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**TO THE PROBLEM OF D-MESON LEPTONIC
AND SEMILEPTONIC DECAYS**

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ABSTRACT

The infinite mass effective theory, when a heavy quark mass goes to infinity, and chiral perturbation theory at the quark level, based on the extended Nambu-Jona-Lasinio model, are applied for the description of form factors of semileptonic decay $D \rightarrow \bar{K} e^+ \nu_e$ and leptonic constant F_D of D -mesons. There is derived the relation, connecting the form factor $f_+(0)$ with the ratio F_D/F_π , which gives the estimate $F_D/F_\pi \simeq 1.20 \pm 0.17$. The slope parameter of the f_+ -form factor is calculated in agreement with experimental data.

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In refs. [1,2] there has been developed chiral perturbation theory at the quark level (CHPT)_q based on the extended Nambu-Jona-Lasinio (ENJL) model with linear realization of chiral $U(3) \times U(3)$ symmetry [3] used as a low-energy approximation of QCD. Indeed as has been shown in ref. [4] QCD with a linearly rising confinement potential realizes the NJL mechanism of spontaneous breaking of chiral symmetry (SBCS). It implies that the low-energy QCD and the ENJL model possess apparently the similar wave functions of nonperturbative vacua [5] and the same spectra of low-lying quark antiquark collective excitations with quantum numbers of low-lying observable mesons [1,2]. The applications of (CHPT)_q for the description of semileptonic and nonleptonic decays of K -mesons were successfully performed in refs. [6,7].

In this paper we are expanding the area of the applicability of (CHPT)_q on the weak decays of D -mesons. We are considering the evaluation of D -meson leptonic constant and form factors of semileptonic decays $D \rightarrow \bar{K} e^+ \nu_e$ ($D^0 \rightarrow \bar{K}^- e^+ \nu_e$ and $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$).

For the description of D -mesons we are applying the infinite mass effective theory (IMET) [8,9], when the c -quark mass M_c goes to infinity, i.e. $M_c \rightarrow \infty$. In IMET the long-distance physics can be described by an effective theory with a static colour source. By our admission this effective theory, taken in the leading order in large N expansion, can be reduced to (CHPT)_q [10] (see also [11] by Bardeen and Hill).

At first let us consider the semileptonic decays $D \rightarrow \bar{K} e^+ \nu_e$. The amplitude of these decays reads

$$M(D \rightarrow \bar{K} e^+ \nu_e) = -\frac{G_F}{\sqrt{2}} V_{cs}^* \langle \bar{K}(p') | \bar{s}(0) \gamma_\mu c(0) | D(p) \rangle l^\mu, \quad (1)$$

where $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant, $|V_{cs}| = 0.975$ is the CKM-mixing matrix element, $s(0)$ and $c(0)$ are the s - and c -current-quark fields having N colour degrees of freedom, $p^\mu = M_D v^\mu$, where v^μ is the velocity of the D -meson with components $v^\mu = (1, 0, 0, 0)$ in the rest frame, and $l^\mu = \bar{u}(k_\nu) \gamma^\mu (1 - \gamma^5) v(k_e)$ is the weak leptonic current. The matrix element

$$M_\mu(D \rightarrow \bar{K}) = \langle \bar{K}(p') | \bar{s}(0) \gamma_\mu c(0) | D(p) \rangle l^\mu, \quad (2)$$

can be parametrized by two form factors [12,13]

$$M(D \rightarrow \bar{K}) = f_+(q^2)(p + p')_\mu + f_-(q^2)(p - p')_\mu, \quad (3)$$

where $q^2 = (p - p')^2$ is the invariant square mass of the leptonic pair. The partial width of the $D \rightarrow \bar{K} e^+ \nu_e$ decay can be expressed in terms of $f_+(0)$ [13,14]

$$\begin{aligned} \Gamma(D \rightarrow \bar{K} e^+ \nu_e) &\simeq \frac{29 G_F^2 |V_{cs}|^2}{192 \pi^3 M_D^3} |f_+(0)|^2 = \\ &= 0.96 \times 10^{-13} |f_+(0)|^2 \text{ GeV}. \end{aligned} \quad (4)$$

The experimental value of $f_+(0)$ obtained at $|V_{cs}| = 0.975$ [14] is

$$|f_+(0)|_{\text{exp}} = 0.7 \pm 0.1 . \quad (5)$$

In order to explore the matrix element (2) within IMET we are applying the reduction technique [12]

$$M_\mu(D \rightarrow \bar{K}) = i \int d^4x e^{-ipx} (\square_x + M_D^2) \langle \bar{K}(p') | T(\bar{s}(0) \gamma_\mu c(0) \phi_D^+(x)) | 0 \rangle, \quad (6)$$

where $\phi_D^+(x)$ is the interpolating operator of the D -meson field.

