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CHIRAL SYMMETRY BREAKING IN QCD WITH LINEARLY RISING CONFINEMENT POTENTIAL

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Abstract

Chiral symmetry breaking is explored within QCD with linearly rising confinement potential at the Nambu-Jona-Lasinio (NJL) prescription. The dynamical quark mass is obtained from the gap-equation in terms of the string tension σ , i.e. $m = 2\sqrt{\sigma}/\pi = 0.33$ GeV at $\sigma = 0.27$ GeV². The phenomenological constants of the NJL local four-quark interactions G_1 and G_2 are found equal and expressed in terms of the string tension and the confinement radius. It is shown that the spectra of low-lying quark-antiquark collective excitations coincide with those predicted by the NJL model.

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1. Introduction

It is well-known that the Nambu-Jona-Lasinio (NJL) model [1,2] realizes a very nice mechanism of spontaneous breaking of chiral symmetry (SBCS). The chiral symmetry breaking, being due to the strong attraction in the quark-antiquark channels produced by the NJL-local-four-quark interactions, leads to the appearance of quark-antiquark collective excitations with quantum numbers of low-lying mesons [1-4]. Within linear realization of chiral $U(3) \times U(3)$ symmetry there appear four nonets of scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) quark-antiquark collective excitations [1-4]. The main shortcoming of the NJL-model, if one likes treating it as a real theory of strong low-energy interactions of low-lying hadrons [3], is the absence of confinement. The latter makes unstable under the decay into quark-antiquark pairs the quark-antiquark collective excitations with masses in excess of the twice mass of constituent quark [3,4]. Actually these are vector and axial-vector collective excitations.

The lattice QCD experience [5] and the dual-superconducting picture of QCD [6] lets us know that confinement in a quark-antiquark pairs can be realized via the linearly rising potential

$$V(\vec{x}_1 - \vec{x}_2) = \sigma |\vec{x}_1 - \vec{x}_2| - 2\sqrt{\sigma}, \quad (1)$$

where $\vec{x}_{1,2}$ are the positions of quark and antiquarks, and σ is the string tension. Below for σ we use the numerical value $\sigma = 0.27$ GeV², obtained from the fit of spectra mass of low-lying mesons within a potential model [7]. Can the quark-antiquark interaction, provided by the potential (1), lead to SBCS and what kinds of quark-antiquark collective excitations can be produced because of it? The answers on these questions we explore in this paper.

First of all, let us remind the main features of the NJL-mechanism of SBCS. The total Lagrangian of quarks interacting in the NJL manner reads

$$L(x) = \bar{\psi}(x) i \gamma^\mu \partial_\mu \psi(x) + L_{\text{NJL}}(x), \quad (2)$$

where $L_{\text{NJL}}(x)$ is the NJL local four-quark interaction invariant under $U(3) \times U(3)$ chiral rotations [1-5]:

$$L_{\text{NJL}}(x) = 2G_1[(\bar{\psi}(x)t^a\psi(x))^2 + (\bar{\psi}(x)i\gamma^5 t^a\psi(x))^2] - 2G_2[(\bar{\psi}(x)\gamma_\mu t^a\psi(x))^2 + (\bar{\psi}(x)\gamma_\mu \gamma^5 t^a\psi(x))^2]. \quad (3)$$

Here $\psi = (\psi_u, \psi_d, \psi_s)$ are massless current quark fields with N colour degrees of freedom, and t^a ($a = 0, 1, \dots, 8$) denote the Gell-Mann matrices of the $SU(3)_{\text{flavour}}$ group normalized by the condition $\text{tr}(t^a t^b) = \delta^{ab}/2$. The strong attraction in the $\bar{\psi}\psi$ -channel, leading to SBCS and the appearance of four nonets of $\bar{\psi}\psi$ -collective excitations with quantum numbers of low-lying scalar, pseudoscalar, vector and axial-vector mesons, is ensured by the corresponding values of phenomenological coupling constants G_1 and G_2 . The G_1 constant takes on SBCS and its value can be determined from the gap-equation $G_1 \simeq$

10 GeV⁻² [4]. The constant G_2 defines the mass spectra of vector and axial-vector $\bar{\psi}\psi$ -collective excitations. Below we shall show that $G_2 = G_1$.

