KEK P 93-188
SW 9412

KEK Preprint 93-188
January 1994
A

CERN LIBRARIES, GENEVA

P00021919

**Effect of the Wire on the Electromagnetic
Field in an Accelerating Cavity in the
Coaxial-Wire Method**

T. TOYOMASU, M. IZAWA and Y. KAMIYA

Submitted to Part. Accel.

very large value compared to the metallic wire case. After the same procedures as those given in Section 2, we obtain

$$\frac{\delta\omega}{\omega_{0mp}} \simeq \frac{1}{\{1 + (p\pi/u_{0m})^2/f_L^2\}} \frac{N_0(u_{0m})}{u_{0m} J_1(u_{0m})} \frac{\pi(1 - 1/\epsilon_r)}{2(\log - 2/\epsilon_r v_{0m}^2)}. \quad (27)$$

Though the frequency shift in the metallic wire case is positive and rather large, these frequency shifts are negative and are very small in our case (order of $10^{-3} \sim 10^{-4}$).

However, the Q -values change due to the large permittivity of the wire. Indeed, using Eq. (21) and after carrying out the same procedures as given in Section 2 for the Q -values, we obtain

$$\frac{Q}{Q_0} \simeq \sqrt{\frac{\omega}{\omega_{0mp}}} \frac{1 + k_0(u_{0m}, v_{0m})}{1 + \sqrt{2\sigma/\omega\epsilon_W} h_0(u_{0m}, v_{0m}) f_W}, \quad (28)$$

with

$$k_0(u_{0m}, v_{0m}) \equiv \left(\frac{2}{\pi u_{0m} J_1(u_{0m}) \Delta_0(v_{0m})} \right)^2 (2 \log - 1), \quad (29)$$

$$h_0(u_{0m}, v_{0m}) \equiv \left\{ \frac{1 - 1/\epsilon_r}{J_1(u_{0m}) v_{0m} (2/\epsilon_r v_{0m}^2 - \log)} \right\}^2 \frac{f_L(1 + \delta_{0p})}{2 + f_L(1 + \delta_{0p})}, \quad (30)$$

where ϵ_W is the permittivity of the wire. In Eq. (28) we may put $\omega \simeq \omega_{0mp}$ and ignore $k_0(u_{0m}, v_{0m})$ due to the large value of $\Delta_0(v_{0m})$. In Eq. (30) we may also ignore $1/\epsilon_r$ in the numerator and the log in the denominator compared to the other terms.

In our cases, this effect leads to a decrease in the Q -values. We should notice here that, if we consider only the Bessel component as a good approximation of the fields and if we do not assume the Neumann component for the fields, the decrease in the Q -values is negligibly small.

Acknowledgement The authors would like to thank Prof. Y. Kimura and Prof. H. Kobayakawa for many encouragements. The authors also thank Dr. T. Koseki for helpful discussions and his advice from the experimental side.

References

- [1] F. Caspers, *Frontiers of Particle Beams: Intensity Limitations* (Lecture Notes in Physics 400, Springer-Verlag, Berlin, 1992), p.80
- [2] J.A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), Chapter VI, IX.
- [3] J.C. Slater, *Microwave Electronics* (D. Van Nostrand, New York, 1950).
- [4] Handbook of Mathematical Functions, Dover, (1970).
- [5] L.C. Maier, Jr. and J.C. Slater, Journal of Applied Physics **23** No.1, (1952) 68-77.
- [6] H. Hahn and H.J. Halma, Review of Scientific Instruments **36** No.12, (1965) 1788-1796.
- [7] M. Sands and J. Rees, PEP-95 (1974).
- [8] M. Izawa, T. Kiuchi, S. Tokumoto, Y. Hori, S. Sakanaka, M. Kobayashi, and H. Kobayakawa, Review of Scientific Instruments **63** No.1, (1992) 363-366.
- [9] S.M. Hanna and P.M. Stefan, PAC-93 (1993).
- [10] M. Bartsch, M. Dehler, M. Dohlus, P. Hahne, R. Klatt, F. Krawczyk, Z. Min, T. Propper, P. Schutt, T. Weiland, S. Wipf, M. Marx, F. Ebeling and B. Steffen, Proc. 1990 Linac Conf., Albuquerque, ed. S.O. Schriber, p.372 (1990).

