Supersymmetric predictions for the inclusive 
$b \to s \gamma$ decay

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ABSTRACT

We study the penguin induced transition $b \to s \gamma$ in the minimal N=1 supersymmetric extension of the Standard Model with radiative breaking of the electroweak group. We include the effects of one-loop corrections to the Higgs potential and scalar masses. We show that the present upper and lower experimental limits on the inclusive decay sharply constrain the parameter space of the model in a wide range of tan $\beta$ values. The implications of the recently advocated relation $|B| \geq 2$ for the bilinear SUSY soft breaking parameter in grand unified theories are also analyzed.
1 Introduction

Since the recent measurement of the first exclusive \( B \to X_s \gamma \) decay, namely [1]
\[
BR(B \to K^*(892) \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}
\]
(1)
and the improved bounds on the inclusive branching ratio [2]
\[
0.8 \times 10^{-4} < BR(B \to X_s \gamma) < 5.4 \times 10^{-4}
\]  
(2)
(at 95% confidence level) there has been a renewed interest in the theoretical status of the predictions for this process in the standard model and beyond [4–13].

In particular it has been emphasized that the upper bound in eq. (2) can sharply constrain non supersymmetric two Higgs doublet models [3–5], whereas its impact on supersymmetric (SUSY) extensions of the model crucially depends on the value of the ratio of the two vacuum expectation values \( \tan \beta = v_2/v_1 \) [7–11].

The measurement of \( B \to K^*(892) \gamma \) represents the first evidence of penguin induced decays. The peculiarity of this loop-induced process, \( b \to s \gamma \) at the quark level, is that it “dominated” by higher order QCD corrections, which soften logarithmically the power-like GIM [14] suppression present in the pure electroweak contribution. This fact was first pointed out in refs. [15], and subsequently confirmed by accurate renormalization group (RG) analysis [16].

The logarithmic flavour-changing suppression of the QCD corrected amplitude makes the process less sensitive to the top quark mass, but enhances for \( m_t \approx 150 \, GeV \) by a factor of 3-4 the branching ratio with respect to the purely electroweak prediction.

The first detailed analysis of this and other \( \Delta B = 1 \) processes in the context of supersymmetric extension of the SM (MSSM) has been carried out in ref. [17]. In that paper the hypothesis of an underlying grand unified theory (GUT) was employed in order to reduce the number of arbitrary parameters, together with the assumption of radiative breaking of the electroweak group. The predictions of the MSSM are there parameterized in terms of \( m_t, \tan \beta \) and two independent SUSY masses.

Charged Higgs, chargino, gluino and neutralino exchange were thoroughly studied and the results were presented for given ranges of the four dimensional parameter
space. The analysis of \(b \to s \gamma\) and the other rare \(b \to s\) transitions considered in ref. [17] showed as a general feature that, for squarks and gluinos heavier than 100 GeV, gluino and neutralino induced contributions were negligible with respect to those induced by charged Higgs and chargino exchange.

Whereas the charged Higgs amplitude for \(b \to s \gamma\) interferes always constructively with the SM (W-induced) one, the chargino component of the amplitude may interfere either constructively or destructively with the previous two depending on the region of the SUSY parameter space considered. The region of destructive interference in the range of \(\tan \beta\) considered in ref. [17] \((\tan \beta \leq 8)\) was observed to be confined to a limited area of the parameter space. It has been recently observed [7–10] that for larger \(\tan \beta\) the chargino contribution increases in size allowing for branching ratios much below the SM prediction, thus weakening substantially the potentiality of \(B \to X_s \gamma\) in constraining SUSY models.

With the present paper we want to present a detailed analysis of the \(b \to s \gamma\) transition in the MSSM with radiative electroweak breaking, by extending and completing the study of ref. [17]. With respect to the latter, we include here the corrections due to the one-loop effective potential in the determination of the minimum and shifts of the scalar masses. In the effective potential we consider fully the contributions of stop, sbottom and stau scalars. We have also released the GUT scale constraint \(B = A - 1\) between the trilinear and bilinear soft breaking parameters and studied the response of the model to varying independently the two parameters. The unification of the gauge couplings is obtained including two-loop effects in the gauge running and the request that bottom and tau Yukawa couplings unify at the GUT scale (within 30%) is imposed. For the bottom and top quarks we include the shifts induced by QCD corrections between the pole masses and the \(\overline{MS}\) running masses (for a recent review see ref. [18]). Considering the bottom pole mass \(M_b\) between 4.6 and 5.2 GeV, we find that the requirement of tau-bottom Yukawa unification generally pushes \(M_b\) to the upper edge of the range. We finally discuss in detail the correlation between \(\tan \beta\) and the size of the chargino amplitude through numerical studies and analytic results.

Recent works on the topic have covered only partially the aforementioned aspects, either for instance only a phenomenological approach is taken in limiting the range of the many SUSY parameters [10], or radiative corrections to the tree-level potential are neglected [11].
In particular, releasing the GUT constraint \( B = A - 1 \) turns out to have important implications for the MSSM GUT model. We consider interesting to examine the case \( |B| \geq 2 \) which is advocated in ref. [19] as a consequence of the integration of the heavy dynamical degrees of freedom in SUSY GUT theories. In agreement with the “preferred” range \( |B| < 1 \) found in ref. [10], the constraint \( |B| \geq 2 \) turns out to lower the possibility of obtaining the correct vacuum and therefore reduces substantially the accessible parameter space.

