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AND THE NATURE OF DARK MATTER

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Abstract

One of the promising ways to determine the nature of dark matter is to study correlations in the distribution of galaxies. We analyze the influence of peculiarities in the inflation potential on the spectrum of perturbations and on the galaxy correlation function in CDM models. We show that a break in the inflation potential creates a distinct feature in the correlation function. It turns out that an appropriate choice of the potential parameters can change, and in particular, increase the power of the spectrum on a given scale. We compare the galaxy correlation function in a CDM model with peculiarities in the inflation potential with the galaxy correlation function in a universe with baryonic dark matter, which also possesses peculiar features. We show that the galaxy correlation function in the CDM model considered here imitates quite well the galaxy correlation function in baryonic models.

1. Introduction

Observational data strongly indicate that a large fraction of matter in the universe is in the form of dark matter. Very little is known about the properties of the dark matter component of the universe. Recently there have been attempts to gain some information on the nature of dark matter by studying correlations in the distribution of galaxies, clusters of galaxies, and quasars.

Primordial density perturbations evolve differently in models with different types of dark matter, thereby leading to different present distributions of light emitting matter. For example, dark matter and baryonic components evolve differently in cold dark matter (CDM) models and in pure baryonic models, and this leads to different correlation functions of visible matter [1]. Therefore properties of the observed correlation function can provide information on the nature of dark matter.

Evolution of the baryonic component is quite complicated. At the moment when the energy density of matter becomes equal to the energy density of radiation the initial entropy perturbations generate secondary adiabatic perturbations [2, 3, 4, 5]. In models with standard thermal histories these adiabatic perturbations evolve as acoustic waves in

a plasma until the recombination epoch. Small scale adiabatic perturbations are damped during the radiation epoch and at the moment of recombination only perturbations larger than the Silk scale r_S and up to the scale of the acoustic horizon $R_{ac} = R_h v_{ac}/c = R_h/\sqrt{3(1+3\rho_b/4\rho_r)}$ survive (v_{ac} is the speed of sound, R_h - size of the horizon, and ρ_b and ρ_r are correspondingly the densities of baryonic matter and radiation) [6, 7]. After recombination, in models dominated by baryonic matter ($\Omega_b \gg \Omega_{CDM}$), primordial perturbations which survived the radiation epoch grow and form the large scale structure. In these models the two point correlation function $\xi(r)$ possesses a characteristic feature in the form of two pronounced maxima at scales $r = R_{ac}$ and $r = 2R_{ac}$ (so called Long Distance Correlation [8]). In models allowing secondary ionization (further referred as PBI models) [9] due to energy release by the first generation of gravitationally bound objects [10] the situation is similar. However there is an important difference, namely in the PBI models r_S is on the order of R_{ac} and this leads to the appearance of a wide bump in the correlation function at scales comparable to R_{ac} [11] instead of two resonance features.

When $\Omega_b \ll \Omega_{CDM}$ the oscillating spectrum of baryonic perturbations is quickly smoothed out after recombination by gravitational interactions with which by then are much more evolved inhomogeneities of the CDM component [2, 4, 12, 13, 14]. Further evolution of the CDM inhomogeneities determines the large scale distribution of visible matter. In standard CDM models the two point correlation function does not possess any anomalies. In these models evolution of the initial smooth spectrum does not create an oscillating spectrum but leads to a monotonic correlation function. Therefore the two point correlation functions $\xi(r)$ in baryonic models and in standard CDM models are different.

However if in the CDM model a nonstandard inflation is considered then there appears a possibility of creating oscillations already in the initial spectrum of perturbations [15, 16]. In CDM models the initial spectrum preserves its form in the process of evolution because the growth of perturbations is monotonic. Therefore oscillations in the initial spectrum are present also later. The correlation function generated by such perturbations possesses anomalies similar to anomalies of correlation function in baryonic models. However the correlation function in baryonic models possesses two maxima while in CDM models with non standard inflation proposed by Starobinsky [16] it has only one.

