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A Simple Relation among the Quark Masses

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ABSTRACT

Based on our recent suggestion that the first and second generations of quarks and leptons can be taken as almost and quasi Nambu-Goldstone fermions, respectively, derived is a simple relation among the quark masses, $m_u m_s^3 m_t^2 \cong m_d m_c^3 m_b^2$, from which the top quark mass is predicted to be $m_t \cong (180 \pm 100) - (200 \pm 110) \text{ GeV}$.

To predict the masses of the top quark (t) and the Higgs scalar (H) is now one of the most urgent problems in particle theory. Among many attempts, the most natural way to do it seems to be in composite models [1] in which the Higgs scalar is almost a bound state of the top-antitop pair (or a "top-antitop condensate") [2, 3]. In 1980, the present author [2] predicted them as

$$m_t \cong \sqrt{8/3} m_W \cong 131 \text{ GeV for } m_W = 80 \text{ GeV,} \quad (1)$$

$$m_H \cong 2m_t \cong 261 \text{ GeV} \quad (2)$$

from the sum rules for quark and lepton masses derived earlier in our unified composite model of the Nambu-Jona-Lasinio (NJL) type [4], where not only the Higgs scalar but also the weak bosons ($W^\pm$ and Z) are composite. Much later, similar predictions have been made by Nambu, Miransky et al. and Bardeen et al. [3] in similar models of the NJL type.

All the recent analyses of the constraints on m_t from the experimental data on m_W , m_Z and the neutral currents (and muon life time) by many authors [5] strongly indicate that the old prediction of $m_t \cong 131 \text{ GeV}$ is roughly correct. It is even more remarkable that the more recent analyses of the CDF candidate event for top-antitop production [6] have suggested $m_t \cong 131 \left(\begin{smallmatrix} +22 \\ -11 \end{smallmatrix} \right) \text{ GeV}$ or $119-151 \text{ GeV}$ (68% C.L.), which just coincides with the prediction. Does this already indicate that our unified composite model of the NJL type correctly describes the nature and dynamics of quarks and leptons? In this picture, the top quark is the only one of the quarks and leptons that is reasonably heavy (as heavy as m_W and m_Z), and all the other quarks and leptons are abnormally light. Why? Very recently, Nambu [7] has proposed a new mechanism for splitting the mass of the top quark and those of all the other quarks and leptons by minimizing the vacuum energy, which may be called "spontaneous breakdown of quark-lepton democracy."

In composite models of quarks and leptons, the problem of "why are the quark and lepton masses so small compared to the compositeness mass scale (A which is presumably larger than 100 GeV)?" is rather old and has been extensively discussed in the literature [1]. There are the

following three solutions proposed so far: quarks and leptons are (i) almost chiral fermions [8], (ii) almost Nambu-Goldstone (NG) fermions [9], or (iii) quasi NG fermions [10]. The latter two pictures may not be independent since quarks and leptons are taken as almost massless NG fermions due to spontaneous breakdown of (approximate) supersymmetry (SUSY) in (ii) and as almost massless superpartners of NG bosons due to spontaneous breakdown of (approximate) flavor-color global symmetry in (iii).

In our previous first paper [11], we have derived the mass formulas for quarks and leptons as either almost NG fermions or quasi NG fermions from the partially conserved induced supercurrent (PCIS) hypothesis. We have also shown the sum rule of [12]

$$m_e^{1/2} \equiv m_d^{1/2} - m_u^{1/2} \quad (3)$$

holds if the first generation of quarks and leptons can be taken as almost NG fermions. It is remarkable that this sum rule is well satisfied by the experimental value of m_e and the estimates of m_u and m_d . In fact, for $m_e = 0.511\text{MeV}$, $m_u = 4.5 \pm 1.4\text{MeV}$ and $m_d = 7.9 \pm 2.4\text{MeV}$ [13], the left-hand side of eq. (3) is $0.71\text{MeV}^{1/2}$ while the right-hand side is $0.69 \pm 0.22\text{MeV}^{1/2}$. Of course, since the estimate of the current quark masses for the first generation is ambiguous [14], this might indicate a mere coincidence. However, if this is taken seriously (as by us), it may indicate that the first generation of quarks and leptons are close to the almost NG fermions.