For the analysis of the r.h.s. of (6) at the quark level we admit that the operator $\phi_D^+(x)$ satisfies the equation

$$(\square_x + M_D^2) \phi_D^+(x) = i g_D \bar{c}(x) \gamma^5 q(x), \quad (7)$$

where $q(x) = u(x)$ or $d(x)$ for D^0 - and D^+ - meson, respectively, g_D is a constant which we shall fix bellow. By substituting (7) into the r.h.s. of (6) one obtains

$$M_\mu(D \rightarrow \bar{K}) = i \int d^4x e^{-ipx} i g_D \langle \bar{K}(p') | T(\bar{s}(0) \gamma_\mu c(0) \bar{c}(x) \gamma^5 q(x)) | 0 \rangle. \quad (8)$$

The application of IMET [8,9] to the r.h.s. of (8) allows us to turn it into the form

$$M_\mu(D \rightarrow \bar{K}) = i \int_{-\infty}^{\infty} dx_0 i g_D C(\mu) \theta(x_0) \times \\ \times \langle \bar{K}(p') | T([e^{-i \int_0^{x_0} dz_0 g_s G_0(z_0, 0)}]_{ab} \bar{s}(0)_a \gamma_\mu \left(\frac{1 + \gamma^0}{2} \right) \gamma^5 q(x_0, 0)_b) | 0 \rangle. \quad (9)$$

Here $C(\mu)$ is the Wilson coefficient, $G_0(z_0, 0)$ is the gluon potential, and μ is a normalization scale [9]. Also we have omitted the exponential $\exp i(M_C - M_D)x_0$, appearing in the integrand and being equal to unity at $M_C = M_D$. Below we are keeping to this equality. In this case numerically M_C is of order of the constituent c -quark mass, i.e. $M_C = M_D = 1.87$ GeV. Physically this restriction on M_C can be justified by the obvious fact that the c -quark mass cannot exceed of the D -meson one.

In the leading order in large N expansion the r.h.s. of (9) acquires the form

$$M_\mu(D \rightarrow \bar{K}) = i \int_{-\infty}^{\infty} dx_0 i \tilde{g}_D(\mu) \theta(x_0) \times \\ \times \langle \bar{K}(p') | T(\bar{s}(0) \gamma_\mu \left(\frac{1 + \gamma^0}{2} \right) \gamma^5 q(x_0, 0)) | 0 \rangle, \quad (10)$$

where we have summed up over colour indices and denoted $\tilde{g}_D(\mu) = g_D C(\mu)$. For the evaluation of the matrix element

$$i \int_{-\infty}^{\infty} dx_0 \theta(x_0) \langle \bar{K}(p') | T(\bar{s}(0) \gamma_\mu \left(\frac{1+\gamma^0}{2} \right) \gamma^5 q(x_0, 0)) | 0 \rangle, \quad (11)$$

formally involving only the light-quark fields, we are applying (CHPT)_q. Resultely we get [1] (see also [11]):

$$\begin{aligned} & i \int_{-\infty}^{\infty} dx_0 \theta(x_0) \langle \bar{K}(p') | T(\bar{s}(0) \gamma_\mu \left(\frac{1+\gamma^0}{2} \right) \gamma^5 q(x_0, 0)) | 0 \rangle = \\ &= -\frac{N}{16\pi^2} \frac{ig_1 Z^{1/2}}{\sqrt{2}} \int \frac{d^4 k}{\pi^2 i} \text{tr} \left\{ \frac{1}{m - \hat{k}} \gamma^5 \frac{1}{m - \hat{k} - \hat{p}'} \gamma_\mu \left(\frac{1+\gamma^0}{2} \right) \frac{1}{k^0} \gamma^5 \right\} = \\ &= -\frac{N}{16\pi^2} \frac{ig_1 Z^{1/2}}{\sqrt{2}} \int \frac{d^4 k}{\pi^2 i} \text{tr} \left\{ \frac{1}{m - \hat{k}} \gamma^5 \frac{1}{m - \hat{k} - \hat{p}'} \gamma_\mu \left(\frac{1+\hat{v}}{2} \right) \frac{1}{v \cdot k} \gamma^5 \right\} = \\ &= \frac{2i}{g_1} \left[\left(\frac{\bar{v}'}{2} - \frac{M_D^2 - q^2}{M_D} \ln \left(1 + \frac{\bar{v}' M_D}{2} \frac{1}{M_D^2 - q^2} \right) \right) v_\mu + \right. \\ & \quad \left. \ln \left(1 + \frac{\bar{v}' M_D}{2} \frac{1}{M_D^2 - q^2} \right) p'_\mu \right]. \end{aligned} \quad (12)$$