In the one-quark-loop approximation the NJL-interaction (3) generates the following contribution to the dynamical quark mass [1-4]:

$$\delta L_{\text{eff}}^{(\text{mass})}(x) = -4G_1 \text{tr}_{\text{C+L+F}}[iS_F(0)t^a]\bar{\psi}(x)t^a\psi(x) = -2G_1 \text{tr}_{\text{C+L}}[iS_F(0)]\bar{\psi}(x)\psi(x). \quad (4)$$

Here indices C, F and L denote the evaluation of traces over colour, flavour and Lorentz indices. $S_F(x)$ is the Green's function of a Dirac particle with mass m , considered as the dynamical quark mass. The quantity $\text{tr}_{\text{C+L}}[iS_F(0)]$ reads

$$\text{tr}_{\text{C+L}}[iS_F(0)] = \frac{N}{16\pi^2} \int \frac{d^4k}{\pi^2 i} \text{tr}_L \left(\frac{1}{m - \hat{k}} \right) = 4mI_1(m), \quad (5)$$

where $\hat{k} = \gamma^\mu k_\mu$ and $I_1(m)$ is a quadratically divergent integral

$$I_1(m) = \frac{N}{16\pi^2} \int \frac{d^4k}{\pi^2 i} \frac{1}{m^2 - k^2} = \frac{N}{16\pi^2} \left[\Lambda_\chi^2 - m^2 \ln \left(1 + \frac{\Lambda_\chi^2}{m^2} \right) \right]. \quad (6)$$

Here Λ_χ is the Euclidean 4-momentum cut-off playing the role of the SBCS scale [1-4].

By following the NJL-prescription and assuming that the dynamical mass m is fully determined by the one-quark-loop contribution [1], one gets the gap-equation

$$m = m8G_1I_1(m). \quad (7)$$

In the case $G_1 = 0$ the gap-equation has only the trivial solution $m = 0$. For $G_1 \neq 0$ there may exist, in addition to the trivial solution $m = 0$, corresponding to the chirally invariant phase of the quark system with local four-quark interaction, the non-trivial solution $m = \Lambda_\chi \phi(G_1\Lambda_\chi^2)$. This solution describes the chirally broken phase of the quark system with the interaction (3). In this phase the quark system is unstable under the creation of $\bar{\psi}\psi$ -collective excitations with quantum numbers of low-lying mesons, and it possesses the ground state, which is the non-perturbative many-particle vacuum of superconductivity type [1].

Of course, one can characterize the SBCS-phase of the NJL-quark system in terms of the quark condensate, having the meaning of the order parameter of SBCS in QCD. At the one-quark-loop approximation the quark condensate reads [3,4,8]

$$\langle 0|\bar{\psi}\psi|0\rangle = -\text{tr}_{\text{C+L}}[iS_F(0)] = -4mI_1(m) = -\frac{Nm}{4\pi^2}[\Lambda_\chi^2 - m^2 \ln(1 + \frac{\Lambda_\chi^2}{m^2})]. \quad (8)$$

The detailed exploration of the quark condensate properties in the NJL-model has been performed in refs.[3,4,8]. By using the numerical values $m = 0.33\text{GeV}$ and $\Lambda_\chi = 0.94\text{GeV}$,

determined within the NJL model [3,4,8], one can calculate the numerical value of the quark condensate

$$\langle 0|\bar{\psi}\psi|0\rangle = -(0.255\text{GeV})^3. \quad (9)$$

describing correctly the mass spectrum of pseudoscalar mesons within $(CHPT)_q$ [4] via the Gell-Mann-Oaks-Renner formulae at the current quark masses ($m_{0u} = 4\text{MeV}$, $m_{0d} = 7\text{MeV}$ and $m_{0s} = 135\text{MeV}$), obtained within QCD at the normalization point $\mu = 1\text{ GeV}$ [9].

2. Chiral symmetry breaking

From the brief outline of the NJL mechanism of SBCS one can see that in order to analyse this effect in QCD we must regard the four-quark interaction, formally describing the $\bar{\psi}\psi$ (or $\psi\psi$)-scattering. Of course, the evaluation of such interaction within QCD is very complicated problem which can hardly be resolved at present. However, for our aim we need the interaction defined at large distances only. In this case by following the Adler's theory of static quark forces [10] we can admit that the main contribution to the $\bar{\psi}\psi$ -interaction at large distances comes from the interaction between colour quark charges. The total effective action of quark fields, having only the $\rho \times \rho$ interaction, where ρ is a quark colour charge density, reads

$$-iS_{\text{st. QCD}}^{(\bar{\psi}\psi)} = \int d^4x \bar{\psi}(\vec{x}, t) i\gamma^0 \frac{\partial}{\partial t} \psi(\vec{x}, t) - iS_{\text{st. int}}^{(\bar{\psi}\psi)}. \quad (10)$$

The second term describes the $\rho \times \rho$ interaction

$$\begin{aligned} -iS_{\text{st. int}}^{(\bar{\psi}\psi)} &= -\frac{1}{2} \int d^3x_1 d^3x_2 \int_{-\infty}^{\infty} dt \rho^A(\vec{x}_1, t) V(\vec{x}_1 - \vec{x}_2) \rho^A(\vec{x}_2, t) = \\ &= -\frac{1}{2} \int d^3x_1 d^3x_2 \int_{-\infty}^{\infty} dt \bar{\psi}(\vec{x}_1, t) \gamma^0 t_c^A \psi(\vec{x}_1, t) V(\vec{x}_1 - \vec{x}_2) \psi(\vec{x}_2, t) \gamma^0 t_c^A \psi(\vec{x}_2, t). \end{aligned} \quad (11)$$

Here t_c^A ($A = 1, \dots, N^2-1$) are the matrices of the $SU(N)_c$ group normalized by $\text{tr}_c(t_c^A t_c^B) = \delta^{AB}/2$.