National Laboratory for High Energy Physics, 1994

KEK Reports are available from:

Technical Information & Library
National Laboratory for High Energy Physics
1-1 Oho, Tsukuba-shi
Ibaraki-ken, 305
JAPAN

Phone: 0298-64-1171
Telex: 3652-534 (Domestic)
(0)3652-534 (International)
Fax: 0298-64-4604
Cable: KEK OHO
E-mail: LIBRARY@JPNKEKVX (Bitnet Address)
library@kekvax.kek.jp (Internet Address)

Effect of the Wire on the Electromagnetic Field in an Accelerating Cavity in the Coaxial-Wire Method

T. Toyomasu , M. Izawa and Y. Kamiya *

KEK

1-1 Oho, Tsukuba, Ibaraki 305, Japan,

January 19, 1994

Contents

1	Introduction	3
2	Pill-box model analysis	4
3	Discussion	14

Abstract

The wire used in the coaxial-wire method to measure the characteristics of an accelerating cavity cannot be treated as a perturbation. Using a pill-box model, we analytically studied the electromagnetic field of a cavity with a wire. By this analysis, the effect of the wire on the resonance frequencies and Q -values of the cavity modes was clarified.

*The Institute for Solid State Physics, The University of Tokyo

1 Introduction

The coaxial-wire method[1] proposed by Sands and Rees[7] is a well-known technique for measuring the loss parameters of accelerating cavities and beam-duct components. It gives satisfactory results in most cases. On the other hand, we can find the field strengths of the resonant modes of cavities using the perturbation method proposed by Maier and Slater[5]. Recently, it was also reported that the resonance characteristics can be measured by a coaxial-wire method [8,9].

Theoretically, the wire should be treated as a disconnected region within a cavity. It is clear that the electromagnetic fields are different from those of a cavity without the wire (wire-less case), and cannot be constructed with ordinal functions of a wire-less case, since the physical system is different. The problem thus naturally arises whether we can discuss the properties of the resonant mode of cavities by using the coaxial-wire method. We present in this paper an analytic solution of the electromagnetic fields of a cavity with a wire using a simple pill-box model, the features of which are fairly well known. We consider how the wire changes the electromagnetic fields and we show the effect of the wire on the resonance frequency and Q-values due to a change in the electromagnetic fields.

In this analysis it is shown that the wire cannot be directly treated as a perturbation, such as a metallic needle in the perturbation methods of Slater[3, 5,6]. Especially, a difference clearly appears in the resonance frequency shift.

The contents of this paper are as follows: in the section 2 we consider the effects of a wire using a simple pill-box model; the section 3 includes discussions about the effects of radius of a wire and the dielectric wire case.

2 Pill-box model analysis

We now introduce a simple pill-box for a resonant cavity and assume that the cavity has three regions i.e a wire region (region 1), an empty region (region 2) and a wall region (region 3). For this model we denote the radius of the cavity as a , the length of the cavity as L and the radius of the wire as a_w . In cylindrical coordinates (r, θ, z) these regions are expressed by $r < a_w$, $a > r > a_w$ and $r > a$, respectively. We denote every quantity in these regions by suffices 1, 2 and 3. Then, the electromagnetic fields of the TM mode in each region becomes

$$H_{\theta 1} = j \frac{\gamma_{01}^2}{\omega \mu_1 \beta_{c1}} J'_n(\beta_{c1} r) F_n, \quad E_{z1} = J_n(\beta_{c1} r) F_n, \quad (1)$$

$$H_{\theta 2} = j \frac{\gamma_{02}^2}{\omega \mu_2 \beta_{c2}} \{A_n J'_n(\beta_{c2} r) + B_n N'_n(\beta_{c2} r)\} F_n, \quad (2)$$

$$E_{z2} = \{A_n J_n(\beta_{c2} r) + B_n N_n(\beta_{c2} r)\} F_n, \quad (2)$$

$$H_{\theta 3} = C_n j \frac{\gamma_{03}^2}{\omega \mu_3 \beta_{c3}} H_n^{(1)}(\beta_{c3} r) F_n, \quad E_{z3} = C_n H_n^{(1)}(\beta_{c3} r) F_n, \quad (3)$$

with

$$F_n = e^{jn\theta} \otimes e^{-j\theta} \otimes e^{j\omega t} \quad \text{and} \quad k_p = \frac{p\pi}{L}, \quad (4)$$

where $H_n^{(1)}$, J_n and N_n are the n-th order Hankel function of the first kind, the n-th order Bessel function and the n-th order Neumann function, respectively; the γ_0 's are the propagation constants in free space, μ 's the permeabilities, β_c 's the cut-off constants and p an integer. A_n, B_n and C_n are constants which are to be determined. Using k_p, β_c and c the speed of light, the resonance frequency ω is determined by

