TESTING THE VELOCITY FIELD IN NON-SCALE INVARIANT COLD DARK MATTER MODELS
Abstract

We analyze the cosmic peculiar velocity field as traced by a sample of 1184 spiral, elliptical and S0 galaxies, grouped in 704 objects. We perform a statistical analysis, by calculating the bulk flow, Cosmic Mach Number and velocity correlation function for this sample and for mock catalogs extracted from a set of N-body simulations. We run four cold dark matter (CDM) simulations: two tilted models (with spectral index $n = 0.6$ and $n = 0.8$), the standard model ($n = 1$) and a “blue” one ($n = 1.2$), with different values of the linear bias parameter $b$. By means of a Maximum Likelihood analysis we estimate the ability of our models to fit the observations, as measured by the above statistics, and to reproduce the Local Group properties. On the basis of this analysis we conclude that the best model is the unbiased standard model ($n, b = (1, 1)$), even though the overall flatness of the joint likelihood function implies that one cannot strongly discriminate models in the range $0.8 \leq n \leq 1$, and $1 \leq b \leq 1.5$. Models with $b \geq 2.5$ are rejected at the 95% confidence level. For $n = 0.8$ the values of $b$ preferred by the present analysis, together with the COBE data, require a negligible contribution to $\Delta T/T$ by gravitational waves. Finally, the blue model, normalized to COBE, does not provide a good fit to the velocity data.


1 Introduction

It is well known that the standard cold dark matter (hereafter SCDM) scenario for structure formation possesses a high predictive power in explaining many observed properties of the large-scale structure of the universe. The model is characterized by a primordial scale–invariant power–spectrum, $P(k) \propto k^n$, with spectral index $n = 1$, of Gaussian adiabatic perturbations in an Einstein–de Sitter universe with vanishing cosmological constant. As usual, one can parametrize the amplitude of the primordial perturbations by the linear bias parameter $b$, defined as the inverse of the rms mass fluctuation on a sharp–edged sphere of radius $R_s \equiv 8 h^{-1}$ Mpc. In this paper we adopt the value $h = 0.5$ for the Hubble constant $H_0$ in units of 100 km s$^{-1}$. The COBE DMR detection (Smoot et al. 1992; Bennett et al. 1994) of large angular scale Cosmic Microwave Background (CMB) anisotropies can be used to normalize the SCDM power–spectrum, resulting in $b \approx 0.9$. However, the SCDM model has met increasing problems, mostly due to the high ratio of small to large-scale power; in particular the COBE normalized model predicts excessive velocity dispersion on Mpc scale and is unable to reproduce the slope of the galaxy angular correlation function obtained from the APM survey (Maddox et al. 1990).

Many alternative models have been proposed to overcome the difficulties of the SCDM
model. In order of increasing number of changes of the standard assumptions one can quote: i) “tilted” (i.e. $n < 1$) CDM models (hereafter TCDM); ii) hybrid (i.e. hot plus cold dark matter) models; iii) CDM models with a relic cosmological constant; iv) CDM models with non-Gaussian initial conditions.

In this paper we will mostly consider TCDM models. This can be considered the most natural way to decrease the small-scale power relative to the large-scale one: in fact, tilting the spectral index of the primordial perturbations boosts power from small to large scales (e.g. Vittorio, Matarrese, & Lucchin 1988; Tormen, Lucchin, & Matarrese 1992; Adams et al. 1992; Cen et al. 1992; Tormen et al. 1993, hereafter TMLM). Moreover, these models can be easily motivated in the frame of the inflationary origin of perturbations (Lucchin & Matarrese 1985; Adams et al. 1992). The first year data from the COBE DMR experiment renewed the interest in these models: the observed anisotropy is in fact consistent with $n = 1.1 \pm 0.5$ on scales $\geq 10^8$ $h^{-1}$ Mpc. It is evident that the COBE normalization of TCDM models implies reduced power on all scales below $10^8$ Mpc. Moreover, a large number of post-COBE analyses (see, e.g., Crittenden et al. 1993 and references therein) pointed out that properly accounting for the gravitational-wave contribution to the Sachs–Wolfe effect may lead to a relevant modification of the linear biasing factor. One can write