In this paper we construct a CDM model with nonstandard inflation which generates a spectrum of primordial adiabatic perturbations with two or more characteristic scales of peculiarities. We use only one scalar field φ but allow a few local anomalies in the inflation potential $V(\varphi)$. Qualitatively similar results can be obtained in more complicated models of inflation, for example, models with several scalar fields (see for example [17]) or in models with a polynomial inflation potential [18]. Note however that differences in the type of scalar fields or in the form of the potential lead to different types of spectrum. We show that peculiarities of the correlation function on scales of the acoustic horizon at the moment of recombination for CDM models with an anomalous inflation potential are similar to peculiarities of the correlation function at the same scale in pure baryonic models.

However the behavior of the correlation function close to the resonances appearing on scales comparable to the acoustic horizon at the moment of recombination allows one

to distinguish the CDM models from purely baryonic models. Therefore in principle it is possible to determine the nature of the dark matter by careful analysis of the correlation function.

We also show that, by appropriate choice of the parameters of the potential peculiarities, we can control the amplitude of the correlation function on large scales and therefore imitate the correlation function also for the PBI models.

2. Adiabatic perturbations in inflationary model with breaks in the inflaton potential

Evolution of adiabatic inflaton perturbations is described by a gauge invariant scalar quantity v defined by [19, 20]

$$v = a(\delta\varphi - \varphi' \frac{\Psi}{\alpha}), \quad (2.1)$$

where $\delta\varphi$ describes the perturbation of the inflaton field φ , Ψ is a metric perturbation in the conformally Minkowskian gauge, a is the scale factor, and $\alpha = a'/a$. The prime denotes differentiation with respect to the conformal time $\eta = \int dt/a$.

In an inflationary universe, evolution of the Fourier transform of v is governed by the equation [20]

$$v_k'' + (k^2 - \frac{2}{\eta^2} + V_{\varphi\varphi}a^2)v_k = 0. \quad (2.2)$$

The last term in the bracket is important only in the vicinity of breaks in the potential.

In the vicinity of breaks of the potential $V(\varphi)$, for sufficiently small region of smoothing out, the potential can be described by

$$V(x) = V_0 + A_j^- x + (A_j^+ - A_j^-)\theta(x)x, \quad (2.3)$$

where $x = \varphi - \varphi_j$, φ_j is the value of φ at the break point j , A_j^- and A_j^+ describe the j -th break, and $\theta(x)$ is the Heaviside step function. Numbering of breaks is consistent with the increasing value of the scalar field.

In what follows, for simplicity, we consider only the case of a potential with two breaks and we assume that $A_1^- \approx A_2^+ = A$ (see Fig.1). From (2.3) we have

$$V_{\varphi\varphi} = \frac{(A - A_1^+)}{\varphi'}\delta(\eta - \eta_1) + \frac{(A_2^- - A)}{\varphi'}\delta(\eta - \eta_2), \quad (2.4)$$

where $\eta_{1,2}$ denotes the moment when the field φ passes the breaks.

To calculate φ' we use the evolution equation of the field φ [16]

$$\ddot{x} = -\frac{A_2^+}{3H} \quad \eta < \eta_2, \quad (2.5a)$$

$$\ddot{x} = -\frac{(A + (A_2^+ - A)e^{-3Ht})}{3H} \quad \eta_2 < \eta < \eta_1. \quad (2.5b)$$

Here the dot denotes differentiation with respect to the physical time t and $H = \dot{a}/a$ is the Hubble constant. Substituting (2.5) into (2.4) for $\eta = \eta_1$ and $\eta = \eta_2$ we obtain

$$V_{\varphi\varphi} = \{C_2\delta(\eta - \eta_2) + C_1\delta(\eta - \eta_1)\}\frac{3H}{a}, \quad (2.6)$$

where $C_2 = (1 - \frac{A}{A_2^+})$, $C_1 = \frac{A - A_1^-}{A + (A_2^+ - A)(\frac{k_1}{k_2})^3}$. When $A \ll A_2^+(k_2/k_1)^3$, $C_j = 1 - A_-^j/A_+^j$, $C_2 = 1$, $C_1 \sim -\frac{A_1^-}{A_2^+}(\frac{k_1}{k_2})^3$. Here $k_j = H(\eta_j)a(\eta_j)$ is the wave number corresponding to the mode entering the horizon at the moment when the inflaton field passes the break point of the potential.

