A Solution to the Polonyi Problem in the Minimum SUSY-GUT

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Abstract

We show that the Polonyi problem is solved in the minimum SUSY-GUT model in which a self-coupling strength for a heavy Higgs $\Sigma$, $\lambda \Sigma^3$, is very small $\lambda \sim 10^{-6}$. It is stressed that with this small $\lambda$ the mass of the physical $\Sigma$ becomes $m_{\Sigma} \sim 10^{13} \text{GeV}$ and the unification scale is raised up to the gravitational one, $M \simeq 2 \times 10^{18} \text{GeV}$. A potential problem, however, is also pointed out in this GUT model.

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The Polonyi problem[1] is one of serious problems in $N=1$ supergravity models in which the supersymmetry (SUSY) is spontaneously broken in hidden sector[2]. In these theories there is necessarily a light massive field (Polonyi field) $\phi$ of the mass $m_\phi = O(m_{3/2})[1,3]$. The Polonyi field $\phi$ couples only gravitationally to particles of observed sector and hence it decays at a very late time. Under quite general assumptions$\Gamma$ the coherent mode of $\phi$ dominates the energy density of the universe until $T \sim 10^{-2} MeV$ and its decay releases an unacceptable amount of entropy[1]. No convincing mechanism has been found to solve this serious problem.

In this letter we show that there is indeed no Polonyi problem in supergravity models where a pair of light Higgs multiplets is obtained by a fine tuning of parameters$\Gamma$ just like in the minimum SUSY-SU(5) model and the self-coupling $\lambda$ (defined in eq.(6)) is very small.

Let us start with the Polonyi superpotential[4]

$$W(z) = \mu^2 (z + a)$$

with $a = (2 - \sqrt{3}) M$ and $M$ being the gravitational scale $M = M_{\text{planck}}/\sqrt{8\pi} \simeq 2.4 \times 10^{18} GeV$. With the Kähler potential $K(z, z^*) = zz^* \Gamma$ the scalar potential is given by

$$V(z) = \exp \left( \frac{zz^*}{M^2} \right) \left\{ \left| \frac{z^* W}{M^2} + \mu^2 \right|^2 - 3 \left| \frac{W}{M^2} \right|^2 \right\}. \quad (2)$$

This potential has a minimum at $\langle z \rangle = (\sqrt{3} - 1) M$ with a vanishing cosmological constant $\Lambda_{cos} = 0$. At the minimum $z = \langle z \rangle$ the SUSY is broken$\Gamma$ giving the gravitino mass

$$m_{3/2} = \exp \left( \frac{\langle z \rangle \langle z^* \rangle}{2M^2} \right) \left| \frac{W(\langle z \rangle)}{M^2} \right| \simeq \frac{\mu^2}{M} e^{2 - \sqrt{3}}. \quad (3)$$

There is a flat direction called Polonyi field $\phi_0$ that is defined as

$$\phi_0 \equiv z - \langle z \rangle. \quad (4)$$

Its mass is at the same order of the gravitino mass$\Gamma$ 

$$m_{\phi_0} \simeq \sqrt{2\sqrt{3}m_{3/2}}. \quad (5)$$

We now introduce Higgs multiplets. In order to demonstrate our main point clearly$\Gamma$we
take all Higgs fields $\Sigma H$ and $\bar{H}$ to be singlets and assume the following superpotential;

$$W = \lambda \left( \frac{1}{3} \Sigma^3_0 - \frac{1}{2} v \Sigma^2_0 + \frac{1}{6} v^3 \right) + g R \Sigma_0 H - M_H \bar{H} H + W(z). \quad (6)$$

In the global SUSY limit the $\Sigma_0$ has a vacuum-expectation value $\langle \Sigma_0 \rangle = v$. With a fine tuning $M_H \approx g v$ we get a pair of light Higgs multiplets whose mass is set as

$$m_H = g \langle \Sigma_0 \rangle - M_H \approx O(m_{3/2}). \quad (7)$$

We choose $m_{3/2} \approx 1 TeV$ in this letter.

Total scalar potential is given by

$$V = \exp \left( \frac{z S + \Sigma_0 S_0 + \cdots}{M^2} \right) \left\{ \left. \frac{\Sigma^*_0 W}{M^2} + \left( \frac{\partial W}{\partial \Sigma_0} \right)^2 + \left( \frac{H^* W}{M^2} + \left( \frac{\partial W}{\partial H} \right)^2 \right) \right| \left. \frac{\bar{H}^* W}{M^2} + \left( \frac{\partial W}{\partial \bar{H}} \right)^2 + \left( \frac{z^* W}{M^2} + \mu^2 \right)^2 - 3 \left| W \right|^2 \right\}. \quad (8)$$

An important observation is that the first term in the bracket possesses a non-negligible mixing term between $\phi_0$ and $\Sigma_0$ fields.