From the first glance the action (10) is valid only for heavy quarks, since the kinetic term of quark fields does not involve the coordinate derivatives. The neglect of the coordinate derivatives of the light quark fields in (10) can be justified as follows. We admit that the four-quark interaction (11) produces the strong tension between quarks in the $\bar{\psi}\psi$ -channels. By virtue of this tension the quarks are being kept at small relative velocities as a matter of fact. As a result we deal with the light quark fields weakly varying at large relative distances. The latter can provide the neglect of the coordinate derivatives of the light quark fields in the action (10).

By changing the variables

$$\vec{x}_1 = \vec{x}, \quad \vec{x}_2 = \vec{x} - \vec{z},$$

where $\vec{z}$ is the relative radius-vector of the $\bar{\psi}\psi$ -system, we obtain

$$-iS_{\text{st. int}}^{(\bar{\psi}\psi)} = -\frac{1}{2} \int d^4x \int d^3z [\bar{\psi}(\vec{x}, t) \gamma^0 t_c^A \psi(\vec{x}, t) V(z) [\bar{\psi}(\vec{x} - \vec{z}, t) \gamma^0 t_c^A \psi(\vec{x} - \vec{z}, t)]]. \quad (12)$$

Now we need to rewrite this interaction in terms of colourless quark densities. For this aim we have to perform Fierz transformations

$$\begin{aligned} (\gamma^0)_{\alpha\beta}(\gamma^0)_{\gamma\delta} &= \frac{1}{4}(1)_{\alpha\delta}(1)_{\gamma\beta} + \frac{1}{4}(i\gamma^5)_{\alpha\delta}(i\gamma^5)_{\gamma\beta} - \\ &- \frac{1}{4}(\gamma^\mu)_{\alpha\delta}(\gamma_\mu)_{\gamma\beta} - \frac{1}{4}(\gamma^\mu\gamma^5)_{\alpha\delta}(\gamma_\mu\gamma^5)_{\gamma\beta} + \\ &+ \frac{1}{2}(\gamma^0)_{\alpha\delta}(\gamma^0)_{\gamma\beta} + \frac{1}{2}(\gamma^0\gamma^5)_{\alpha\delta}(\gamma^0\gamma^5)_{\gamma\beta}, \end{aligned} \quad (13a)$$

and

$$\begin{aligned} \sum_{A=1}^{N^2-1} (t_c^A)_{i'j'}(t_c^A)_{l'k'}\delta_{ij}\delta_{kl} &= \sum_{a=0}^8 (t^a)_{il}(t^a)_{kj}\delta_{i'k'}\delta_{l'j'} - \\ - \frac{1}{2N}\delta_{i'j'}\delta_{l'k'}\delta_{ij}\delta_{kl} &\xrightarrow{N \rightarrow \infty} \sum_{a=0}^8 (t^a)_{il}(t^a)_{kj}\delta_{i'k'}\delta_{l'j'}. \end{aligned} \quad (13b)$$

By keeping only the leading terms in large N expansion we can rewrite the action (12) as follows

$$-iS_{\text{st. int}}^{(\bar{\psi}\psi)} = \frac{1}{8} \int d^4x \int d^3z [\bar{\psi}(\vec{x}, t) \Gamma \psi(\vec{x} - \vec{z}, t) V(z) [\bar{\psi}(\vec{x} - \vec{z}, t) \Gamma \psi(\vec{x}, t)]], \quad (14)$$

where the sum over $\Gamma = t^a, i\gamma^5 t^a, i\gamma^\mu t^a, i\gamma^\mu\gamma^5 t^a, \sqrt{2}\gamma^0 t^a$ and $\sqrt{2}\gamma^0\gamma^5 t^a$ must be performed. The r.h.s. of (14) contains the noncovariant terms at $\Gamma = \sqrt{2}\gamma^0 t^a$ and $\Gamma = \sqrt{2}\gamma^0\gamma^5 t^a$. The appearance of these terms is due to the noncovariance of the approximation (10) that we have exploited in order to simplify the exploration of the problem under consideration, i.e. the realization of SBCS via confinement. In fact as there will be seen below the noncovariant terms do not contribute to SBCS.