The appearance of the q^2 -dependence is due to the quantity $p'_0 = (M_D^2 - q^2)/2M_D$, being the energy of the massless $\bar{K}$ -meson in the rest frame of the D -meson. Then $g_1 = 2m/F_0 Z^{1/2}$ is the coupling constant satisfying the relation [15]

$$\frac{Ng_1^2}{8\pi^2} = 1, \quad (13)$$

$m = 0.33$ GeV and $F_0 = 0.092$ GeV are the mass of constituent light quarks and the PCAC constant of light pseudoscalar mesons, respectively, calculated in the chiral limit [1], $Z = 2$ is the renormalization constant of the pseudoscalar meson nonet caused by SBCS [1]. At last $\bar{v}' = 4\Lambda = 2.62$ GeV where Λ is the cut-off in Euclidean 3-momentum space connected with the SBCS scale in (CHPT)_q by the relation $\Lambda = \Lambda_\chi/\sqrt{2} = 0.66$ GeV. By using the formulae (12), (13) and $Z = 2$ we can reduce $M_\mu(D \rightarrow \bar{K})$ to the form

$$\begin{aligned} M_\mu(D \rightarrow \bar{K}) = & -\frac{1}{\sqrt{2}} \frac{\hat{g}_D}{M_D} \frac{\sqrt{N}}{2\pi} \frac{\bar{v}'}{2} \left[\left(1 + \frac{q^2}{M_\star^2} \ln \left(1 + \frac{M_\star^2}{M_D^2 - q^2} \right) \right) (p + p')_\mu - \right. \\ & \left. \left(2 \frac{M_D^2}{M_\star^2} \left(1 - \frac{q^2}{2M_D^2} \right) \ln \left(1 + \frac{M_\star^2}{M_D^2 - q^2} \right) - 1 \right) (p - p')_\mu \right], \end{aligned} \quad (14)$$

where $M_\star = \sqrt{M_D \bar{v}'/2}$. It is seen that $M_\mu(D \rightarrow \bar{K})$ still contains the unknown parameter $\hat{g}_D(\mu)$. We propose to express $\hat{g}_D(\mu)$ in terms of F_D being the PCAC constant of D -mesons and determined by the matrix element

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma^5 c(0) | D(p) \rangle = i\sqrt{2} F_D M_D v_\mu, \quad (15)$$

where $q(0) = u(0)$ or $d(0)$ for D^0 - and D^+ -meson, respectively.

By applying IMET and keeping only the leading order in large N expansion one can reduce the l.h.s. of (15) to the vacuum expectation value (v.e.v.) involving the light quark fields only

$$\begin{aligned} i \int_{-\infty}^{\infty} dx_0 \, i\tilde{g}_D(\mu)\theta(x_0)\langle 0|T(\bar{q}(0)\gamma_\mu\gamma^5\left(\frac{1+\gamma^0}{2}\right)\gamma^5 q(x_0,0))|0\rangle = \\ = i\sqrt{2}F_D M_D v_\mu. \end{aligned} \quad (16)$$

The evaluation of this v.e.v. within (CHPT)_q gives us

$$\begin{aligned} i \int_{-\infty}^{\infty} dx_0 \, \theta(x_0)\langle 0|T(\bar{q}(0)\gamma_\mu\gamma^5\left(\frac{1+\gamma^0}{2}\right)\gamma^5 q(x_0,0))|0\rangle = \\ = -\frac{N}{16\pi^2} \int \frac{d^4 k}{\pi^2 i} \operatorname{tr} \left\{ \frac{1}{m - \hat{k}} \gamma_\mu \gamma^5 \left(\frac{1+\gamma^0}{2} \right) \frac{1}{k^0} \gamma^5 \right\} = \\ = -\frac{N}{16\pi^2} \int \frac{d^4 k}{\pi^2 i} \operatorname{tr} \left\{ \frac{1}{m - \hat{k}} \gamma_\mu \gamma^5 \left(\frac{1+\hat{v}}{2} \right) \frac{1}{v \cdot k} \gamma^5 \right\} = \\ = -\frac{F_0}{g_1 Z^{1/2}} (\bar{v} + \bar{v}') v_\mu = -\frac{\sqrt{N}}{4\pi} F_0 (\bar{v} + \bar{v}') v_\mu, \end{aligned} \quad (17)$$