$$\omega = c \sqrt{\beta_c^2 + k_p^2}. \quad (5)$$

We can then connect the electromagnetic fields at each boundary (i.e. $r = a$ and $r = a_w$) using a radial impedance [2] defined by

$$Z_r = -\frac{E_z}{H_\theta}. \quad (6)$$

The connection condition leads to the following determination equation:

$$-\frac{A_n}{B_n} = \frac{N_n(v) - p P_n(x) N'_n(v)}{J_n(v) - p P_n(x) J'_n(v)}$$

$$= \frac{N_n(u) - qQ_n(y)N'_n(u)}{J_n(u) - qQ_n(y)J'_n(u)}, \quad (7)$$

where $u = \beta_{ca}a$, $v = \beta_{ca}aw$, $x = \beta_{ca}aw$ and $y = \beta_{ca}a$, respectively. p , $P_n(x)$, q and $Q_n(y)$ are defined by

$$\begin{aligned} p &= \frac{\mu_1\beta_{ca}\gamma_{02}^2}{\mu_2\beta_{ca}\gamma_{01}^2}, \quad P_n(x) = -\frac{J_n(x)}{J'_n(x)}, \\ q &= \frac{\mu_3\beta_{ca}\gamma_{02}^2}{\mu_2\beta_{ca}\gamma_{01}^2}, \quad Q_n(y) = -\frac{H_n^{(1)}(y)}{H_n^{(1)'}(y)}. \end{aligned}$$

Taking the limit of infinite conductivity for regions 1 and 3 in Eq. (7), we

obtain

$$-\frac{A_n}{B_n} = \frac{N_n(v)}{J_n(v)} = \frac{N_n(u)}{J_n(u)}. \quad (8)$$

For convenience in the following we define

$$\Delta_n(v) \equiv -\frac{N_n(v)}{J_n(v)}. \quad (9)$$

For instance the fields $H_{\theta 2}$ and E_{z2} are expressed as

$$H_{\theta 2} = j\frac{\gamma_{02}^2}{\omega\mu_2\beta_{ca}}A_n \left\{ J'_n(\beta_{ca}r) + \frac{1}{\Delta_n(v)}N'_n(\beta_{ca}r) \right\} \cos(n\theta) \cos(k_p z) e^{j\omega t}, \quad (10)$$

$$E_{z2} = A_n \left\{ J_n(\beta_{ca}r) + \frac{1}{\Delta_n(v)}N_n(\beta_{ca}r) \right\} \cos(n\theta) \cos(k_p z) e^{j\omega t}. \quad (11)$$

Here, we choose the phase such that E_{z2} is real at $t = 0$. This simply means that electromagnetic fields within the cavity under consideration are expressed by linear combinations of Bessel and Neumann functions. However, one should note that the combination depends on the radius of the wire.

When we need to know the resonance frequency in this cavity, we must solve Eq. (8) and determine β_{ca} . In the following we consider the electromagnetic fields in only region 2 and omit the suffix 2 for simplicity.

Now, the quantities to be analyzed are the frequency shifts,

$$\frac{\delta\omega}{\omega_{nmp}} \equiv \frac{\omega - \omega_{nmp}}{\omega_{nmp}}, \quad (12)$$

where ω_{nmp} is the resonance frequency of the wire-less case and is represented as the resonance frequency of a pill-box model,

$$\omega_{nmp} = c\sqrt{\left(\frac{u_{nm}}{a}\right)^2 + k_p^2}, \quad (13)$$

where u_{nm} is a m -th root of the n -th order Bessel function. Using Eq. (5) and Eq. (13), Eq. (12) becomes

$$\frac{\delta\omega}{\omega_{nmp}} = \frac{\delta u/u_{nm}}{1 + (p\pi/u_{nm})^2/f_L^2}, \quad (14)$$

where

$$f_L = \frac{L}{a} \quad \text{and} \quad u = \beta_{ca}a. \quad (15)$$

We can thus solve Eq. (8) for u under some suitable approximations, and can then obtain the frequency shifts from Eq. (14).