In our previous second paper [15], we have further suggested that the first and second generations of quarks and leptons can be taken as almost and quasi NG fermions, respectively, and derived simple (order-of-magnitude) relations such as

$$m_\tau \sim \left(m_\mu^3/m_e\right)^{1/2} = 1520\text{MeV for } m_\mu = 105.7\text{MeV}, \quad (4)$$

$$m_t \sim \left(m_c^3/m_u\right)^{1/2} \equiv 23\text{GeV for } m_c = 1.35 \pm 0.05\text{MeV} [13], \quad (5)$$

$$m_b \sim \left(m_s^3/m_d\right)^{1/2} \equiv 0.69\text{GeV for } m_s = 155 \pm 50\text{MeV} [13], \quad (6)$$

which should be compared to the experimental value of $m_\tau \equiv 1780\text{MeV}$, our prediction [2] or the CDF suggestion [6] of $m_t \equiv 131\text{GeV}$ and the estimate of $m_b = (5.3 \pm 0.1) - (5.9 \pm 0.1)\text{GeV}$ [13]. These relations seem to be well satisfied experimentally as far as the orders of magnitude are concerned. Especially, it seems remarkable that the relation for the charged leptons (4) is very well satisfied with the coefficient very close to unity. What does this mean? Is it mere accidental? To answer this, we must find a dynamical model in which all the masses of an almost NG fermion, a quasi NG fermion and an ordinary composite fermion can be calculated in a consistent way. In this paper, instead of searching for such a model, we shall still further proceed in a current-algebraic way to derive another simple relation among the quark masses,

$$m_u m_s^3 m_t^2 \equiv m_d m_c^3 m_b^2, \quad (7)$$

from which we shall predict the top quark mass as

$$m_t \equiv \left(m_d m_c^3 m_b^2 / m_u m_s^3\right)^{1/2} = (180 \pm 100) - (200 \pm 110)\text{GeV}. \quad (8)$$

Let us first briefly review the essential contents of our previous papers [11, 15]. Suppose there exist n sets of $N=1$ supermultiples $\Phi_i = (\phi_i, \chi_i)$ ($i = 1, \dots, n$) which represent subquarks, the constituents of quarks and leptons, whose masses are (μ_i, M_i) . Let $\psi = (x, \chi^{C\dagger})^T$ be the four-component spinor and $\eta = \phi_L + \phi^{C\dagger} R$ be the corresponding scalar, where $L = \frac{1}{2}(1 - \gamma_5)$ and $R = \frac{1}{2}(1 + \gamma_5)$. The supercurrent s_μ and supercharge S are given by

$$s_\mu = i(\partial^\nu \eta^\dagger) \gamma_\nu \gamma_\mu \psi - \eta^\dagger \gamma_\mu M \psi, \quad (9)$$

$$S = \int d^3x s_0, \quad (10)$$

where $(M)_{ij} = M\delta_{ij}$. The flavor-color currents j_μ^A and charges Q^A are given by

$$j_\mu^A = \bar{\psi} \gamma_\mu \lambda^A \psi + i \eta^\dagger \bar{\partial}_\mu \lambda^A \eta + i \eta^C \bar{\partial}_\mu \lambda^A \eta^C, \quad (11)$$

$$Q^A = \int d^3x j_0^A, \quad (12)$$

where $\eta^C = \phi^C_L + \phi^\dagger_R$ and $\lambda^A (A = 1, \dots, n^2)$ are the $n \times n$ matrices for $U(n)$ flavor-color symmetry. Then, the supercurrents with the flavor-color quantum numbers can be induced by the commutators

$$s_\mu^A = [S, J_\mu^A], \quad (13)$$

which can be written in terms of subquark fields as

$$s_\mu^A = (i \eta^\dagger \gamma_\mu \gamma_\nu \partial^\nu - \eta^\dagger \gamma_\mu M) \lambda^A \psi + \partial^\nu (\eta^\dagger \sigma_{\mu\nu} \lambda^A \psi), \quad (14)$$

