetry is good, and all SU_3

relations between amplitudes continue to hold, but there is a small mixing (1) between the SU_3 eigenstates Λ_8, Σ_8^0 . This mixing will then affect amplitudes of processes involving a Λ or Σ^0 in the initial or final state [8]. To find the corrected amplitude for a process involving a physical Λ state, equation (1) is used together with the SU_3 relations for the amplitudes of Λ_8, Σ_8^0 for the same process. This will first be illustrated for the decay $\Sigma^- \rightarrow \Lambda e \bar{\nu}$.

In the absence of isospin breaking the hadronic amplitudes for the Σ^- decay into Λ_8, Σ_8^0 have the form:

$$\Sigma^- \rightarrow \Lambda_8 e \bar{\nu} : \quad A_\Lambda = \sqrt{\frac{2}{3}} D \gamma_\mu \gamma_5 \cos \theta \quad (2)$$

$$\Sigma^- \rightarrow \Sigma_8^0 e \bar{\nu} : \quad A_\Sigma = \sqrt{2} [\gamma_\mu + F \gamma_\mu \gamma_5] \cos \theta$$

In these equations θ is the Cabibbo angle, F and D are axial SU_3 couplings; the vector SU_3 coupling $F_V \equiv 1$, $D_V \equiv 0$. Form factors are replaced by unity. For the physical Λ state we take the linear combination given in (1)

$$\Sigma^- \rightarrow \Lambda e \bar{\nu} \quad A = \left\{ \sqrt{2} \gamma_\mu \sin \phi + \left(\sqrt{\frac{2}{3}} D \cos \phi + \sqrt{2} F \sin \phi \right) \gamma_\mu \gamma_5 \right\} \cos \theta \quad (3)$$

The vector coupling vanishes for $\phi = 0$, as it follows from CVC [10], but to first order in ϕ :

$$(g_V/g_A)_{\Sigma^- \rightarrow \Lambda} = \frac{\sqrt{3}}{D} \tan \phi = \frac{\sqrt{3}}{D} \phi \quad (4)$$

In the naïve quark model $D = 1$, and equation (4) can be verified directly [8]. With the choice of $\phi \simeq -0.015$ and $D = 0.8$ [3] we find

$$(g_V/g_A)_{\Sigma^- \rightarrow \Lambda} = -0.03 \quad (5)$$

The estimate (5) is within one standard deviation of the best experimental determination available

$$(g_V/g_A)_{\Sigma^- \rightarrow \Lambda} = +0.034 \pm 0.08 \quad (6)$$

from the SPS charged hyperon beam [11]. There is a discrepancy in sign between (5), (6); both follow the same convention of (g_A/g_V) positive in neutron beta decay. I shall comment further on the discrepancy in sign between (5) and (6), but I should stress at the outset that the discrepancy is not significant statistically.

Aside from the vector coupling (4) the mixing (1) also changes the size of the axial amplitude which in turn affects the decay rate of $\Sigma^- \rightarrow \Lambda e \bar{\nu}$. The small vector coupling only affects the rate in second order in ϕ , which we neglect. The decay rate corrected to first order in ϕ is

$$\Gamma(\Sigma^- \rightarrow \Lambda e \bar{\nu}) = \Gamma_0^{(-)} \left(1 + \frac{2\sqrt{3}F}{D} \phi \right) \quad (7)$$

where $\Gamma_0^{(-)}$ is the value of the rate for $\phi = 0$. Since ϕ is negative and F, D are positive, this is a diminution of the rate by about 3%, which helps improve the SU_3 fit to this rate

(according to reference [3], where there is an 8% shortfall in an SU_3 fit).

For the positively charged Σ^+ decay the coupling to Σ_8^0 changes sign (because the Σ is isovector) while coupling to Λ_8 is unchanged, so that

$$(g_V/g_A)_{\Sigma^+ \rightarrow \Lambda} = +0.03 \quad (8)$$

The correction to the rate changes sign as well compared to (5), so that the ratio of the two decay rates has a 6% correction due to isospin breaking

$$R = \frac{\Gamma(\Sigma^+ \rightarrow \Lambda \bar{e} \nu)}{\Gamma(\Sigma^- \rightarrow \Lambda e \bar{\nu})} = \left[\frac{m(\Sigma^+) - m(\Lambda)}{m(\Sigma^-) - m(\Lambda)} \right]^2 \left(1 + 3 \frac{m(\Sigma^-) - m(\Sigma^+)}{m(\Sigma) + m(\Lambda)} \right) \left(1 - \frac{4\sqrt{3}F}{D} \phi \right) = 0.65 \quad (9)$$

where for $\phi = 0$ the ratio of the rates is taken from reference [1b]. The ratio (9) is in slightly better agreement with the ratio of experimental rates 0.645 ± 0.18 but this "improvement" is also of little statistical significance since the semileptonic decay rate of Σ^+ is based on a total sample of 21 events [1a].