$$b(n) \approx b_0(n) \times G(n),$$

where the quantity $b_0(n) = 0.9 \times 10^{1.2(1-n)}(1 \pm 0.25)$ accounts for the effect of modifying the spectral slope. The possible contribution to CMB anisotropies by gravitational waves is taken into account by the function $G(n)$; this effect is especially relevant in power-law inflation: Lucchin, Matarrese, & Mollerach (1992) obtained the approximate relation $G(n) = [(14 - 12n)/(3 - n)]^{1/2}$ in the range $0.5 \lesssim n \lesssim 1$. Models where the value of $n$ is smaller than unity but the gravitational-wave contribution is negligible, i.e. $G(n) \approx 1$, are the typical outcome of “natural” inflation (e.g. Adams et al. 1992). Even though second year COBE data have raised the range of preferred values of the spectral index to $n = 1.59^{+0.49}_{-0.53}$ (Bennett et al. 1994), we can safely continue to use the above estimate of $b_0(n)$ outside the new range, due to the flatness of the likelihood function for these observables (e.g. Scaramella & Vittorio 1993).

For the sake of comparison and completeness, in the present paper we will also consider an “anti-tilted” or “blue” ($n > 1$) CDM model (hereafter BCDM). Recently, many observational data seem to suggest values of $n$ larger than unity from large-scale CMB anisotropies (Bennett et al. 1994; Wright et al. 1994; Hancock et al. 1994) and analyses of the matter distribution (Piran et al. 1993; Lauer & Postman 1994). Contrary to the widespread belief that inflation always implies $n \leq 1$, blue spectra may be also easily motivated in the frame of inflation (e.g. Mollerach, Matarrese, & Lucchin 1994). These models predict a negligible contribution from gravitational waves to CMB anisotropies, i.e. $G(n) \approx 1$. According to the above equation, BCDM models, normalized to COBE, exacerbate the problem of SCDM: too much power on
small and intermediate scales. The simplest solution is to invoke a large free-streaming effect on these scales, as can be due to a suitable amount of hot dark matter (Lucchin et al. 1994a).

In a recent work (TMLM) we analyzed the peculiar velocity field traced by the same sample considered here, to probe the primordial spectrum up to scales $\sim 100 \ h^{-1} \ Mpc$. The results were then compared to similar analyses carried out on mock catalogs extracted from Monte Carlo simulations. These were obtained from linear theory in $n \leq 1$, and/or $\Omega_0 \leq 1$ CDM models, for different values of the bias factor, with the assumption that the galaxy velocity field gives an unbiased signal of the underlying mass distribution. In the present paper the same type of analysis is performed on mock catalogs extracted from N-body simulations in $\Omega_0 = 1$ CDM models. A similar method has been applied also to simulations with skewed (i.e. non-Gaussian) CDM initial conditions: the results will be presented in (Lucchin et al. 1994b).

The plan of the paper is as follows. Section 2 is devoted to the description of the real catalogs of galaxy peculiar velocities and of the method used to construct the mock catalogs from the N-body simulations. In Section 3 the results of the different statistical tests adopted in the study of the velocity field are presented and a Maximum Likelihood analysis is performed. In Section 4 brief conclusions are drawn.

2 Real and simulated catalogs

The catalog of peculiar velocities we consider here is the same used in TMLM, where more details can be found. It was compiled from the “Mark II” data sample, which is a collection of different samples including spirals, ellipticals and S0 galaxies. In order to reduce distance uncertainties, we grouped the galaxies following the rules in the original papers (Lynden-Bell et al. 1988; Faber et al. 1989). Our sample finally consists of 1184 galaxies grouped in 704 objects (see Table 1 in TMLM).

To simulate the large-scale peculiar velocity field traced by optical galaxies we performed N-body simulations of the matter distribution. We used a particle-mesh code with $128^3$ particles on $128^3$ grid points; the simulation box is $260 \ h^{-1} \ Mpc$ large, much larger than the typical depth of the real galaxy sample: in fact the maximum galaxy distance is $\approx 12,000 \ km \ s^{-1}$, but about 90% of the galaxies in the sample has a distance smaller than $6,000 \ km \ s^{-1}$.

The initial velocity field is assumed to be Gaussian distributed with power-spectrum $P_v(k) \propto k^{n-2} T^2(k)$, where $T(k)$ is the CDM transfer function $T(k) = [1 + 6.8k + 72.0k^{3/2} + 16.0k^2]^{-1}$ (Davis et al. 1985).