After the field φ passes all the breaks of the potential, the general solution of the equation (2.2) can be written as

$$v_k = \alpha(k)v_{k\text{in}} + \beta(k)v_{k\text{in}}^*, \quad (2.7)$$

where

$$v_{k\text{in}} = \frac{1}{(2k)^{1/2}}e^{-ik\eta}(-1 + \frac{i}{k\eta}) \quad (2.8)$$

is normalized with respect to the vacuum solution of equation (2.2) with a potential without breaks (see [21] and references therein). It was shown by Starobinsky [16] that the spectrum of initial density fluctuations can be expressed in terms of α and β :

$$P(k) \propto k|\alpha - \beta|^2. \quad (2.9)$$

From linearity and the invariance under complex conjugate of the evolution equation (2.2) we have

$$\begin{aligned} \alpha &= \alpha_2\alpha_1 + \beta_2\beta_1^*, \\ \beta &= \alpha_2\beta_1 + \beta_2\alpha_1^*, \end{aligned} \quad (2.10)$$

where α_j and β_j are coefficients in the expansion (2.7) in the case of only one, namely the j -th break. These coefficients were calculated by Starobinsky [16]

$$\begin{aligned} \alpha_j &= 1 - \frac{3i}{2}C_j y_j^{-1}(1 + y_j^{-2}), \\ \beta_j &= \frac{3i}{2}C_j \exp(2iy_j)y_j^{-1}(1 + iy_j^{-1})^2, \end{aligned} \quad (2.11)$$

where $y_j = k/k_j$.

Inserting (2.11) into (2.10) and (2.10) into (2.9) we obtain

$$P(k) = kD(k) = k(D_1D_2 + I), \quad (2.12)$$

where

$$I = 4(A_1 - C_1)(A_1 - C_1)(B_2 + A_2C_2) - (1 - B_1)(C_2 - A_2B_2),$$

$$D_j = |\alpha_j - \beta_j|^2 = 1 + 3C_j y_j^{-1} [(1 - y_j^{-2}) \sin 2y_j + 2y_j^{-1} \cos 2y_j] + \frac{9}{2} C_j^2 y_j^{-2} [(1 + y_j^{-2}) \cos 2y_j - 2y_j^{-1} \sin 2y_j],$$

$$A_j = \text{Im}(\alpha_j); \quad B_j = \text{Re}(\beta_j); \quad C_j = \text{Im}(\beta_j). \quad (2.13)$$

From (2.13) it follows that

$$D(0) = \left(\frac{A_1^-}{A_1^+}\right)^2; \quad D(\infty) = 1.$$

When $C_j \gg 1$, in the vicinity of breaks of the potential a region of false vacuum appears. In this case the simple calculation presented above is not applicable because in the vicinity of $\varphi \sim \varphi_j$ a potential barrier appears and therefore it is necessary to take into account the process of quantum tunneling to the region of smaller values of the scalar field. In what follows we assume that $C_j \leq 1$.

When the region of the field $\Delta\varphi$ containing breaks is smaller than the value of the field (this condition is satisfied in the standard inflationary model, for example in the $\lambda\varphi^4$ theory during inflation all the present scales beginning from 1 Mpc and up to the horizon scale crossed the horizon while $\Delta\varphi/\varphi \sim 0.3$ [22]) it is possible to obtain a recurrence relation for the modulation function $D(k)$. Let after the scalar field φ passes the j -th break v has the following expansion in terms of v_{in}

$$v = \alpha_j v_{in} + \beta_j(k) v_{in}^*, \quad (2.14)$$

Applying the same procedure as above we obtain

$$\begin{aligned} \alpha_{j-1} &= \alpha_j \alpha_{(j-1)} + \beta_j \beta_{(j-1)}^*, \\ \beta_{j-1} &= \alpha_j \beta_{(j-1)} + \beta_j \alpha_{(j-1)}^*, \end{aligned} \quad (2.15)$$

and

$$D_{j-1} = D_{(j-1)} D_j + I, \quad (2.16)$$

$$I = 4(A_{(j-1)} - C_{(j-1)})(A_{(j-1)} - C_{(j-1)})(B_j + A_j C_j) - (1 - B_{(j-1)})(C_j - A_j B_j),$$

where quantities with the index j are determined by the expansion (2.14) and quantities with the index in bracket, for example $(j-1)$, are obtained from a similar expansion in the case of only one break but with parameters corresponding to the $j-1$ break in the multi break case.