Table I gives the matrix elements for all weak currents in the octet affected by the mixing (1), to first order in the angle ϕ . I have omitted $\Sigma^- \rightarrow \Sigma^0 e \bar{\nu}$ which is unlikely to be measured. Table II lists the relevant formulae for the axial-to-vector ratios corrected to first order in the mixing angle ϕ . These are obtained from the corresponding formulae in Table I. Table II also contains, for convenience, the formulae for the decays $n \rightarrow p \bar{e} \nu$ and $\Sigma^- \rightarrow n \bar{e} \nu$ which are not affected by the mixing (1).

(iii) Data fitting: Given the formulae of Tables I and II one should now refit all the available data on hyperon semileptonic decays with these formulae, as done for example

in reference [3]. This ambitious program is not attempted here. I shall only fit the subset of data on axial-to vector ratios, with the formulae of Table II. There are two possible ways of fitting the data: we can use the theoretical value of $\phi \simeq -0.015$ and ignore the measurement of the vector coupling in $\Sigma^- \rightarrow \Lambda e \nu$ reported by reference [11]. The other possibility is to include this measurement and consider the angle ϕ as a parameter just like F and D. But as I already mentioned the determination of the mixing angle ϕ through the vector coupling in Σ^- semileptonic decay, eqn. (6), or through the ratio of Σ^+/Σ^- decay rates eqn. (9) is not statistically significant. I shall take the theoretical value of ϕ and fit the data with it. The resulting best chi-squared fit is given in Table III. The parameters F,D are very near those found from the same data by Close and Roberts [2]. Their best value for F/D is 0.575 as compared with 0.573 in Table III. As the accuracy of the data will improve the parameter ϕ should also be determined experimentally together with F and D.

(iv) Comments: From the viewpoint of the mixing effects discussed here the quality of the present data is just insufficient to determine the mixing angle ϕ . A modest improvement in the branching ratio of $\Sigma^+ \rightarrow \Lambda \bar{e} \nu$ would allow the unambiguous confirmation of equation (7) and with it the mixing angle. A sample size of about one thousand semileptonic decays should be sufficient for this purpose, if the estimate of $\phi \sim -0.015$ is correct. This would measure the isospin mixing ϕ directly, and give a new determination of $(m_d - m_u)/m_s$ in the baryon octet.

(v) Acknowledgements: I would like to thank Professor V. Gupta for discussions on

semileptonic decays. This work was started while I was on leave at the Ludwig-Maximilians-Universität in Munich, and I wish to thank the Alexander von Humboldt Stiftung for their support and Professor Harald Fritzsch for this kind hospitality. I thank very warmly Dr. N. Chan and Guelph General Hospital, for their vital help which allowed this note to be completed.

References

- 1) For reviews see: (a) Physical Review D45, June 1st (1992)
(b) Jean-Marc Gaillard and Gilles Sauvage, Ann. Rev. Nucl. Part. Science 34, 351 (1984)
(c) M. Roos, Phys. Lett. 246B, 179 (1990)
(d) R.L. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990)
- 2) F.E. Close, RAL-93-034 (Rutherford Lab Preprint, June 1993)
F.E. Close and R. Roberts, Phys. Letters 316B, 165 (1993)
- 3) P.M. Gensini and G. Violini, in Proceedings of the 5th International Symposium on Meson Nucleon Physics and the Structure of the Nucleon, volume II, p. 80 [π N-Newsletter No 9, December 1993, ISSN 0942-4148; Editors G. Höhler, W. Kluge and B.M.K. Nefkens]
- 4) R.H. Dalitz and F. von Hippel, Phys. Lett. 10, 153 (1964)
A.J. MacFarlane and E.C.G. Sudarshan, Nuovo Cimento 31, 1176 (1964)
- 5) Nathan Isgur, Physical Review D21, 779 (1980)