where we have denoted

$$\bar{v} = -\langle 0|\bar{q}q|0\rangle/F_0^2 = 1.92 \text{ GeV} [1] \quad (18)$$

Also in the formula (17) we have used the relation (13) and $Z = 2$.

From the formulae (16) and (17) we can obtain $\tilde{g}_D(\mu)$

$$\tilde{g}_D(\mu) = -\sqrt{2} \frac{F_D}{F_0} \frac{2\pi}{\sqrt{N}} \frac{2M_D}{\bar{v} + \bar{v}'}. \quad (19)$$

By substituting (19) in (14) we define $M_\mu(D \rightarrow \bar{K})$ in terms of F_D and (CHPT)_q parameters

$$\begin{aligned} M_\mu(D \rightarrow \bar{K}) = \frac{F_D}{F_0} \frac{\bar{v}'}{\bar{v} + \bar{v}'} \left[\left(1 + \frac{q^2}{M_\star^2} \ln \left(1 + \frac{M_\star^2}{M_D^2 - q^2} \right) \right) (p + p')_\mu - \right. \\ \left. \left(2 \frac{M_D^2}{M_\star^2} \left(1 - \frac{q^2}{2M_D^2} \right) \ln \left(1 + \frac{M_\star^2}{M_D^2 - q^2} \right) - 1 \right) (p - p')_\mu \right] \end{aligned} \quad (20)$$

By matching the formula (20) with the formula (3) we get

$$f_+(q^2) = f_+(0) \left[1 + \frac{q^2}{M_\star^2} \ln \left(1 + \frac{M_\star^2}{M_D^2 - q^2} \right) \right] = f_+(0) \left[1 + \frac{q^2}{\Lambda_\star^2} + O(q^4) \right], \quad (21)$$

$$f_+(0) = \frac{F_D}{F_0} \frac{\bar{v}'}{\bar{v} + \bar{v}'} = 0.58 \frac{F_D}{F_0} , \quad (22)$$

$$f_-(0) = -f_+(0) \left[2 \frac{M_D^2}{M_*^2} \left(1 - \frac{q^2}{2M_D^2} \right) \ln \left(1 + \frac{M_*^2}{M_D^2 - q^2} \right) - 1 \right], \quad (23)$$

$$f_-(0) = -f_+(0) \left[2 \frac{M_D^2}{M_*^2} \ln \left(1 + \frac{M_*^2}{M_D^2} \right) - 1 \right] = -0.52 f_+(0). \quad (24)$$

The parameter Λ_* , which reads

$$\Lambda_* = \frac{M_*}{\sqrt{\ln \left(1 + \frac{M_*^2}{M_D^2} \right)}} = 2.15 \text{ GeV} , \quad (25)$$

can be considered as a slope parameter. The theoretical value of which $\Lambda_* = 2.15 \text{ GeV}$ is in good agreement with the experimental value $(\Lambda_*)_{\text{exp}} = 2.1_{-0.2}^{+0.4} \text{ GeV}$ [16]. From the formula (22) we give the ratio F_D/F_0 in terms of $f_+(0)$ and the (CHPT)_q parameters

$$\frac{F_D}{F_0} = f_+(0) \frac{\bar{v} + \bar{v}'}{\bar{v}'} = 1.73 f_+(0). \quad (26)$$

This formula can be valued as a kind of Callan-Treiman theorem [17] for the $D \rightarrow \bar{K} e^+ \nu_e$ decay. By using the experimental value of $f_+(0)$ (5) we estimate

$$\frac{F_D}{F_0} = 1.21 \pm 0.17 . \quad (27)$$

This result agrees with estimates obtained within different approaches [13,18] and the experimental restriction $\sqrt{2} F_{D^+} < 310 \text{ MeV}$ ($CL = 90 \%$) [14].

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