The PCIS hypothesis introduced in ref. [11] has been revised in ref. [15] in the following way: The induced supercurrents consist of three parts as

$$s_\mu^A = s_\mu^{A(1)} + s_\mu^{A(2)} + s_\mu^{A(3)}. \quad (15)$$

In terms of subquark fields, these parts behave as

$$s_\mu^{A(1)} = -\eta^\dagger \gamma_\mu M \lambda^A \psi, \quad s_\mu^{A(2)} = i \eta^\dagger \lambda^A \partial_\mu \psi, \quad s_\mu^{A(3)} = \eta^\dagger \sigma_{\mu\nu} \lambda^A \partial^\nu \psi \quad (16)$$

and they are dominated by the first, second and third generation of quarks and leptons, $f_\Lambda^{(l)}$ ($l = 1, 2, 3$), as

$$s_\mu^{A(l)} = -F_\Lambda^{(l)} \gamma_\mu f_\Lambda^{(l)}, \quad s_\mu^{A(2)} = i F_\Lambda^{(2)} \partial_\mu f_\Lambda^{(2)}, \quad s_\mu^{A(3)} = F_\Lambda^{(3)} \sigma_{\mu\nu} \partial^\nu f_\Lambda^{(3)}, \quad (17)$$

where the F 's are the "decay constants" of quarks and leptons. The PCIS relations now become

$$\partial^\mu s_\mu^{A(1)} = i F_\Lambda^{(2)} m_{f_\Lambda^{(1)}} f_\Lambda^{(1)}, \quad \partial^\mu s_\mu^{A(2)} = -i F_\Lambda^{(2)} m_{f_\Lambda^{(2)}} f_\Lambda^{(2)}, \quad \partial^\mu s_\mu^{A(3)} = 0, \quad (18)$$

where the m 's are the quark and lepton masses. It should be noted that the automatic vanishing of $\partial^\mu s_\mu^{A(3)}$ is closely related to the non-suppression of $m_{f_\Lambda^{(3)}}$. Namely, the masses of the third generation of quarks and leptons as ordinary composite fermions are of the order of the compositeness mass scale Λ .

In ref. [15], we have derived the following mass formula from the PCIS relations (18) by repeating the same procedure as in ref. [11]:

$$m_{f_\Lambda^{(l)}}^{2l-1} = F_\Lambda^{2l-6} (M_l - M_j)^2 \left(M_j \langle \phi_1^C \phi_1 \rangle_0 + M_l \langle \phi_j^C \phi_j \rangle_0 \right) \quad (l = 1, 2), \quad (19)$$

for the first and second generations of quarks and leptons specified by $f_\Lambda^{(l)} = f_{ij}^{(l)}$ for $(\lambda^A \phi)_i = \phi_j$ that behaves as $f_{j\bar{l}} \sim \phi_1^C \psi_{j\bar{l}} + (\psi_1^C)_l \phi_j$ and $f_{jR} \sim \phi_1^+ \psi_{jR} + (\psi_1^C)_R \phi_j^C$. This mass formula indicates that the masses of the first and second generations of quarks and leptons become of the order of M^3/Λ^2 and M , respectively if both the "decay constant" F and the vacuum expectation value $\langle \phi_1^C \phi_1 \rangle_0$ are determined by the compositeness mass scale Λ as $F \sim \Lambda$ and $\langle \phi_1^C \phi_1 \rangle_0 \sim \Lambda^2$:

$$m_{f_\Lambda^{(1)}} \sim M^3/\Lambda^2, \quad m_{f_\Lambda^{(2)}} \sim M \quad (\text{and } m_{f_\Lambda^{(3)}} \sim \Lambda). \quad (20)$$

For $M \ll \Lambda$, this estimate not only explains why the masses of the first and second generations of quarks and leptons are so small compared to the compositeness mass scale but also produces the gross structure in the mass hierarchy of three generations of quarks and leptons, i. e. $m_{f_\Lambda^{(1)}} \ll m_{f_\Lambda^{(2)}} \ll m_{f_\Lambda^{(3)}}$. Furthermore, it suggests the following simple relations among the masses of three generations of quarks and leptons:

$$m_{f_\Lambda^{(3)}} \sim (m_{f_\Lambda^{(2)}}/m_{f_\Lambda^{(1)}})^{1/2}. \quad (21)$$

More explicitly, they are the simple (order-of-magnitude) relations in eqs. (4), (5) and (6).