For breaks occurring sufficiently far from each other $k_{j+1}/k_j \ll 1$, and, when approaching all the breaks, evolution of φ can be described by the slow roll down approximation (the slope of the potential is not too large), the interference term I in (2.16) can be neglected and $D(k)$ is then represented by a product of modulation functions of the one break case, so

$$D(k) = \prod_{j=1}^n D_j(y_j); \quad y_j = k/k_j, \quad (2.17)$$

and in this case $C_j = 1 - A_j^-/A_j^+$. Using the high frequency approximation for $j > l$, and low frequency approximation for $j < l$, we obtain $D_j(y_j)|_{y_j \rightarrow \infty} = 1 + 3C_j y_j^{-1} \sin(2y_j)$, and $D_j(y_j)|_{y_j \rightarrow 0} = (A_j^-/A_j^+)^2$. Inserting these relations into (2.17) we obtain

$$D(k) = \prod_{j < l} \left(\frac{A_j^-}{A_j^+}\right)^2 [1 + 3 \sum_{j > l} C_j y_j^{-1} \sin(2y_j)] D_0(y_l). \quad (2.18)$$

Assuming that $A_{j+1}^- \sim A_j^+$ we finally obtain

$$D(k) = \left(\frac{A_1^-}{A_1^+}\right)^2 \left(1 + 3 \sum_{j > l} C_j y_j^{-1} \sin(2y_j)\right) D_0(y_l). \quad (2.19)$$

Let us note that irrespective of the position of breaks and the slope of the potential in the region including breaks the low and high frequency asymptotes of the modulation function (2.16) are determined only by the ratio of the slopes of the potential at the entrance and exit from the region containing breaks so

$$D(k \rightarrow 0) = \left(\frac{A_1^-}{A_n^+}\right)^2; \quad D(k \rightarrow \infty) = 1. \quad (2.20)$$

This is so because in the low frequency region (for models which are far outside the horizon when the scalar field traverses the region with breaks) and in the high frequency region (modes which expanded past the horizon after the scalar field passed through the region with breaks) the spectrum and therefore also $D(k)$ are determined by the relation (see for example [23, 24, 25])

$$k^3 \Phi^2(k) \sim V^3 / \left(\frac{\partial V}{\partial \varphi}\right)^2. \quad (2.21)$$

This relation can be used also when the potential has breaks but only for the asymptotic regions of the spectrum because this relation is valid only if the slow roll approximation is satisfied at the moment when the mode crosses the horizon.

3. Peculiarities of the inflaton potential and the correlation function of galaxies

Let us now discuss general properties of the correlation function for the CDM model with nonstandard inflation. As was shown in the previous section, appropriate choice of the inflaton potential can generate characteristic peculiarities in the spectrum of primordial adiabatic density perturbations. This spectrum is different from the spectrum obtained recently by Starobinsky [16] and it contains two modes of oscillations in the case of two peculiarities in inflation potential, what leads to the appearance of two local maxima in the correlation function. When there are many breaks, the number of local peculiarities in the correlation function is equal to the number of peculiarities in the potential. Without loss of generality, in the rest of the paper we consider a potential with only two breaks.

A potential with two breaks produces a spectrum which generates a correlation function very similar to the correlation function of purely baryonic universe with isocurvature perturbations when the characteristic scales R_1 and R_2 are appropriately chosen. This means that if R_1 and R_2 are much larger than the correlation radius then the correlation function will have two maxima similar to the features describing long distance correlations in purely baryonic models.

The correlation function $\xi(r)$ is given by

$$\xi(r) = \frac{b^2}{2\pi^2 r} \int_0^\infty k P(k) T(k) e^{-k^2 R^2} \sin(kr) dk, \quad (3.1)$$

where b is the bias parameter, $T(k)$ - the transfer function, $P(k)$ the spectrum of initial perturbations, and R_* describes the scale of a Gaussian filter [26].