We ran simulations with four values of the primordial spectral index ($n = 0.6, 0.8, 1, 1.2$), considering different values for the linear biasing parameter $b$. In particular we consider $b = 2, 2.5$ for $n = 0.6$; $b = 1, 1.5, 2, 2.5$ for $n = 0.8$ and $n = 1$; $b = 0.5, 0.75, 1, 1.5$ for $n = 1.2$. These values are in the range allowed by COBE data, when considered with their error bars. In the following analysis we will only plot the results for the “basic” models, i.e. those with the
value of $b$ closest to the best fit from COBE data, when the gravitational–wave contribution is neglected (i.e. $G(n) = 1$): these are $(n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)$ and $(1.2, 0.5)$. For all the other models we report the results only in the tables for completeness.

In order to recover the velocity field from each simulation, we follow a standard procedure (e.g. Kofman et al. 1994). First, we interpolate the mass and momentum from the particle distribution onto a cubic grid with $128^3$ grid points, using a TSC algorithm (e.g. Hockney & Eastwood 1981); then, we smooth with a further Gaussian filter with width $2\, h^{-1}$ Mpc, to ensure a non zero density at every grid point. Finally, we define the velocity at each grid point as the momentum divided by the mass density.

In Figures 1a and 1b we plot the projected particle positions and the smoothed peculiar velocity taken from a slice of depth $4\, h^{-1}$ Mpc, for the basic models. A first glance suggests that the dominant effect is given by the value of $b$. The tilted models, which are characterized by high power on large scales, are unable to fully display this feature, because of the low evolution implied by the COBE normalization. Conversely, the blue model, due to the long period of non–linearity occurred, appears as the one with the largest and densest structures accompanied by coherent flows.

**FIGURES 1a and 1b**

Our simulated catalogs are built up after choosing from each simulation 500 “observers”, corresponding to grid points with features similar to those of the Local Group (LG) (e.g. Gorski et al. 1989; Davis, Strauss, & Yahil 1991; Strauss, Cen, & Ostriker 1993). The requirements (slightly updated with respect to those applied in TMLM) are the following:

i) The peculiar velocity $v$ is in the range of the measured LG motion, $v_{LG,obs} = 627 \pm 22$ km s$^{-1}$ (Kogut et al. 1993).

ii) The local flow is quiet, as expressed by the the small value of the local ‘shear’, $S \equiv |v - \langle v \rangle|/|v| < 0.2$, where $\langle v \rangle$ is the average velocity of a sphere of radius $R = 750$ km s$^{-1}$ centered on the LG.

iii) The density contrast in the same sphere is in the range $-0.2 \leq \delta \leq 1.0$.

Radial peculiar velocities are measured by sampling the velocity field from the LG position. We fix the axes of our reference frame by imposing that the velocity of each simulated Local Group singles out the CMB dipole direction ($l = 276 \pm 3^\circ$, $b = 30 \pm 3^\circ$; Kogut et al. 1993), with the direction of the remaining axis randomly selected; we account for observational errors in the direction determination by adding a Gaussian random shift with the given uncertainy. Next we built up our simulated catalogs by collecting, for each of the 704 positions of the selected objects, the closest particle in the simulation and projecting its velocity along the line of sight of the mock LG; this method is applied in order to avoid the velocity smoothing, which would be automatically present if the velocity field was reconstructed using a grid (see, e.g., TMLM).

Random galaxy distance errors are then considered by perturbing each distance and radial peculiar velocity with Gaussian noise (e.g. Dekel, Bertschinger, & Faber 1990): $r_{i,p} = r_i + \xi_i \Delta r_i$.
and \( u_{i,p} = u_i - \xi_i \Delta r_i + \eta_i \sigma_f \), where \( \xi_i \) and \( \eta_i \) are independent standard Gaussian variables; \( \Delta r_i \) is the estimated galaxy distance error and \( \sigma_f = 200 \text{ km s}^{-1} \) is the Hubble flow noise.

### 3 Statistical tests and Maximum Likelihood analysis

In this Section we will compare the real sample with the mock catalogs using different statistics: the Local Group constraints, the bulk flow, the Cosmic Mach Number and the velocity correlation function.

#### 3.1 Local Group statistics

The first statistic we applied to our mock catalogs is based on characterizing the observers (Local Groups) by their velocity, local shear and local density contrast, as previously described. We found that the ensemble of these LG constraints changes the estimates of the other statistics: for instance, by imposing the quietness of the local flow and allowing for a narrow range for the local velocity we exclude grid points with large velocity. Moreover, by considering points with a density contrast very close to the mean avoids the choice, as origin of our simulated catalogs, of structures like (Great) attractors, where the flow is very peculiar.