To see this we neglect the exponential term in eq.(8) for simplicity. The relevant part of $V$ is written as

$$V = \left| \frac{\Sigma^*_0}{M^2} \mu^2 (\phi_0 + M) + \lambda \left( \frac{\Sigma^2_0}{3} - v \Sigma_0 \right) \right|^2$$

$$+ \left| \frac{\phi^*_0 + \langle z^* \rangle}{M^2} \mu^2 (\phi_0 + M) + \mu^2 \right|^2$$

$$- \frac{3}{M^2} \left| \lambda \left( \frac{1}{3} \Sigma^3_0 - \frac{1}{2} v \Sigma^2_0 + \frac{1}{6} v^3 \right) + \mu^2 (\phi_0 + M) \right|^2. \quad (9)$$

We see that the physical $\Sigma$ and the Polonyi field $\phi$ is defined as

$$\Sigma \simeq \Sigma_0 + \frac{\mu^2}{\lambda M^2} \phi_0, \quad (10)$$

$$\phi \simeq \phi_0 - \frac{\mu^2}{\lambda M^2} \Sigma_0. \quad (11)$$
Substituting eq.(10) and eq.(11) into eq.(9) we obtain the potential \( V \) as

\[
V \approx \lambda^2 v^2 \left| \Sigma - \left( v - \frac{\mu^2}{\lambda M} \right) \right|^2 \nonumber \\
+ 2 \left( \frac{\mu^2}{M} \right)^2 |\phi|^2 + (\sqrt{3} - 1) \left( \frac{\mu^2}{M} \right)^2 \left( \phi^2 + \phi^* \phi \right).
\]  

(12)

Here we have assumed all parameters are real. It is now clear that vacuum-expectation values for the fields \( \Sigma \) and \( \phi \) are given by

\[
\langle \Sigma \rangle \approx v - \frac{\mu^2}{\lambda M},
\]

(13)

\[
\langle \phi \rangle \approx \frac{\mu^2}{2\lambda M}.
\]

(14)

and \( \phi \) is indeed the physical Polonyi field. Then the Polonyi field \( \phi \) couples to fermion partners of \( H \) and \( \bar{H} \) denoted by \( \psi_H \) and \( \bar{\psi}_H \) through the above mixing (10) and (11). The effective Yukawa coupling of \( \phi \) is given by

\[
\mathcal{L}_{yukawa} = \left( \frac{m_{3/2}}{\lambda M} \right) g \phi \psi_H \bar{\psi}_H + h.c.
\]

(15)

Notice that if \( \lambda \) is very small \( \sim 10^{-6} \) this Yukawa coupling is not negligible and induces a fast decay of \( \phi \) enough to solve the Polonyi problem as explained below.

The rate of the decay \( \phi \rightarrow \psi_H + \bar{\psi}_H \) is calculated as\(^1\)

\[
\Gamma_{\phi} \approx \frac{\alpha}{4} \left( \frac{m_{3/2}}{\lambda M} \right)^2 m_{\phi},
\]

(16)

\(^1\)At the decay time the amplitude of \( \phi \) is

\[
\phi \sim \frac{\alpha}{4} \left( \frac{m_{3/2}}{\lambda M} \right)^2 M.
\]

With this \( \phi \) the Higgs fermion \( \psi_H \) get a mass

\[
m_{\psi_H} \sim \frac{ga}{4} \left( \frac{m_{3/2}}{\lambda M} \right)^3 M, \quad (\alpha = \frac{g^2}{4\pi}).
\]

Requiring the \( 2m_{\psi_H} < m_{\phi} \) so that the decay of \( \phi \rightarrow \psi_H + \psi_H \) is possible, we get a constraint on \( \lambda \) with \( g \sim O(1) \)

\[
\lambda > \left( \frac{m_{3/2}}{M} \right)^{2/3} \sim 10^{-10},
\]

for \( m_{3/2} = 1TeV \).
and the reheating temperature is

\[ T_R \approx 0.1 \sqrt{\Gamma_\phi M_{\text{planck}}} \approx 0.1 \left( \frac{\sqrt{\alpha m_{3/2}}}{2\lambda M} \right) \sqrt{m_\phi M_{\text{planck}}}. \] (17)

If one requires \( T_R \gtrsim O(100 GeV) \) so that the electroweak baryogenesis is possible[5], one gets for \( \alpha \sim O(1) \) and \( m_{3/2} \approx 1 TeV \)

\[ \lambda \lesssim 10^{-6}. \] (18)

However, we do not need to create baryon asymmetry at the electroweak scale since the dilution factor \( D \) of the baryon asymmetry from the \( \phi \) decay is only\(^2\)

\[ D \approx 10^{-6}, \] (19)

with \( \lambda \approx 10^{-6} \) and the initial value \( \phi \approx M \). This requires the primordial baryon asymmetry to be \( \Delta B/s \approx 10^{-4} - 10^{-5} \) which may be produced in a much early epoch of the universe.