Let us now analyze the correlation function for a CDM model with the spectrum of initial perturbations given by

$$P(k) = Ak D(k, k_1, k_2), \quad (3.2)$$

where A is the amplitude, and $D(k, k_1, k_2)$ is a modulation function. When $D(k, k_1, k_2) = 1$, $P(k) \sim k$ describes the Harrison-Zel'dovich spectrum. The integral kernel in (3.1) for the CDM model at low frequencies is proportional to $k^2 \sin kr$ and for $C_j \sim 1$ its value is determined by the high frequency region $k \gg R_j^{-1}$. Therefore to estimate the correlation function $\xi(r)$ it is sufficient to take into account only the first few terms in an expansion of $D(k)$ in powers of y_j^{-1} . From (2.12) we see that for $y_1 \geq 4.5$ $D(k, k_1, k_2)$ is well approximated by

$$D \approx 1 + 3C_1 \frac{\sin(2y_1)}{y_1} + 3C_2 \frac{\sin(2y_2)}{y_2}. \quad (3.3)$$

Using (3.3) we rewrite (3.1) in the form

$$\xi(r) = \xi_0(r) + \xi_1(r) + \xi_2(r), \quad (3.4)$$

where

$$\begin{aligned} \xi_0(r) &= \frac{\tilde{A}}{r} \int f(k) \sin(kr) dk, \\ \xi_j(r) &= \frac{3C_j \tilde{A}}{2R_j r} \int f(k) \frac{\cos((r - 2R_j)k)}{k} dk \\ &\quad - \frac{3C_j \tilde{A}}{2R_j r} \int f(k) \frac{\cos((r + 2R_j)k)}{k} dk = \xi_j^- + \xi_j^+ \end{aligned} \quad (3.5)$$

with $j = 1, 2$; $\tilde{A} = \frac{b^2}{2\pi^2}$; $f(k) = T(k)k^2 e^{-k^2 R^2}$. In our calculations we use the analytic approximation of the transfer function in CDM models obtained by Holtzman [27].

There are useful approximate relations for $\xi(r)$, namely

$$\begin{aligned} \xi(r) &\approx \xi_0(r) \quad \text{for } r \leq r_c \\ \xi(r) &\approx \xi_j(r) \quad \text{for } \frac{r - 2R_j}{2R_j} \ll 1 \quad j = 1, 2 \end{aligned} \quad (3.6)$$

From (3.5) it is apparent that the three different functions ξ_0 , ξ_1 and ξ_2 defined by (3.4) describe correlations on different scales. $\xi_0(r)$ describe correlations on a scale less than the correlation radius, and $\xi_1(r)$ and $\xi_2(r)$ describe correlations close to the resonance points $r = 2R_j$, $j = 1, 2$. Fig. 2 illustrates this property.

Taylor expanding the functions $\xi_1(r)$ and $\xi_2(r)$ in the vicinity of the resonance points, we obtain

$$\xi(r) \quad r \rightarrow 2R_j \approx \frac{3C_j \xi_0(0) r_0^2}{4R_j^2} \left(1 - \frac{(r - 2R_j)^2}{2r_0^2}\right) \quad (3.7)$$

where

$$r_0^2 = \frac{\int \frac{f(k)}{k} dk}{\int k f(k) dk}. \quad (3.8)$$

For our model $r_0 \sim 2.5 \text{Mpc}$ [27]. From (3.5) and (3.6) it follows that in the vicinity of the j -th resonance $\xi(r)$ has a maximum for $C_j > 0$, or a minimum for $C_j < 0$, with an amplitude $\frac{3C_j \xi_0(0) r_0^2}{4R_j^2}$ and width r_0 . However in our case $C_j < 1$ and therefore the amplitude of this bump is bounded from above:

$$\xi(2R_j) < \frac{3\xi_0(0)r_0^2}{4R_j^2}. \quad (3.9)$$

Let us note that there exists only an upper bound for the local maximum of the correlation function while the minimum ($C_j < 0$) is not bounded from below.