In Figure 2 we show for our basic models \((n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)\) and \((1.2, 0.5)\) the probability distribution of the quantities \(v, \delta\) and \(S\): the shaded regions in the histograms show the range allowed by the assumed LG constraints.

**FIGURE 2**

Table 1 reports the percentage of grid points, for all the considered models, that fulfil each constraint separately and altogether \(\mathcal{P}(LG)\).

**TABLE 1**

Even though the allowed range for the local shear has been restricted with respect to TMLM (from \(S < 0.5\) to \(S < 0.2\)), we still find that this constraint is poorly effective: the differences among the considered models are very small and do not help in discriminating among them. On the contrary, the constraints on the density and velocity of the LG turn out to be strongly dependent on the bias parameter but almost independent of the spectral index: higher is the value of \(b\), higher is \(\mathcal{P}(\delta)\) and lower \(\mathcal{P}(v)\), for all the considered \(n\); the only exception to this trend is for the most evolved model \((n, b) = (1.2, 0.5)\). Finally, the velocity constraint is the most effective one. These results generally confirm the linear analysis performed in TMLM. The total probability \(\mathcal{P}(LG)\) selects as best models those in the region \((n, b) = (0.8 - 1, 1.5)\).

In Figure 2, we also plot the probability distributions, for SCDM with \(b = 1\), obtained within linear theory by Monte Carlo simulations in TMLM. Note that, besides being unable to
reproduce the actual skewness of the mass distribution, the linear theory tends to overestimate both the velocity and the shear.

### 3.2 Bulk Flow

The velocity dipole or *bulk flow* for a galaxy catalog with $N$ objects, with peculiar velocities $v_i$, can be defined through a least-squares fit (e.g. Regős & Szalay 1989)

$$
\vec{v}_{\text{bulk}}^\alpha = \frac{(M^{-1})^{\alpha\beta} \sum_i w_i \vec{u}_i^\beta}{\sum_i w_i}
$$

(summation over repeated indices is understood); $u_i^\alpha \equiv (\vec{v}_i \cdot \hat{r}_i)\hat{r}_i^\alpha$ is the $\alpha$ component ($\alpha = 1, 2, 3$) of the radial peculiar velocity and $w_i$ the weight assigned to the $i$-th galaxy. The weighted projection matrix $M^{\alpha\beta} = \sum_i w_i \vec{r}_i^\alpha \vec{r}_i^\beta / \sum_i w_i$ accounts for the sample geometry.

As discussed in TMLM, where different choices for the weight were considered, the “number weighting” scheme, $w_i = 1$, always provides a better estimate of the true velocity dipole: for this reason we will apply in this paper this scheme also to the other statistics considered (Cosmic Mach Number and velocity correlation function).

In Figure 3 we plot the distribution of bulk flow amplitudes calculated from our mock catalogs. The continuous vertical line refers to the observed value: for our composite galaxy sample we find $v_{\text{bulk}} = 306 \pm 72$ km s$^{-1}$, with a misalignment angle $\alpha = 54^\circ \pm 13^\circ$ with respect to the direction of the CMB dipole. The plotted error bars take into account the uncertainties due to both sparse geometry (obtained by bootstrap resamplings of the real catalog) and distance errors (obtained by Monte Carlo error propagation on the observed sample). The figure suggests that the leading effect is once more determined by the value of $b$: higher the normalization (lower $b$) higher the mean bulk flow and larger the spread of the distribution.

**FIGURE 3**

In linear theory the three components of the peculiar velocity field are independent Gaussian fields; it follows that the probability distribution for the bulk flow of a sample is a Maxwellian,

$$
\mathcal{P}(x)dx = \frac{32}{\pi^2} \frac{x^2}{\langle x \rangle^3} \exp \left( - \frac{4}{\pi} \frac{x^2}{\langle x \rangle^2} \right) dx.
$$

(Suto & Fujita (1990), using N-body simulations, found that the bulk flow data can be fitted by a Maxwellian distribution also in the mildly nonlinear regime. By a Kolmogorov–Smirnov test, we checked that this curve provides a good representation also for the mock data extracted from our simulations. The resulting curves are also shown in Figure 3.