Now, a point in this paper is the following: From our mass formula (19), we can derive the three rigorous relations of

$$\begin{aligned} \left(m_\mu^3/m_e\right)^{1/2} &= F_e^2/F_\mu (\equiv \Lambda_1), & \left(m_c^3/m_u\right)^{1/2} &= F_u^2/F_c (\equiv \Lambda_U), \\ \left(m_s^3/m_d\right)^{1/2} &= F_d^2/F_s (\equiv \Lambda_D). \end{aligned} \quad (22)$$

Let us define the dimensionless coefficients, c_l , c_U and c_D , by

$$m_\tau = c_l \Lambda_1, \quad m_t = c_U \Lambda_U, \quad m_b = c_D \Lambda_D \quad (23)$$

Then, what we have implicitly assumed in order to derive the simple (order-of-magnitude) relations, (4), (5), and (6), is that these coefficients are all of the order of unity, i. e. $c_l, c_U, c_D \sim 1$. In fact, from the experimental values of m_e, m_μ and m_τ , from the estimates of m_u, m_d, m_c, m_s and m_b [13] and from our prediction [2] or the CDF suggestion [6] of $m_t \equiv 131\text{GeV}$, we can find that these coefficients may well be of the order of unity as

$$\begin{aligned} c_l &= \left(m_e m_\tau^2 / m_\mu^3\right)^{1/2} = 1.17, & c_U &= \left(m_u m_t^2 / m_c^3\right)^{1/2} = 5.7 \pm 2.2, \\ c_D &= \left(m_d m_b^2 / m_s^3\right)^{1/2} = (7.7 \pm 6.2) - (8.6 \pm 6.9). \end{aligned} \quad (24)$$

Although it is difficult to calculate these coefficients which are dynamical and model-dependent, it seems natural to assume the approximate iso-spin independence of

$$c_U \equiv c_D \quad (\equiv c_q) \quad (25)$$

If this is the case, we can immediately derive the simple relation among the quark masses in eq. (7) and predict the top quark mass as in eq. (8).

What is left is to explain why the ratio of c_q/c_l is at least as large as 3. A possible explanation is that the ratio is enhanced by the mass renormalization effect mostly due to strong interactions of quarks although it would be unity, i. e. $c_q^{(0)} = c_l^{(0)}$, without such effect taken into account. This reminds us of the possible explanation for the ratio of $m_b/m_t (\equiv 3)$ by Buras et. al. [17]: The ratio is enhanced to become as large as 3 at the low mass scale ($\sim \text{GeV}$) by the mass renormalization effect due to strong and electroweak interactions, provided that the equation of $m_b^{(0)} = m_t^{(0)}$ holds at the grand unification mass scale ($\sim 10^{15} \pm 1\text{GeV}$). However, the situation looks somewhat different since the compositeness mass scale Λ may not be so large as the grand unification mass scale but as small as the order of, say, 100GeV .

Our new prediction of $m_t \equiv \left(m_d m_c^2 m_b^2 / m_u m_s^3\right)^{1/2} \equiv (180 \pm 100) - (200 \pm 110)\text{GeV}$ is consistent not only with our old prediction of $m_t \equiv \sqrt{8/5} m_W \equiv 131\text{GeV}$ but also with the constraints from the experimental data [5] and the CDF suggestion [6]. Both of these predictions suggest that the long desired discovery of the top quark is now within an immediate reach of the TEVATRON $p\bar{p}$ Collider experiments at Fermilab. Note also that these predictions are quite independent of each other: The old prediction is derived from the picture in which the quark and lepton masses come from the $\bar{t}$ condensation while the new one is from the picture in which they come from the subquark-antisubquark condensation. In other words, the Higgs scalar can be taken either as a bound state of the $\bar{t}$ pair in the old picture or as a bound state of the subquark-antisubquark pair in the new one. This further strengthens the validity of "triplicity hypothesis" [18] that a certain physical quantity such as the weak current can be taken equally well as either one of a composite operator of hadrons, that of quarks and that of subquarks. In conclusion, it should be emphasized that having the new relation among the quark masses may drive us one step forward, toward the goal of our program [19] to solve the quark and lepton mass matrices by deriving as many sum rules and relations as possible in a current-algebraic way (without depending on the unknown underlying dynamics) and by solving a set of the sum rules and relations.

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