In order to analyse SBCS by using the action (14) and within the NJL-prescription we have to evaluate the one-quark-loop contribution. As a result we get

$$-iS_{\text{st. int}}^{(\bar{\psi}\psi)}|_{\text{one-loop}} = -\frac{1}{4} \int d^4x \int d^3z [\bar{\psi}(\vec{x}, t) \Gamma \psi(\vec{x} - \vec{z}, t)] V(z) \text{tr}_{\text{C+L+F}}[iS_F(\vec{z}) \Gamma] =$$

$$\begin{aligned} &= -\frac{1}{8} \int d^4x \int d^3z [\bar{\psi}(\vec{x}, t) \psi(\vec{x} - \vec{z}, t)] V(z) \text{tr}_{\text{C+L}}[iS_F(\vec{z})] - \\ &- \frac{1}{8} \int d^4x \int d^3z [\bar{\psi}(\vec{x}, t) \vec{\gamma} \psi(\vec{x} - \vec{z}, t)] V(z) \text{tr}_{\text{C+L}}[iS_F(\vec{z}) \vec{\gamma}]. \end{aligned} \quad (15)$$

The appearance of the first and the second terms are caused by the scalar and vector four-quark interactions in (14), respectively. The pseudoscalar, axial-vector and noncovariant interactions do not give any contribution, formally, by virtue of the γ -matrix algebra. The quantities $\text{tr}_{\text{C+L}}[iS_F(\vec{z})]$ and $\text{tr}_{\text{C+L}}[iS_F(\vec{z}) \vec{\gamma}]$ take the form

$$\begin{aligned} \text{tr}_{\text{C+L}}[iS_F(\vec{z})] &= \frac{4mN}{(2\pi)^4 i} \int d^4k \frac{e^{i\vec{k}\cdot\vec{z}}}{m^2 - k_0^2 + \vec{k}^2} = \frac{N m^2}{\pi^2 z} K_1(mz) \\ \text{tr}_{\text{C+L}}[iS_F(\vec{z}) \vec{\gamma}] &= -\frac{i \vec{z}}{m z} \frac{d}{dz} \left[\frac{N m^2}{\pi^2 z} K_1(mz) \right]. \end{aligned} \quad (16)$$

The Mc Donald's function $K_1(mz)$ has an exponential behaviour for large z

$$K_1(mz) \xrightarrow{z \rightarrow \infty} \frac{e^{-mz}}{\sqrt{z}}. \quad (17)$$

It means that $K_1(mz)$ localizes the integrand of the $\vec{z}$ -integration in (15) to the region $|\vec{z}| \leq 1/m$. It is known [11] that the wave-functions of the dynamical quarks represented by the operators $\psi(\vec{x}, t)$ and $\bar{\psi}(\vec{x} - \vec{z}, t)$, should be localized to the regions $|\vec{x}| \leq 1/m$ and $|\vec{x} - \vec{z}| \leq 1/m$, respectively. Resultely it implies that we can expand the operator $\bar{\psi}(\vec{x} - \vec{z}, t)$ in powers of $\vec{z}$ by keeping only the lowest terms nonvanishing after integration over $\vec{z}$.

Let us write down the r.h.s. of (15) by keeping only non-vanishing terms in $\vec{z}$ -expansion:

$$\begin{aligned} -iS_{\text{st. int}}^{(\bar{\psi}\psi)}|_{\text{one-loop}} &= - \int d^4x \bar{\psi}(x) \psi(x) \int d^3z \frac{Nm^2}{8\pi^2} V(z) \frac{K_1(mz)}{z} + \\ &+ \int d^4x \bar{\psi}(x) (-i\gamma_i \nabla_i \psi(x)) \int d^3z \frac{Nm^2}{8\pi^2} \frac{z_i z_j}{z} V(z) \frac{d}{dz} \left[\frac{K_1(mz)}{z} \right] = \\ &= - \int d^4x \left(\frac{N\sigma}{\pi m} - \frac{1}{2} N \sqrt{\sigma} \right) \bar{\psi}(x) \psi(x) + \\ &+ \int d^4x \left(\frac{4}{3} \frac{N\sigma}{\pi m^2} - \frac{N\sqrt{\sigma}}{2m} \right) \bar{\psi}(x) i \vec{\gamma} \cdot \vec{\nabla} \psi(x). \end{aligned} \quad (18)$$

Within the NJL-prescription the first term in the r.h.s. of (18) defines the total value of the dynamical quark mass m . In the framework of this assumption we get the gap-equation

$$m = \frac{N\sigma}{\pi m} - \frac{1}{2} N \sqrt{\sigma}, \quad (19)$$

which has the following solution

$$m = \frac{N\sqrt{\sigma}}{4} \left(\sqrt{1 + \frac{16}{\pi N}} - 1 \right) \xrightarrow{N \rightarrow \infty} \frac{2\sqrt{\sigma}}{\pi}. \quad (20)$$