We now assume that the radius of the wire is very small and that the resonance frequency has approximately the same value as in the wire-less case, so that $v \ll 1$ and $u \simeq u_{nm} + \delta u$ for the first order. In calculating $\Delta_n(v)$ defined by Eq. (9), we also assume that $v \simeq v_{nm} \equiv u_{nm}f_W$, where we put

$$f_W = \frac{a_W}{a}. \quad (16)$$

Then, from Eq. (8) we obtain

$$\delta u \simeq \frac{N_n(u_{nm})J_n(v_{nm})}{J'_n(u_{nm})N'_n(v_{nm})} = \frac{1}{\Delta_n(v_{nm})J_{n+1}(u_{nm})}. \quad (17)$$

Using limiting forms of the Bessel and Neumann functions[4], we can also approximate $\Delta_n(v_{nm})$ as

$$\Delta_0(v_{0n}) \simeq \frac{2}{\pi} \log\left(\frac{2}{Cv_{0n}}\right) \quad \text{for } n = 0, \quad (18)$$

$$\Delta_n(v_{nm}) \simeq \frac{n!(n-1)!}{\pi} \left(\frac{2}{v_{nm}}\right)^{2n} \quad \text{for } n \neq 0, \quad (19)$$

where $C = 1.78107$.

Note that Eq. (19) diverges more rapidly than Eq. (18) as v becomes infinitesimally small, i.e., the wire-less limit, $f_W \rightarrow 0$. Thus in most cases in

which we are concerned, we may ignore the second term of the right-hand side of Eqs. (10) and (11) for the $n \neq 0$ order and can approximate it only by the first term; then, the frequency shifts are negligible.

On the other hand, Eq. (18) diverges only logarithmically. Equations (10) and (11) for the $n = 0$ order therefore must include both Bessel and Neumann components. In this case the frequency shifts are given by Eq. (14) together with Eqs. (17) and (18) as

$$\frac{\delta\omega}{\omega_{0mp}} = \frac{1}{\{1 + (\pi/u_{0m})^2/f_L^2\}} \frac{N_0(u_{0m})}{u_{0m}J_1(u_{0m})2\log(2/Cu_{0m}f_W)} \cdot \quad (20)$$

From Eq. (20) we can reduce the effect by decreasing the radius of the wire. However the effect of the wire still remains finite, in practice, due to the fact that the contribution of its radius is only logarithmic.

The frequency shifts as functions of the diameter of the wire are given in Figs. 1, 2 and 3, where we assume that $a = 2.3 \times 10^{-1}$ (m) and $L = 2.6 \times 10^{-1}$ (m) for a cavity of which the fundamental mode is near to 500 (MHz). We also give plots of the results of MAFIA[10] for the $m = 1, p = 0$ and $m = 1, p = 1$ modes in Figs. 1 and 2. The theoretical values agree with them quite well. We find that the frequency shifts are positive and that the wire effect is largest on the fundamental mode and becomes smaller for the higher modes. We also find that the frequency shifts are reduced by decreasing the radius of the wire but it still has an appreciable value.

In addition to the frequency shifts, we can also calculate the Q -values easily for the sake of a simple model.

As usual, the Q -value is defined by

$$\frac{1}{Q} = \frac{1}{\omega} \left(\frac{1}{2} \sqrt{\frac{\omega\mu}{2\sigma}} \int_S \mathbf{H} \cdot \bar{\mathbf{H}} dS + \frac{\mu}{2} \int_V \mathbf{H} \cdot \bar{\mathbf{H}} dV \right), \quad (21)$$

where σ is the conductivity of the cavity wall, S the area of the cavity wall and V its volume. $\bar{\mathbf{H}}$ is the complex conjugate of $\mathbf{H}$. We may assume here that since the fields at the cavity wall are not much affected by the wire, the power losses are almost the same form as for the wire-less case. We may then write

for the case of the $n = 0$ order

$$\begin{aligned} \frac{1}{Q} &\simeq \frac{1}{Q_0} \sqrt{\frac{\omega_{0mp}}{\omega}} \left(\int_V \mathbf{H}_0 \cdot \bar{\mathbf{H}}_0 dV + \int_S \mathbf{H} \cdot \bar{\mathbf{H}} dV \right) \\ &\times \left\{ \int_S \mathbf{H} \cdot \bar{\mathbf{H}} dS + \int_S \mathbf{H}_0 \cdot \bar{\mathbf{H}}_0 dS \right. \\ &\left. + \sqrt{\frac{\sigma}{\sigma_W}} \int_W \mathbf{H} \cdot \bar{\mathbf{H}} dS + \int_S \mathbf{H}_0 \cdot \bar{\mathbf{H}}_0 dS \right\}, \quad (22) \end{aligned}$$

where the suffix 0 means the wire-less case; W is the wire surface and σ_W the conductivity of the wire.