It is a straightforward task to incorporate our general mechanism into the minimum SUSY-SU(5) model. The superpotential is given by

\[
W = \frac{1}{3} \lambda tr \Sigma^3 + \frac{1}{2} \lambda vtr \Sigma^2 - 5\lambda v^3 \\
+ g \tilde{H}^a \Sigma^\beta H_\beta + M_H \tilde{H}^a H_a + W(z),
\] (20)

where \( \Sigma, H \) and \( \tilde{H} \) are 24\,\overline{5} and 5\,\overline{5} of SU(5). In the global SUSY limit the \( \Sigma \) field has a vacuum-expectation value

\[
\langle \Sigma \rangle = v \left( \begin{array}{ccc} 2 & 2 & \vspace{0.5cm} \\
2 & 2 & -3 \vspace{0.5cm} \\
-3 & -3 & \end{array} \right),
\]

and masses of triplet and doublet Higgs multiplets \( H_c \) and \( H_f \) are

\[
m_{H_c} = 2gv + M_H, \quad m_{H_f} = 2\lambda v + M_{\phi}, \]

\(^2\)In the previous analysis[1], the decay rate of \( \phi \) is assumed as \( \Gamma_\phi \approx \frac{m_\phi^3}{2M_{\text{planck}}} \) which leads to the dilution factor \( D \approx 10^{-14} \).

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By a fine tuning \( M_H \simeq 3gV \) we obtain a pair of light Higgs doublets \( H_f \) and \( \tilde{H}_f \) and a pair of heavy Higgs triplets \( H_c \) and \( \tilde{H}_c \). We have checked that the similar mixings between \( \Sigma \) and \( \phi \) takes place in the SU(5) broken phase as in the singlet model inducing a Yukawa coupling of \( \phi \):

\[
L_{\text{yukawa}} = 3 \left( \frac{m_{\phi}^2}{\lambda M} \right) g \phi \psi_{H_f} \bar{\psi}_{H_f} + h.c.
\]  

Finally we stress that the small self-coupling \( \lambda \simeq 10^{-6} \) suggests that the mass of physical \( \Sigma \) \((24)\) in SUSY SU(5) model is relatively small \( m_{\Sigma} \sim \lambda \langle \Sigma \rangle \sim O(10^{12} GeV) \). As pointed out by Hisano, Murayama and Yanagida[6] this is still consistent with the unification of three gauge coupling constants if the GUT scale \( \langle \Sigma \rangle \sim O(10^{18} GeV) \). (This may be rather interesting for superstring theories.) The crucial point in this letter is that the Polonyi field \( \phi \) couples to \( \psi_H \) and \( \psi_{\bar{H}} \) with the strength \( m_{\phi}^2/m_{\Sigma} \) as shown in eq.(15) which arises from the mixing between the \( \phi \) and \( \Sigma \) fields. The coupling of \( \phi \) is no longer suppressed by \( 1/M \) as expected previously. However there is a potential problem in this GUT model. As seen in eq.(13) the shift of \( \langle \Sigma \rangle \) is not small enough to maintain the hierarchy achieved in the global SUSY model. Therefore we must rechoose \( M_H \) so that the physical \( m_{H_f} \) is \( O(m_{\phi}/M) \) after the shift of \( \langle \Sigma \rangle \). Moreover the shift also produces a large soft-SUSY breaking term \( \sqrt{3g} \phi \frac{4}{\lambda M^2} \tilde{H}H \). The coefficient of this dangerous term however depends strongly on the form of Kähler potential and in fact we have found a non-minimum Kähler potential with which such a soft-SUSY breaking term vanishes.\(^3\) Since more fine tuning is required the GUT model with small \( \lambda \) is less attractive. Nevertheless it is very much encouraging that a fine tuning for guaranteeing the large mass hierarchy in Higgs sector solves automatically the serious cosmological problem.

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\(^3\)In this model we take the Kähler potential \( K = z^* z \exp(\Sigma^* \Sigma/M^2) + \Sigma^* \Sigma \) and the superpotential for \( z \),

\[
W(z) = \mu z + c_1 (z/M) + c_2 (z/M)^2 + \cdots
\]

The coefficients \( c_i \) are fixed from the requirement that the scalar potential \( V(z, \Sigma) \) has a minimum at \( \langle z \rangle = 0 \) with a vanishing cosmological constant.
References