We have kept the gap-equation solution at the leading order in large N expansion, since the action (14) has been obtained at the same order. From the formula (20) we can calculate the numerical value of the dynamical quark mass, at $\sigma = 0.27 \text{ GeV}^2$ we get

$$m = \frac{2\sqrt{\sigma}}{\pi} = 0.33 \text{ GeV}. \quad (21)$$

Our result for the dynamical quark mass agrees with the prediction of the so-called naive nonrelativistic quark model [12] and with the value of m , calculated in the chiral limit within chiral perturbation theory at the quark level (CHPT)_q based on the NJL model [4] (see also refs.[11]). By using the gap-equation solution (20) we can calculate the coefficient in the second term of the r.h.s. of (18)

$$\frac{4}{3} \frac{N\sigma}{\pi m^2} - \frac{N\sqrt{\sigma}}{2m} = \frac{\pi N}{12} \approx 1 \quad (\text{at } 20\% \text{ error}) \quad (22)$$

at $N = 3$. Resultely at the leading order in large N expansion the r.h.s. of (14) reads

$$-iS_{\text{st. int}}^{(\bar{\psi}\psi)}|_{\text{one-loop}} = \int d^4x [\bar{\psi}(x)i\vec{\gamma} \cdot \vec{\nabla} \psi(x) - m\bar{\psi}(x)\psi(x)]. \quad (23)$$

Thus we have obtained that at the one-quark-loop approximation and at the leading order in large N expansion the interaction (14) fully restores the free Lagrangian of dynamical quarks and fixes the dynamical quark mass value via the gap-equation (19). It implies that the interaction in the $\bar{\psi}\psi$ -channels, realized via the linearly rising confinement potential (1), provides SBCS within the NJL prescription.

3. Local four-quark interactions

In terms of the dynamical quark mass (21) we can define the quark condensate, which is the order parameter for SBCS. It should be stressed that in QCD the quark condensate depends on the normalization point μ , being chosen of order of $O(1\text{GeV})$ as usually [9,13]. Without loss of generality one can set $\mu = \Lambda_\chi = 0.94\text{GeV}$. Then in the one-quark-loop approximation and by following [14], where the normalization point μ was considered as an upper limit of momentum integrals, one can determine the quark condensate, being in agreement with the NJL one (8)

$$\langle 0|\bar{\psi}\psi|0\rangle = -\text{tr}_{\text{C+L}}[iS_{\text{F}}(0)] = -4mI_1(m),$$

where $I_1(m)$ is defined in (6).

The knowledge of the dynamical quark mass and the value of the normalization point $\mu = \Lambda_\chi$, fixed in terms of the SBCS scale in the NJL model, allows to express the NJL constant G_1 in terms of the SBCS scale Λ_χ and the string tension σ

$$G = \frac{1}{\Lambda_\chi^2} f\left(\frac{\sigma}{\Lambda_\chi^2}\right), \quad (24)$$

where $f(\sigma/\Lambda_\chi^2)$ is a well-known function (see the formulae (5) and (6)).

From the different point of view we can define the constant G_1 as well as G_2 independently from the interaction (14) in terms of the string tension σ and the confinement radius r_c . Concerning the confinement radius one knows that its value is of order of $O(1/m_\pi)$, i.e. $O(1.4 \text{ fm})$, where $m_\pi = 0.14 \text{ GeV}$ is the pion mass. In our consideration r_c is introduced as an upper limit for the region of the integration over the relative coordinates $\vec{z}$. In order to express G_1 and G_2 in terms of the confinement radius we suggest to expand the action (14) in powers of the relative radius $\vec{z}$. After this expansion and the integration over $\vec{z}$ we obtain

$$-iS_{\text{st. int}}^{(\bar{\psi}\psi)} = \int d^4x L'_{\text{NJL}}(x) + \int d^4x L_{(\text{non-NJL})}(x), \quad (25)$$

where

$$L'_{\text{NJL}}(x) = 2 \left(\frac{\sigma\pi}{16} r_c^4 - \frac{\sqrt{\sigma}\pi}{6} r_c^3 \right) [\bar{\psi}(x)\Gamma\psi(x)][\bar{\psi}(x)\Gamma\psi(x)]. \quad (26)$$

By matching (26) with (3) we get

$$G_2 = G_1 = \frac{\sigma\pi}{16} r_c^4 - \frac{\sqrt{\sigma}\pi}{6} r_c^3. \quad (27)$$

Thus we have obtained that $G_2 = G_1$. This agrees with the results of the previous attempts to evaluate the NJL-phenomenological constants within QCD [15]. The local four-quark interaction (25) we have denoted as $L'_{\text{NJL}}(x)$, since it differs from the NJL interaction (3) by the noncovariant terms at $\Gamma = \sqrt{2}\gamma^0 t^a$ and $\sqrt{2}\gamma^0\gamma^5 t^a$. Beneath we shall show that the $\bar{\psi}\psi$ -collective excitations produced by the noncovariant local four-quark interactions can be fallen into the heavy mass states. Indeed in the chirally invariant hadronic phase they acquire the masses in excess of Λ_χ . Let us remind that in the chirally invariant hadronic phase of the NJL-model the $\bar{\psi}\psi$ -collective excitations, described by the NJL model, have the masses not exceeding the ρ -meson mass which is less than Λ_χ [3,4].