Then, by using Eqs. (10) and (22) and the same assumption as in the calculation of the frequency shifts and also using limiting forms of the Bessel and Neumann functions, the Q -value of the cavity under consideration becomes

$$\frac{Q}{Q_0} \simeq \sqrt{\frac{\omega}{\omega_{0mp}}} \frac{1 + k_0(u_{0m}, v_{0m})}{1 + \sqrt{\sigma/\sigma_W} h_0(u_{0m}, v_{0m}) f_W}, \quad (23)$$

with

$$k_0(u_{0m}, v_{0m}) \equiv \left(\frac{2}{u_{0m} J_1(u_{0m})} \right)^2 \frac{1}{\pi \Delta_0(v_{0m})} \left(1 - \frac{1}{\pi \Delta_0(v_{0m})} \right), \quad (24)$$

$$h_0(u_{0m}, v_{0m}) \equiv \left(\frac{2}{v_{0m} J_1(u_{0m}) \pi \Delta_0(v_{0m})} \right)^2 \frac{f_L(1 + \delta_{0p})}{2 + f_L(1 + \delta_{0p})}, \quad (25)$$

where δ_{0p} is Kronecker's delta and $\Delta_0(v_{0m})$ is given by Eq. (18). In Eq. (23) we have neglected the contribution from the integral of the cross terms of the Bessel and Neumann functions as well as the contribution from the Neumann function at the cavity wall.

In the cases of the $n \neq 0$ orders, the changes in the Q -value are very small since $\Delta_n(v_{nm})$ has very large value (Eq. (19)).

The Q/Q_0 as functions of the diameter of the wire are given in Fig. 4 for the $m = 1, p = 0$ and $m = 1, p = 1$ modes together with the corresponding results of MAFIA, where we assume the same values as in Fig. 1 and that the material of the wire is the same as that of the cavity; thus $\sigma = \sigma_W$.

In spite of some approximations, the theory well predicts the behavior of the Q -values calculated by MAFIA. We find here that Q/Q_0 increases together

with the radius of the wire. Particularly, we should note that the Q/Q_0 becomes smaller and does not approach unity upon decreasing the radius of the wire.

Figure 1: $\delta\omega/\omega_{0mp}$ for $m=1$ and $p=0, 1, 2, 3$. The results of MAFIA are also plotted for $m=1$ and $p=0, 1$.

Figure 2: $\delta\omega/\omega_{0mp}$ for $m = 1, 2, 3, 4$ and $p = 0$. The results of MAFIA are also plotted for $m = 1$ and $p = 0$.

Figure 3: $\delta\omega/\omega_{0mp}$ for $m = 2$ and $p = 0, 1, 2, 3$.

Figure 4: Q/Q_0 for $m = 1$ and $p = 0, 1$ and the corresponding results of MAFIA.

3 Discussion

There are some interesting themes in spite of the simple model which we treated above.

Radius of a wire. When we consider the fields for $n = 0$ in region 2 expressed by Eqs. (10) and (11), each component is regular, except for $\Delta_0(v)$. Even under the condition $v < u_0$, $\Delta_0(v)$ has a zero point; in this case the fields are represented only by the Neumann component.

The value of v_C which satisfies $\Delta_0(v_C) = 0$ is given by the root of $N_0(v_C) = 0$ or $v_C \simeq 0.894$. Practically, this condition does not appear for real cavities, since the radius of the wire becomes very large in this case. Though the zero point does not appear in our case since v which we consider is much smaller than v_C , it will be important when f_w becomes large.

Dielectric wire case. In the perturbation method[3,5,6] a dielectric wire is usually used to move perturbators, such as a sphere, disc or needle. When the wire is made of a dielectric material although it does not have so much effect on the frequency shifts, it does on the Q -values.

We assume here that the conductivity of the wire is zero and that the permeabilities of each region are almost the same. Then, returning to Eq. (7) and using the above assumptions and limiting forms of the Bessel and Neumann functions for $v, x \ll 1$, we obtain for the $n = 0$ mode

$$\begin{aligned} q &\simeq 0, \quad p \simeq \frac{1}{\epsilon_r v}, \\ \Delta_0(v) &\equiv -\frac{N_0(v) - pP_0(x)N_0'(v)}{J_0(v) - pP_0(x)J_0'(v)} \\ &\simeq \frac{2}{\pi} \left\{ \log -\frac{1}{\epsilon_r v^2} \right\} / \left(1 - \frac{1}{\epsilon_r} \right), \end{aligned} \quad (26)$$

where ϵ_r is the relative permittivity of the wire and $\log \equiv \log(2/Cv)$, which is almost constant compared to the rapid change of v^{-2} . Note that $\Delta_0(v)$ has