In Table 2, we report the probability $\mathcal{P}(v_{\text{err}})$ that the simulated bulk flows have amplitude in the interval $[v_{\text{obs}} - \sigma_{v_{\text{obs}}}, v_{\text{obs}} + \sigma_{v_{\text{obs}}}]$ and misalignment angle $\alpha$ in the analogous interval. These results confirm our guess that the spectral index plays only a limited role in this context.

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3.3 Cosmic Mach Number

Another relevant statistic for the peculiar velocity field is the Cosmic Mach Number $\mathcal{M}$, first proposed by Ostriker & Suto (1990). Given a galaxy sample, $\mathcal{M}$ is defined as the ratio of the center–of–mass velocity of the sample (the bulk flow) to the one-point velocity dispersion around this average motion. Since the bulk flow is caused by density fluctuations on scales larger than the sampled volume, while the velocity dispersion mostly depends on the power on smaller scales, $\mathcal{M}$ actually measures the ratio of large to small scale power in the velocity field.

In its first implementation the Cosmic Mach Number has been studied in linear theory, and analytical results have been obtained based on idealized observations. These results show that $\mathcal{M}$ is independent of the $P(k)$ normalization, i.e. of $b$, and rule out SCDM at the 95% confidence level. In an analysis based on N–body simulations, Suto, Cen, & Ostriker (1992) found that the value of $\mathcal{M}$ is well correlated with the bulk flow amplitude $v_{\text{bulk}}$, but practically uncorrelated with the residual one–point velocity dispersion $\sigma_v$. From this study they were able to reject SCDM at the 99% confidence level. Strauss, Cen, & Ostriker (1993) were the first to consider a working definition of $\mathcal{M}$, taking into account the actual observational limitations and uncertainties. In their analysis they tried different versions of the test applied to three galaxy samples: a subsample of the Aaronson et al. (1986, 1989) spirals, the ellipticals in Faber et al. (1989) and the spiral sample of Willick (1991). Using a procedure quite similar to the one adopted in TMLM and in this work, they analyzed the predictions of different cosmological models and came to the rejection of unbiased SCDM at about the 90% confidence level, when the Aaronson et al. subsample is considered.

As we did for the other statistical tests, we will use an operative definition for $\mathcal{M}$, similar to that in Eq.(4) of Strauss, Cen, & Ostriker (1993) for the uniform weighting scheme, namely

$$\mathcal{M} = \frac{|v_{\text{bulk}}|}{\sqrt{3}\sigma_v} \quad (4)$$

where the bulk flow amplitude $v_{\text{bulk}}$ has been previously defined and $\sigma_v$ is the radial part of the one point velocity dispersion in the reference frame of the bulk motion:

$$\sigma_v^2 = \langle (u_i - v_{\text{bulk}}^\alpha \dot{\alpha}_i)^2 \rangle = \frac{1}{N} \sum_i (u_i - v_{\text{bulk}}^\alpha \dot{\alpha}_i)^2, \quad (5)$$

with the sum extending over all the objects in the sample. The factor $\sqrt{3}$ in this expression follows from the assumption of isotropic dispersion for the residual velocities, used in order to estimate the full three–dimensional $\sigma_v$ from its radial component.

For our composite sample we find $\mathcal{M} = 0.24 \pm 0.06$, where, as before, the quoted error takes into account the uncertainties deriving from sparse geometry and distance errors. In Figure
4 we show the Cosmic Mach Number distribution, calculated from our mock catalogs. The shaded region indicates the observed value and its error bar. The dashed line refers to our best fit with a Maxwellian distribution, which once again provides a good fit of the data, as shown by a Kolmogorov–Smirnov test (see also Suto & Fujita 1990; Strauss, Cen, & Ostriker 1993).

Note that, contrary to the linear theory prediction, the Cosmic Mach Number depends on the value of \( b \), which gives in fact the dominant dependence.

**FIGURE 4**

In Table 2, we report the probability \( P(M_{err}) \) that the simulated Cosmic Mach Number is inside the interval \( M = 0.24 \pm 0.06 \). Even though this statistic is the less stringent one, on its basis we can conclude that lower values of \( b \) are preferred.