In principle, by using the eqs. (24) and (27) one can find r_c as a function of the string tension σ and the SBCS scale Λ_χ

$$r_c = \frac{1}{\sqrt{\sigma}} \rho \left(\frac{\sigma}{\Lambda_\chi^2} \right). \quad (28)$$

Practically, we insert the phenomenological values of $G_1 \simeq 10 \text{ GeV}^{-2}$ and $\sigma = 0.27 \text{ GeV}^2$ into (28) and estimate the value of r_c . As a result we get $r_c = 0.84/m_\pi$, which is compared with the restriction $r_c = O(1/m_\pi)$ mentioned above.

It should be stressed that the formula (27) confirms the behaviour of G_1 and G_2 at $N \rightarrow \infty$. The point is G_1 and G_2 are of order of $O(1/N)$ [2-4]. For G_1 it is seen from the gap-equation (6), and for G_2 it should be seen below from the mass formulae of vector and axial-vector mesons. The large N behaviour of G_1 and G_2 implies that at $N \rightarrow \infty$ the l.h.s. of the relation (27) vanishes and we get the equation for $(r_c)_{N \rightarrow \infty}$ having a very simple nontrivial solution

$$(r_c)_{N \rightarrow \infty} = \frac{8}{3} \frac{1}{\sqrt{\sigma}} = \frac{0.72}{m_\pi} = 1 \text{ fm}. \quad (29)$$

Of course, the 20 % deviation of the quantity (29) from the value $r_c = 0.84/m_\pi = 1.2 \text{ fm}$ is caused by the large N corrections.

The Lagrangian $L_{(\text{non-NJL})}(x)$ is an infinite series of terms, which contain derivatives of the quark fields with respect to its coordinates. These interactions describes the $\bar{\psi}\psi$ -collective excitations with higher total spins, in comparison with those excitations which appear by owing to the $L'_{\text{NJL}}(x)$ interaction. The infinite number of terms reflects the ability of QCD as the correct theory of the strong interactions to account for any colourless $\psi\psi$ -hadronic state with quantum numbers of mesons. It is evident from the consideration above that $L_{(\text{non-NJL})}(x)$ should not contribute to the gap-equation and correspondingly, should not influence on SBSCS.

4. Spectra of low-lying collective excitations

Now we may proceed to the discussion of the mass spectra of the $\bar{\psi}\psi$ -collective excitations, produced by the interaction (28). The nonets of these excitations we shall denote as S , P , V , A and H_V and H_A for scalar, pseudoscalar, vector, axial-vector and noncovariant states, respectively. The effective Lagrangian of these excitations reads [2,4].

$$L_{\text{eff}} = \tilde{L}_{\text{eff}}(x) - \frac{g_1^2}{4G_1} \text{tr}(S^2 + P^2) + \frac{g_2^2}{4G_1} \text{tr}(V_\mu V^\mu + A_\mu A^\mu) - \frac{g_2^2}{8G_1} \text{tr}(H_V^2 + H_A^2), \quad (30)$$

where

$$\begin{aligned} \tilde{L}_{\text{eff}}(x) &= -i \langle x | \ln \frac{\text{Det}(i\hat{\partial} - m + \Phi)}{\text{Det}(i\hat{\partial} - m)} | x \rangle = \\ &= -i \text{tr}_{C+L+F} \langle x | \ln \left(1 - \frac{1}{m - i\hat{\partial}} \Phi \right) | x \rangle. \end{aligned} \quad (31)$$

Here $\Phi = g_1(S' + i\gamma^5 P) + g_2(\hat{V} + \hat{A}\gamma^5) + g_2\gamma^0(H_V + H_A\gamma^5)$, $g_1 S' = m + g_1 S$ and $g_{1,2}$ are the quark-meson coupling constants such that $g_2 \equiv g_\rho$ being the $\rho\pi\pi$ -coupling constant. The vacuum expectation values of the S -fields are not equal to zero while the S' -fields are

defined so that $\langle 0 | S' | 0 \rangle = 0$, m is the dynamical quark mass (21). The wave-functions $|x\rangle$ are normalized by the condition $\langle x | y \rangle = \delta^{(4)}(x - y)$.