4. Peculiarities of the correlation function in pure baryonic universes

Let us now analyze behavior of the correlation function $\xi(r)$ for baryonic models with initial isocurvature perturbations. In this case the correlation function is given by (3.1) where we assume that the spectrum of initial isocurvature perturbations is $P(k) \sim k^{-n}$ and the transfer function is [28]

$$T(k) = \frac{(kR)^4}{(1 + k^2 R^2)^2} (1 + e^{-kr} \sin(kR))^2, \quad (4.1)$$

where r_d , R_* , and R represent correspondingly the damping, the filter, and the horizon scale. This transfer function describes baryonic dark matter models without secondary ionization of matter and PBI models as well. These models are distinguished by different scales of the acoustic horizon at the moment of recombination R_* and the Silk damping scale, r_s . In models without secondary ionization, for $\Omega_b \sim 0.1 - 0.2$, $R \sim 50h^{-1} \text{Mpc}$, and $r_s \sim 4 - 5h^{-1} \text{Mpc} \ll R$. For PBI models [11] the spectrum of density perturbations is well approximated by a spectrum obtained using (4.1) with $R \sim 70h^{-1} \text{Mpc}$, and $r_s \sim 27h^{-1} \text{Mpc}$.

As was mentioned in the introduction, effects of the long distance correlations in $\xi(r)$ in the baryonic universe appear due to the existence of two resonance scales in the transfer function at $r = R$ and $r = 2R$. Assuming that $P(k) \sim k^{-n}$ we estimate the amplitude and the half width of the first resonance in the correlation function for baryonic models

without secondary ionization. Using (3.1) and (4.1) we rewrite the correlation function in the form

$$\xi_\delta(r) = \frac{A}{rR^{2-n}} \int \frac{y^{5-n}}{(1+y^2)^2} e^{-(yR_*/R)^2} (1 + e^{-(yrs/R) \sin y})^2 \sin(r/R) dy, \quad (4.2)$$

where $y = kR$. In Fig. 3 we show, for comparison, observational data of Loveday et al. [29], the correlation function for a baryonic universe with $\Omega_b = 0.2$, $R = 52.0h^{-1}$ Mpc, $rs = 4.6h^{-1}$ Mpc, and the correlation function for a CDM model. From Fig. 3 it is apparent that the observational data are reproduced by the power law spectrum with $1 \leq n \leq 2$. For $n = 2$, taking $R_* = 0$, it is possible to obtain an analytic relation between the amplitude, the width of the first resonance, and rs . Indeed, in this case, in the vicinity of the first resonance the correlation function can be approximated by

$$\xi_\delta(r \rightarrow R) \sim \frac{A}{R} \int \tilde{f}(y) \left[1 - \frac{1}{2} \left(\frac{r-R}{R} \right)^2 \int \tilde{f}(y) dy \right] dy, \quad (4.3)$$

where $\tilde{f}(y) = \frac{y^2 e^{-(yrs/R)}}{(1+y^2)^2}$, and A is determined from the normalization condition $\xi(r_n) = 1$, so

$$\xi_\delta(r_n) = 1 = \frac{A}{r_n} \left[\int \tilde{f}(y) dy + \frac{1}{2} \int \tilde{f}(y) e^{-(yrs/R)} \sin(yr_n/R) dy \right]. \quad (4.4)$$

Taking into account that $rs \ll R$, and $r_n \ll R$, the integrals appearing on the right hand side of (4.3) and (4.4) can be easily evaluated. We obtain

$$\xi_\delta(r \rightarrow R) = \frac{2(C + \ln(R/rs))}{3\pi - \arctan(2rs/r_n)} \frac{r_n}{R} \left(1 - \frac{1}{2} \left(\frac{r-R}{rs} \right)^2 \right) \frac{1}{C + \ln(R/rs)}, \quad (4.5)$$

where C is the Euler constant.

Using similar reasoning for the second resonance we find

$$\xi_\delta(r \rightarrow 2R) = -\frac{1}{3\pi - \arctan(2rs/r_n)} \frac{r_n}{2R} \frac{r-2R}{2rs} \left(1 - \frac{1}{6} \left(\frac{r-2R}{2rs} \right)^2 \right). \quad (4.6)$$

The relations (4.5) and (4.6) describe how amplitudes and widths of the correlation function depend on R and rs . The width of the correlation function depends almost linearly on rs and this property holds for any n .