It is worth to point out that our results differ from those of Strauss, Cen, & Ostriker (1993), mainly because the statistical approaches are different in a number of ways. First, we compute the Cosmic Mach Number for the whole Mark II catalog; we expect that such a measure is more stable and representative of the velocity field than measures performed on subsamples. Second, we are relying on a likelihood analysis to draw our conclusions. This means that we are not rejecting a model on the basis of its absolute inability to reproduce the observed \( M \), but rather comparing the performance of different models “normalized” to the performance of the maximum likelihood model. With this warning in mind, we may try to compare our result with those of Strauss Cen, & Ostriker (1993). Repeating the analysis outlined above, but using only the sample of “Good” spirals of Aaronson et al., with estimated distance less than 3000 km s\(^{-1}\), we found that 6% to 10% of the mock catalogs have a Mach Number larger than the observed one \((0.76 \pm 0.10)\) for the unbiased SCDM model, corresponding to a rejection of the model at the 90% confidence level. This is in complete agreement with Strauss Cen, & Ostriker (1993), considering the residual differences of the two approaches. Rephrasing the same analysis in terms of likelihood, 15% of the simulated catalogs have a Cosmic Mach Number equal (within observational errors) to the one observed for the same spirals. Comparing such figures with the corresponding ones for the total sample we see that the spirals pose a harder challenge to the unbiased SCDM model than the whole Mark II catalog.

### 3.4 Velocity correlation function

The last statistic we consider is the *velocity correlation function*; following Gorski et al. (1989) we define

\[
\Psi_1(r) = \frac{\sum_{\text{pair } s(r)} u_1 \cdot u_2}{\sum_{\text{pair } s(r)} (\mathbf{r}_1 \cdot \mathbf{r}_2)^2},
\]

where the sum extends over galaxy pairs separated by a distance \( r \).

Figure 5 compares the velocity correlation resulting from our mock catalogs to the observed one. We evaluated \( \Psi_1(r) \) for the real data by counting galaxy pairs in ten separation bins of
500 km s$^{-1}$ up to a maximum separation of 5,000 km s$^{-1}$. The error bars, estimated as for the bulk flow, take into account both the sparse sampling of the data and the distance errors.

**FIGURE 5**

The simulated distributions look very different in different models: high tilt and bias shrink the distribution. The widest distribution is for $(n, b) = (1.2, 0.5)$.

In order to compare models with observations, we adopt, among the different possible statistics discussed in TMLM, the linear integral of $\Psi_1(r)$ from the origin to the maximum considered pair separation, $R_{\text{max}} = 5,000$ km s$^{-1}$ (see also Gorski et al. 1989):

$$J_v = \int_0^{R_{\text{max}}} \Psi_1(r) \, dr.$$  \hspace{1cm} (7)

This simple one-dimensional statistic is used to compress in one number the information carried by the function $\Psi_1(r)$. This is done in order to reduce the dimension of the probability space associated to the statistic, thus ensuring a more accurate sampling. In TMLM it was found that $J_v$ gives results in agreement with those from the ten-dimensional sampling of $\Psi_1(r)$, showing that the compressed statistics still retains a significant part of the original information.

For our real catalog we find $J_v/(100$ km s$^{-1})^3 = 237.9 \pm 61.5$, where the quoted error has been estimated as for the previous statistics. We then calculated the percentage $\mathcal{P}(J_v \text{ err})$ of the simulated catalogs whose value of $J_v$ is less than one standard deviation different from the observed one: the results are reported in Table 2. In Figure 6 we show the distribution of $J_v$ calculated from our mock catalogs. The shaded region refers to the observed value and its error bar. As a general trend we can say that low values of $b$ are preferred, in particular in connection with values of $n$ close to unity.

**FIGURE 6**

### 3.5 Joint Statistics

We finally performed a Maximum Likelihood analysis to compare the statistics obtained from different simulations. Calling $\vec{C}$ the random vector of the statistics used to constrain the simulated Local Groups, $\vec{C} = (v_{LG}, S, \delta)$, and $\vec{S}$ the vector of all the other statistics, $\vec{S} = (v_{\text{bulk}}, \alpha, \mathcal{M}, J_v)$, the joint distribution of $\vec{C}$ and $\vec{S}$, under the condition $\vec{C} = \vec{C}_{\text{obs}}$, is $\mathcal{P} (\vec{C}_{\text{obs}}, \vec{S}) = \mathcal{P} (\vec{C}_{\text{obs}}) \mathcal{P} (\vec{S} | \vec{C}_{\text{obs}})$. For a given model $H$ (in our case for given values for $n$ and $b$), the likelihood function reads $\mathcal{L}(H) = \mathcal{P}(\vec{C}_{\text{obs}} | H) \mathcal{P}(\vec{S}_{\text{obs}} | \vec{C}_{\text{obs}}, H)$. The joint conditional likelihood $\mathcal{P}(\vec{S}_{\text{obs}} | \vec{C}_{\text{obs}}, H)$ of $v_{\text{bulk}}$, misalignment angle $\alpha$, Cosmic Mach Number $\mathcal{M}$ and correlation integral $J_v$ has been computed by counting the number of simulated catalogs that have, at the same time, $v_{\text{bulk}}$, $\alpha$, $\mathcal{M}$ and $J_v$ consistent with the observed ones, within quoted error bars. Table 2 reports, for all the considered models, the resulting values for the joint likelihood $\mathcal{L}(H)$.  