Since the scalar and pseudoscalar four-quark interactions in (26) coincide completely with those of the NJL interaction (3) we do not need the revision of the $\bar{\psi}\psi$ -collective excitations of the scalar and pseudoscalar kinds which are very good established within the NJL model [2-4]. Therefore we can restrict ourself by the consideration of the mass spectra of the V , A , H_V and H_A collective excitations. However, for this aim it is not necessary also to evaluate the effective Lagrangian $\tilde{L}_{\text{eff}}(x)$ completely. It is sufficient only to consider the kinetic terms of these collective excitations. The Lagrangian of the free V , A , H_V and H_A collective excitation fields can be described by the expression

$$\begin{aligned} L_{\text{eff}}^{(0)}(V(x), A(x), H_V(x), H_A(x)) &= -\frac{i}{2} \text{tr}_{C+L+F} \sum_U \langle x | \frac{1}{m - i\hat{\partial}} U \frac{1}{m - i\hat{\partial}} U | x \rangle + \\ &+ \frac{g_2^2}{4G_1} \text{tr}_F(V_\mu V^\mu + A_\mu A^\mu) + \frac{g_2^2}{8G_1} \text{tr}_F(H_V^2 + H_A^2), \end{aligned} \quad (32)$$

where the sum runs over $U = g_2 \hat{V}$, $g_2 \hat{A}\gamma^5$, $g_2 H_V \gamma^0$ and $g_2 H_A \gamma^0 \gamma^5$. The first term in (31) can be presented in the form of the one-constituent-quark-loop diagrams with two vertices taking the following analytic representation [4]:

$$\begin{aligned} -\frac{i}{2} \langle x | \frac{1}{m - i\hat{\partial}} U \frac{1}{m - i\hat{\partial}} U | x \rangle &= \int \frac{d^4 x_1}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} e^{-ik_1(x_1 - x)} \times \\ &\times \left(-\frac{1}{2} \frac{N}{16\pi^2} \right) \int \frac{d^4 k}{\pi^2 i} \text{tr}_{L+F} \left\{ \frac{1}{m - \hat{k}} U(x_1) \frac{1}{m - \hat{k} - \hat{k}_1} U(x) \right\}. \end{aligned} \quad (33)$$

The main element of these terms is the quark loop with two vector vertices

$$V_{\mu\nu}(k_1) = \frac{N}{16\pi^2} \int \frac{d^4 k}{\pi^2 i} \text{tr}_L \left\{ \frac{1}{m - \hat{k}} \gamma_\mu \frac{1}{m - \hat{k} - \hat{k}_1} \gamma_\nu \right\}. \quad (34)$$

The integral over k is divergent. By using the cut-off in Euclidean 4-momentum space we get [2,4]

$$\begin{aligned} V_{\mu\nu}(k_1) &= 2[I_1(m) + m^2 I_2(m)]g_{\mu\nu} + \\ &+ (k_1^2 g_{\mu\nu} - k_{1\mu} k_{1\nu}) \frac{4}{3} I_2(m). \end{aligned} \quad (35)$$

By following the prescription [2-4] we have kept in (34) only divergent terms, and $I_2(m)$ is a logarithmically divergent integral [2-4]

$$I_2(m) = \frac{N}{16\pi^2} \int \frac{d^4 k}{\pi^2 i} \frac{1}{(m^2 - k^2)} = \frac{N}{16\pi^2} \ln \frac{\Lambda_X^2}{m^2} + \dots \quad (36)$$

After the evaluation of the quark diagrams we obtain the following mass spectra of the V , A , H_V and H_A collective excitations in the chirally invariant hadronic phase [4]

$$m_A^2 = m_V^2 = \frac{g_2^2}{4G_1} - g_2^2[I_1(m) + m^2 I_2(m)] = (0.62 \text{ GeV})^2, \quad (37a)$$

$$m_{H_A}^2 = m_{H_V}^2 = \frac{g_2^2}{8G_1} + g_2^2[I_1(m) + m^2 I_2(m)] = (1.02 \text{ GeV})^2, \quad (37b)$$

i.e. $m_A = m_V = 0.62 \text{ GeV}$ and $m_{H_A} = m_{H_V} = 1.02 \text{ GeV}$. The latter exceed the SBCS scale $\Lambda_\chi = 0.94 \text{ GeV}$.

In the chirally broken hadronic phase the square masses of the A states acquire the additional contribution $6m^2$, i.e. $m_A^2 = m_V^2 + 6m^2$. The theoretical value of the vector meson masses $(m_V)_{\text{th}} = 0.62 \text{ GeV}$ is compared with the experimental value of the ρ -meson mass $(m_\rho)_{\text{exp}} = 0.77 \text{ GeV}$ within uncertainty of order of 20 %. The same agreement we obtain for the axial-vector meson masses in the chirally broken hadronic phase: $(m_A)_{\text{th}} = 1.02 \text{ GeV}$ and $(m_{a_1})_{\text{exp}} = 1.26 \pm 0.03 \text{ GeV}$ [14]. Thus the equality $G_2 = G_1$ does not contradict the experimental data on vector and axial-vector masses.