- 6) J. Gasser and H. Leutwyler, Phys. Reports 87, 77 (1982)
see also John F. Donoghue, Ann. Revs. of Nucl. and Particle Sci. vol. 39 (1989)
- 7) K. Maltman, Phys. Letts. B313, 203 (1993)
- 8) P. Bracken, A. Frenkel and G. Karl, Phys. Rev. D24, 2984 (1981)
- 9) A. de Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12, 147 (1975)
- 10) N. Cabibbo and R. Gatto, Nuovo Cimento 15, 159 (1960)
V.P. Belov, B.S. Mingalev and V.M. Shekhter, Sov. Phys. J.E.T.P. 11, 392
(1960)
- 11) M. Bourquin et al, Zeitschrift fur Physik C12, 307 (1982)

TABLE I

HADRONIC WEAK CURRENTS TO FIRST ORDER IN $\Lambda - \Sigma$ MIXING:

<u>PROCESS</u>	<u>HADRONIC AMPLITUDE</u>
$\Sigma^- \rightarrow \Lambda e \bar{\nu}$	$\left\{ \sqrt{2} \phi \gamma_\mu + \left(\sqrt{\frac{2}{3}} D + \sqrt{2} F \phi \right) \gamma_\mu \gamma_5 \right\} \cos \theta$
$\Sigma^+ \rightarrow \Lambda \bar{e} \nu$	$\left\{ -\sqrt{2} \phi \gamma_\mu + \left(\sqrt{\frac{2}{3}} D - \sqrt{2} F \phi \right) \gamma_\mu \gamma_5 \right\} \cos \theta$
$\Lambda \rightarrow p e \bar{\nu}$	$\left[-\sqrt{\frac{3}{2}} \left(1 + \frac{\phi}{\sqrt{3}} \right) \right] \left[\gamma_\mu + \left(F + \frac{D}{3} - \frac{4D\phi}{3\sqrt{3}} \right) \gamma_\mu \gamma_5 \right] \sin \theta$
$\Xi^- \rightarrow \Lambda e \bar{\nu}$	$\left[\sqrt{\frac{3}{2}} \left(1 - \frac{\phi}{3} \right) \right] \left[\gamma_\mu + \left(F - \frac{D}{3} + \frac{2D\phi}{3\sqrt{3}} \right) \gamma_\mu \gamma_5 \right] \sin \theta$
$\Xi^- \rightarrow \Sigma^0 e \bar{\nu}$	$\left[\frac{1}{\sqrt{2}} (1 - \sqrt{3} \phi) \right] \left[\gamma_\mu + \left(F + D + \frac{4\sqrt{3}D\phi}{3} \right) \gamma_\mu \gamma_5 \right] \sin \theta$

Notes: θ is the Cabibbo Angle, and ϕ the $\Sigma\Lambda$ mixing angle.

TABLE II

AXIAL TO VECTOR RATIOS TO FIRST ORDER IN Λ - Σ MIXING

<u>PROCESS</u>	<u>QTY</u>	<u>FORMULA</u>
$\Sigma^- \rightarrow \Lambda e \bar{\nu}$	g_V/g_A	$\sqrt{3} \phi/D$
$\Sigma^+ \rightarrow \Lambda e \bar{\nu}$	g_V/g_A	$-\sqrt{3} \phi/D$
$\Lambda \rightarrow p e \bar{\nu}$	g_A/g_V	$F + \frac{D}{3} - \frac{4D\phi}{3\sqrt{3}}$
$\Xi^- \rightarrow \Lambda e \bar{\nu}$	g_A/g_V	$F - \frac{D}{3} + \frac{2D\phi}{3\sqrt{3}}$
$\Xi^- \rightarrow \Sigma^0 e \bar{\nu}$	g_A/g_V	$F + D + \frac{4\sqrt{3}D\phi}{3}$
$n \rightarrow p e \bar{\nu}$	g_A/g_V	$F + D$
$\Sigma^- \rightarrow n e \bar{\nu}$	g_A/g_V	$F - D$

TABLE III

FIT TO (g_A/g_V) DATA, WITH ISOSPIN VIOLATION ($\phi = -0.015$)

<u>DECAY</u>	<u>FORMULA</u>	<u>FIT</u>	<u>DATA (Ref 1a)</u>
$n \rightarrow pe\bar{\nu}$	$F + D$	1.257	1.257 ± 0.0028
$\Sigma^- \rightarrow ne\bar{\nu}$	$F - D$	-0.341	-0.340 ± 0.017
$\Lambda \rightarrow pe\bar{\nu}$	$F + 0.344 D$	0.733	0.718 ± 0.015
$\Xi^- \rightarrow \Lambda e\bar{\nu}$	$F - 0.339 D$	0.187	0.25 ± 0.05

$$F = 0.458 \pm 0.008$$

$$D = 0.799 \pm 0.008$$

$$F/D = 0.573 \pm 0.016$$