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In order to assess merits to the different models, we can use the relative likelihood, that is the ratio \( \lambda \) of the likelihood of a model to the maximum likelihood among them, \( \lambda(L) = L/L_{MAX} \). The quantity \(-2 \ln \lambda\) is asymptotically (i.e. for a number of observations \( N \to \infty \)) distributed as a *Chi-square* around this maximum, with a number of degrees of freedom equal to the number of parameters varied in the maximization (e.g. Kendall & Stuart 1979). Such an approximation is also valid in the case of non-independent observations like ours (e.g. Anderson 1971). In our case, due to the small number of observations this is a rough approximation, but it is nevertheless useful to provide an estimate of the confidence levels around the maximum likelihood model. Table 2 also shows the likelihood ratio \( \lambda \) and the confidence levels (CL) for the joint likelihood \( L(H) \).

On the basis of this analysis we can conclude that the best model is the unbiased SCDM \((n, b) = (1, 1)\). In any case we confirm the results of TMLM on the overall flatness of \( L \) in the range \( 0.8 \leq n \leq 1 \), and \( 1 \leq b \leq 1.5 \). Only the models with \( b \geq 2.5 \) are rejected at the 95% confidence level. The blue model, normalized to COBE, does not provide a good fit to the data.

We also checked that removing the Cosmic Mach Number statistics from the tests used, did not greatly change the discriminatory power of our analysis. This is easily explained on the basis of the analysis reported below.

### 3.6 Correlation among statistical tests

The likelihood analysis we have just carried out assumes that the velocity dipole \((v_{bulk}, \alpha)\), the velocity correlation function \( \Psi_1(r) \) and the Cosmic Mach Number \( M \) of a given sample are, in general, correlated quantities. Following this assumption the joint likelihood is obtained by counting the catalogs that have, *at the same time*, values of the statistics similar to the observed ones. Testing such an hypothesis is of some interest because having independent statistics would reduce the dimension of the sampled probability distribution, which would permit higher counts and lower statistical noise. In this subsection we will perform a qualitative analysis of this problem. Figure 7 shows the actual correlations between \( v_{bulk}, J, M \) and \( \sigma_v \) in the form of scatter plots. For the sake of simplicity in the figure we only display values of these statistics from the simulated catalogs of unbiased SCDM; we found that a similar trend is provided by the other models.

**FIGURE 7**

The plots show that the bulk flow of each sample is positively and very well correlated to the corresponding Cosmic Mach Number; on the contrary, there is little correlation between the latter quantity and the residual velocity dispersion \( \sigma_v \). This feature is not new: Suto et al. (1992) find the same trend in their analysis of \( M \); their results however refer to homogeneous and spherically symmetric velocity samples that did not include the effects of distance errors. We can conclude that these properties also apply when more realistic, anisotropic and
noisy velocity samples are considered. The observed trend is easily explained by noticing that relative departures from the respective mean values are much smaller for $\sigma_v$ than for $v_{\text{bulk}}$; as a consequence, it is the latter which mostly determines the value of $M$. The third panel compares $v_{\text{bulk}}$ and $J_v$: it shows indeed a correlation between the two, in the sense that higher velocities give both a higher amplitude for the velocity dipole and a higher value for $J_v$. The last panel shows $v_{\text{bulk}}$ vs. $\sigma_v$: the result implies no relevant correlation between the two statistics.

From this analysis we can conclude that our treatment of the data in the joint likelihood is appropriate in treating the velocity correlation and the bulk flow as correlated quantities. On the other hand, some improvement could be made by calculating the probabilities for the Cosmic Mach Number from the separate distributions of $v_{\text{bulk}}$ and $\sigma_v$, rather than from their joint distribution. However, that would reduce by one the total dimension of the joint probability space, which is not a big improvement anyway.