The masses of H_V and H_A states, created by noncovariant local four-quark interactions, have the masses exceeding the SBCS scale Λ_χ in the chirally invariant hadronic phase. That is why they can be fallen to the heavy-mass-meson states and deleted from the spectrum of low-lying $\bar{\psi}\psi$ -collective excitations, created by the interaction (14). In order to perform it formally we can merely include the non-covariant four-quark interactions from $L_{\text{NJL}}(x)$ into $L_{\text{non-NJL}}(x)$. Resultely the NJL interaction, obtained in the framework of our approximation, reads

$$L_{\text{NJL}}(x) = 2G_1[(\bar{\psi}(x)t^a\psi(x))^2 + (\bar{\psi}(x)i\gamma^5 t^a\psi(x))^2] - 2G_1[(\bar{\psi}(x)\gamma_\mu t^a\psi(x))^2 + (\bar{\psi}(x)\gamma_\mu\gamma^5 t^a\psi(x))^2]. \quad (38)$$

where G_1 is a phenomenological constant for the definition of which we need formally the knowledge of the deviation of r_c from $(r_c)_{N \rightarrow \infty} = 8/3\sqrt{\sigma}$ (30), i.e. $\delta r_c = r_c - (r_c)_{N \rightarrow \infty} = 27G_1\sqrt{\sigma}/32\pi + O(1/N^2)$, at the leading order in large N expansion.

Conclusion

By resuming the obtained results one can conclude that we have run into our aim. It stands for we have shown that the linearly rising confinement potential, providing a strong attraction in the quark-antiquark pairs at large distances, can realize SBCS via the mechanism being very similar to the NJL one. Unfortunately one cannot count that we have performed the investigation of this phenomenon "from the first principle" in QCD. In fact we have observed it in the framework of the admission that the main contribution to the four-quark interaction at large distances comes from the interaction between colour quark charges. We have accepted this approximation mainly by leaning on the Adler's theory of static quark forces and the large N -expansion.. In order to evaluate the gap-equation we have used the one-quark-loop approximation, having been accepted in accordance with

the NJL prescription. The existence of the non-trivial solution of the gap-equation, giving the constituent quark mass value $m = 2\sqrt{\sigma}/\pi = 0.33 \text{ GeV}$ at $\sigma = 0.27 \text{ GeV}^2$, confirms the presence of the SBCS phenomenon in the framework of our approach. Also it confirms that the mechanism of this SBCS phenomenon coincides with the NJL one. It seems interesting the result concerning the evaluation of G_1 and G_2 in terms of the string tension and the confinement radius (27). This formula confirms both the phenomenological values of these constants and their behaviour at large N . As an additional result of the formula (27) one can consider the definition of the confinement radius at $N \rightarrow \infty$ in terms of the string tension (29): $(r_c)_{N \rightarrow \infty} = 8/3\sqrt{\sigma} = 1 \text{ fm}$. The estimate of r_c from (27) at the phenomenological value $G_1 = 10 \text{ GeV}^{-2}$ gives the value $r_c = 0.84/m_\pi = 1.20 \text{ fm}$. This result can be appraised as a confirmation of the applicability of the large N expansion approximation in this area, since the 20 % correction $\delta r_c = r_c - (r_c)_{N \rightarrow \infty} = 0.20 \text{ fm}$ is fully due to the large N one.

As regards the spectra of low-lying quark-antiquark collective excitations which can be produced by the four-quark interactions (14) one can conclude that the spectra of scalar, pseudoscalar, vector and axial-vector nonets fully coincide with those described by the NJL-model. The mass spectra of vector and axial-vector collective excitations, obtained at $G_2 = G_1$, are compared with the experimental data within 20 % of uncertainty. The appearance of noncovariant local four-quark interactions is, of course, a shortcoming of our consideration, which would lead to plenty amount of troubles about an identification of collective excitations, produced by these interactions, with physical meson states. However, to our point we managed to avoid these troubles by demonstrating that the masses of these excitations, obtained in the chirally invariant hadronic phase, exceed of the SBCS scale $\Lambda_\chi = 0.94 \text{ GeV}$. In accordance with the NJL experience with the low-lying mesons these excitations should be fallen to the heavy meson states. It results that the non-derivative-local-four-quark interactions, obtained by owing to the expansion in powers of the relative radius-vector in the quark-antiquark pairs and the integration over this radius-vector, are chirally and Lorentz invariant. Also these interactions fully coincide with those of the NJL model. The enumerated results should be valued apparently as a justification of the NJL model within QCD with linearly rising confinement potential.

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