It is interesting to compare the general behavior of the correlation function close to resonances in the CDM model and in the baryonic universe. The following differences exist in the behavior of the correlation function in those models in the vicinity of the first and the second resonance: The amplitude of the first resonance in the CDM model considered here is bounded from above ($C_1 < 1$) and decreases as $1/R_1^2$ with increasing R , (see (3.9)) while the amplitude of the resonance in the baryonic model is proportional to $1/R$. As is apparent from Fig. 4, for $R = 50$ Mpc the resonance in the baryonic model is more pronounced, $\xi_{CDM}(2R_1) < \xi_\delta(2R_1)$ (see also Fig. 5).

In the vicinity of the second resonance in the baryonic model correlation and anticorrelation appear (see (4.6)), while in the CDM model, at the same scale, neither correlation

nor anticorrelation are present (see (3.6)). Therefore in principle it is possible to distinguish the baryonic model from the CDM model by observing the existence of resonance scales and the behavior of $\xi(r)$ in the vicinity of resonances.

5. The correlation function in a baryonic model with secondary ionization

In the baryonic model with secondary ionization [9, 11] the Silk damping scale of adiabatic perturbations is comparable with the acoustic horizon R at the moment of recombination. In this case oscillations in the correlation function become smeared out and instead of oscillations the power increases at scales $R \sim 100$ Mpc. As we already mentioned, the transfer function for such models is similar to the transfer function for models with secondary ionization. The relation between R and rs is however different for different models. The width of the resonance in the correlation function λ can be estimated in the following way

$$\lambda^2 = \frac{\int y^{5-n} e^{-rs/R} / (1+y^2)^2 dy}{\int y^{7-n} e^{-rs/R} / (1+y^2)^2 dy} \sim \left(\frac{rs}{R} \right)^2 \quad (5.1)$$

up to $rs \sim 0.75R$, and with the accuracy of 10–15% when the spectral index $n = 0 - 0.5$.

In the PBI models $rs \sim R$, and therefore instead of a resonance in the correlation function its value at large scales increases in comparison with the correlation function in CDM models [11]. It turns out that with appropriate choice of parameters of the modulation function it is possible to increase the power of the CDM spectrum at a given scale and imitate the spectrum of the PBI models (see Fig. 6). Let us consider an inflationary model with two breaks separated by a plateau in the inflation potential located in a region of values of the inflation field corresponding to the scale R . In Fig. 7 we present correlation functions for a PBI model, a standard CDM model, and a CDM model with modified inflation potential. In comparison with the standard CDM model the correlation function in our model has more power at large scales and it closely resembles the correlation function of the PBI model.

There exists another basic difference between the PBI model and the standard CDM model, namely, their spectra on small scales ($k \geq 1$ Mpc $^{-1}$) at the moment just before the beginning of formation of the large scale structure are different. According to Cen et al. [11] already at $z \approx 5$ in PBI models there is much more structure than in the standard CDM model. This fact could be important for explanation of the existence of quasars at high redshifts. It is interesting to see if in CDM models it is possible to simultaneously increase power of perturbations not only in the large scale region but also in localized small scale regions without violating existing observational constraints. Because in CDM models the thermal history of the plasma, for $z < 10^4$, practically does not influence the spectrum of perturbations in the dark matter component, it is necessary to consider production of additional power in the spectrum during inflation. From (2.22) we see that by placing a break in the potential at a scale corresponding to 1 Mpc and smaller we can increase the power at that scale without changing the spectrum in its low frequency region. Therefore appropriate modification of the inflation potential can stimulate earlier formation of small scale structure in CDM models.

6. Conclusions

We have shown that the correlation function of CDM models with an appropriately modified inflation potential can very closely imitate the correlation function of PBI models. Modifications of the inflaton potential make it possible also to increase the power of the spectrum of perturbations at different scales and to reproduce the characteristic features in the correlation function of the PBI models.