### 4 Conclusions

In this paper we reported the results of a statistical analysis of the large-scale velocity field in the context of $n \neq 1$ CDM models. This extends our previous work (TMLM), based on Monte Carlo simulations within linear theory, by the use of N–body simulations, which accounts for the behavior of the velocity field in the non–linear regime. We considered the values $n = 0.6$, 0.8, 1 and 1.2 for the spectral index and different values for the bias parameter $b$. We calculated the probability to have grid points with features similar to the Local Group; next we computed bulk flow, Cosmic Mach Number and velocity correlation function for our mock galaxy catalogs and compared the resulting distributions with the results of a composite sample of 1184 galaxies, grouped in 704 objects. Using a Maximum Likelihood method we calculated the probability of the models to reproduce the observations, as measured by the above statistics. Our results essentially confirm those derived from Monte Carlo simulations in TMLM, as long as the same models are concerned.

In particular, models with high tilt ($n = 0.6$) are rejected by the combination of the COBE results and the present analysis. The best model is the unbiased SCDM one, $(n, b) = (1, 1)$, but the likelihood function is nearly flat in the region $0.8 \leq n \leq 1$ and $1 \leq b \leq 1.5$. Note that for $n = 0.8$ the values of $b$ preferred by the present analysis and consistency with the COBE data require a negligible amount of gravitational waves. On the other hand, tilted models with moderate bias are likely to be preferred on the basis of a comparison with small–scale velocity dispersion data (Davis & Peebles 1983).

As a general result, our more accurate treatment of observational errors shows that tilted CDM models are not excluded by the combination of COBE data and the present analysis of galaxy peculiar velocities. Of course, having larger data samples, such as the “Mark III” compilation, can help to increase the discriminatory power of these statistical tests on the
large-scale velocity field.

Acknowledgments

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Figure captions

**Figure 1a.** Slices with thickness $4 \, h^{-1} \text{Mpc}$ for the models $(n, b) = (0.6, 2.5)$ and $(0.8, 1.5)$, in the top and bottom row respectively. Left column: projected particle positions. Right column: projected peculiar velocity field after smoothing by a Gaussian filter with width $2 \, h^{-1} \text{Mpc}$.

**Figure 1b.** As Figure 1a, for the models $(n, b) = (1, 1)$ and $(1.2, 0.5)$, in the top and bottom row respectively.

**Figure 2.** Probability distribution of the peculiar velocity $v$ (top row), density contrast $\delta$ (central row) and local ‘shear’ $S$ (bottom row), calculated on the grid points from simulations of the models $(n, b) = (0.6, 2.5)$ (first column), $(n, b) = (0.8, 1.5)$ (second column), $(n, b) = (1, 1)$ (third column) and $(n, b) = (1.2, 0.5)$ (last column). The shaded regions show the range allowed by the different Local Group constraints.

**Figure 3.** Probability distribution for the absolute value of the bulk flow, $v_{\text{bulk}}$, for the models $(n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)$ and $(1.2, 0.5)$. The shaded regions refer to the one $\sigma$ range obtained from our real catalog.

**Figure 4.** Probability distribution for the Cosmic Mach Number, $\mathcal{M}$, for the models $(n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)$ and $(1.2, 0.5)$. The shaded regions refer to the one $\sigma$ range obtained from our real catalog.

**Figure 5.** Observed velocity correlation function vs. the separation $r$ (thick solid line with squares; error bars are one standard deviation for each bin) compared to the probability distribution for $\Psi_1$ from the simulated catalogs for the models $(n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)$ and $(1.2, 0.5)$. The different lines refer to the 5%, 25%, 50%, 75% and 95% percentiles.

**Figure 6.** Probability distribution for the correlation integral $J_v$, for the models $(n, b) = (0.6, 2.5), (0.8, 1.5), (1, 1)$ and $(1.2, 0.5)$. The shaded regions refer to the one $\sigma$ range obtained from our real catalog.

**Figure 7.** Correlation between different statistics for the model $(n, b) = (1, 1)$. Top–left panel: the bulk flow $v_{\text{bulk}}$ vs. the Cosmic Mach Number $\mathcal{M}$. Top–right panel: the velocity dispersion $\sigma_v$ vs. $\mathcal{M}$. Bottom–left panel: $v_{\text{bulk}}$ vs. the correlation integral $J_v$. Bottom–right panel: $v_{\text{bulk}}$ vs. $\sigma_v$. 


Table 1. Local Group constraints.

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