The characteristic resonances in CDM models and in baryonic models with a standard thermal history are different. In CDM models the amplitude of the first resonance is smaller than in the baryonic models and in addition the correlation function in PBI models possesses a bump and a depression at scale $\approx 2R$. The existence of the second resonance in CDM models is connected with generation of a secondary adiabatic mode and therefore a similar feature in the correlation function appears also in baryonic models with an initial adiabatic spectrum of perturbations [1]. In our model it is not possible to reproduce the bump and the depression which appear at similar scales in the CDM model because in the limit $(k_2 - k_1)/k_2 \rightarrow 0$ the modulation function (2.12) tends to a modulation function in a theory with only one break in the potential and generates either a bump or a depression in the correlation function. Therefore the structure of the resonance on the scale $2R$ is characteristic for the baryonic models and it can be used as a test of the nature of the dark matter.

Let us notice that long distance correlations for baryonic models are not altered by a non zero cosmological constant.

In this paper we have not discussed temperature anisotropies of the microwave background radiation but let us mention that including a cosmological constant in baryonic models decreases the amplitude of temperature fluctuations because the density perturbations grow faster in flat models than in open models [27].

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Figure captions

Fig.1. Potential of the inflaton field φ with two breaks. In inflationary models φ evolves from some sufficiently large positive value toward zero. A_2^+ denotes the value of the right derivative and A_1^- the value of the left derivative of the potential at corresponding breaks. A is the intermediate inclination.

Fig.2. Correlation function for a CDM model with two characteristic features (solid line). $\xi_1(r) + \xi_2(r)$ - long dashed line. $\xi_0(r)$ - short dashed line. In calculations we assume that $C_1 = 0.9$, $C_2 = -1$, $R_1 = 25\text{Mpc}$, $R_2 = 50\text{Mpc}$, $R_* = 3\text{Mpc}$, $\Omega_b = 0.01$, $\Omega_{CDM} = 0.99$. In all cases we use standard normalization $\xi(r = 5\text{Mpc}) = 1$.

Fig.3. Observed correlation function and correlation functions in CDM and baryonic models. Observational points are taken from [29]. CDM model - dashed line, baryonic model - solid line. Parameters of the CDM model: $C_1 = 0.9$, $C_2 = 0.5$, $R_1 = 25\text{Mpc}$, $R_2 = 2R_1$. Parameters of the baryonic model: $n = 1.8$, $R = 52\text{Mpc}$, $r_s = 4.6\text{Mpc}$, $R_* = 3\text{Mpc}$, $\Omega = 0.2$.

Fig.4. Correlation functions for a pure baryonic model (parameters as on Fig. 3) - solid line, and for a CDM model (parameters as on Fig. 2) - short dashed line.

Fig.5. The first resonance in the correlation function for three different baryonic models - solid line, and three CDM models - dashed line at the same resonance scale $R_1 = R/2$. Parameters of the baryonic models: 1. $R = 32.8\text{Mpc}$, $r_s = 2.9\text{Mpc}$, for $\Omega = 0.3$; 2. $R = 52.0\text{Mpc}$, $r_s = 4.6\text{Mpc}$, for $\Omega = 0.2$; 3. $R = 81.4\text{Mpc}$, $r_s = 14.5\text{Mpc}$, for $\Omega = 0.1$. For the CDM models we assumed $C_1 = 0.99$, $C_2 = 0.5$.

Fig.6. Spectrum of perturbations in a PBI model calculated in linear approximation (adopted form [11]) - solid line, and in a CDM model generated by inflation with two breaks in the inflation potential - dashed line. Parameters of the CDM model: $R_1 = 133\text{Mpc}$, $R_2 = 400\text{Mpc}$, $C_1 = 4.48$, $C_2 = 0.2$. Both spectra are normalized and have the same value at 8Mpc (normalizing the CDM spectrum we neglected the oscillating term).

Fig.7. Correlation function for a CDM model (parameters as in Fig. 6, and $R_* = 0.1\text{Mpc}$) - long dashed line compared with the correlation function for a PBI model obtained by Cen et al. [11] - solid line. Correlation functions are normalized and have the same value at 8Mpc . Correlation function in the standard CDM model is represented by short-dashed line. The circles represent error bars for the PBI correlation function.

Fig. 2

Fig. 3

Fig.6

Fig.7