Our results are presented in figures that show the behaviour of \( b \to s \gamma \) amplitudes and branching ratio as a function of various SUSY masses. We have considered the whole ranges of \( \tan \beta \) allowed by the requirement of radiative breaking. The effects of the present experimental constraints on the allowed SUSY parameter space are shown. We find that in the whole range of \( \tan \beta \) the GUT MSSM with radiative breaking is already strongly constrained by the present CLEO limits (specially in the case of \( |B| \geq 2 \)), and that a positive evidence for the inclusive decay might render the MSSM quite predictive for SUSY particle searches at the hadron colliders of the next generation. The present analysis shows that in most of the \( \tan \beta \) range allowed, the indirect constraints on gluino, squarks and light Higgs scalars are stronger than those obtained by direct searches at LEP1 and Tevatron.

The paper is organized as follows. In Sect. 2 we recall some of the basics of the minimal supersymmetric extension of the SM, focussing on the structure of the scalar potential and of the soft SUSY breaking sector. In Sect. 3 we study the role of the chargino induced contribution in the process amplitude and its characteristic dependence on \( \tan \beta \). We then analyze our numerical outcomes for the inclusive \( BR(b \to s \gamma) \) and show their dependence on the relevant SUSY parameters. The implications of the recent experimental observations are finally discussed.
2 The MSSM

In this section we shall set the framework for our analysis and introduce the necessary definitions. After recalling the particle content and the lagrangian of the theory we review the parameters space of the MSSM, focussing in particular on the structure of the model at the GUT scale, and on the soft breaking sector. Finally, we discuss in some details how the renormalization group scaling of the parameters yields us a predictive supersymmetric extension of the Standard Model.

2.1 Particle content and lagrangian

In any SUSY extension of the standard model the particle spectrum must be extended by at least doubling the number of the Higgs fields, and introducing a supersymmetric partner for each standard field. One can describe conveniently the particle content of the theory using the superfield formalism; for instance, each chiral superfield corresponds to a complex scalar and a Weyl spinor, so that the supersymmetric standard model includes the chiral superfields $Q, U^c, D^c, L, E^c$, that extend the standard fermionic sector, and $H_1$ and $H_2$, that extend the standard scalar sector. These superfields transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as follows:

\[ Q \equiv (3, 2, 1/6); \quad U^c \equiv (\bar 3, 1, -2/3); \quad D^c \equiv (\bar 3, 1, 1/3); \]
\[ L \equiv (1, 2, -1/2); \quad E^c \equiv (1, 1, 1); \]
\[ H_1 \equiv (1, 2, -1/2); \quad H_2 \equiv (1, 2, 1/2). \]  

(3)

The chiral matter superfields $Q_i, U^c_i, D^c_i$ are multiplets in generation space; we will consider in the following the case of three generation ($i = 1, 2, 3$). Analogously each vector boson has a supersymmetric fermionic partner. The scalar partners of the quarks (resp. leptons) are called squarks (resp. sleptons), while the fermionic partners of gauge vector bosons (resp. scalars) are called gluino, wino, zino, photino (resp. higgsino). The spontaneous breaking of the SM gauge symmetry is obtained by letting the scalar component of the $H_1$ and $H_2$ fields get vacuum expectation values $v_1$ and $v_2$ respectively; in the model under consideration this mechanism is triggered by the running of the relevant parameters of the Higgs potential, as discussed in the following. Eq. (3) shows the minimal matter content needed in any SUSY extension of the SM, and it is the one considered in the present analysis.
We assume that the supersymmetric standard model under consideration is the low energy manifestation of a Grand Unified Theory, minimally $SU(5)$. To the degrees of freedom of eq. (3) one should add the heavy GUT superfields, which are to be integrated away when considering the low energy regime. Let us recall that the $SU(5)$ GUT assumption leads to a consistent gauge coupling constant unification in the SUSY context at a scale $M_{GUT} \approx 3 \times 10^{16} \text{ GeV}$, provided that the light Higgs doublet content is the minimal one (two Higgs doublets).

The supersymmetric extension of the gauge sector of the standard model is straightforward. By converse the supersymmetrization of the Yukawa interactions deserves some attention. We build up this part of the lagrangian by constructing the most general gauge invariant products and sums of chiral superfields, consistent with renormalizability (a cubic polynomial called superpotential). This procedure leads in general to the presence of potentially dangerous baryon (and lepton) number violating terms. We forbid the appearance of such terms by assuming a further symmetry in the theory, the matter parity, or R-parity. Under this symmetry superparticle transform, but not ordinary particles. Among else the presence of such an unbroken symmetry implies the stability of the lightest superparticle.

When building up the superpotential one is faced with the need of introducing two different Higgs superfields with opposite hypercharge, to allow for mass terms for both up and down quarks and cancel the higgsino anomaly. A potentially dangerous $U(1)$ global symmetry in the Higgs sector can be avoided introducing the so called $\mu$ term in the superpotential, which couples the two different Higgs doublet superfields.

According to the previous discussion we will henceforth refer to the following superpotential:

$$W = -h_0^U H_2 Q_1 U^c_2 + h_0^D H_1 Q_1 D^c_2 + h_0^L H_1 L^c_1 E^c_2 + \mu H_1 H_2$$

(4)

In each monomial of the superpotential any pair of $SU(2)_L$ doublets must be contracted with the matrix $\epsilon \equiv i \tau_2$. The $3 \times 3$ matrices $h_U$, $h_D$ and $h_L$ are the complex conjugates of the usual Yukawa matrices, namely

$$h_x = Y^*_x \quad x = U, D, L$$

(5)

where $Y_U \overline{Q}_L H_2 u_R$ defines the up-quark Yukawa coupling, as it appears usually in the SM lagrangian. The highest dimension field component of the superpotential transforms via a total derivative under a supersymmetric transformation, and can therefore be used to build an invariant action.
An important point on which different kinds of models differ is the specific structure of the sector responsible for supersymmetry breaking. A popular class of realistic SUSY models is that in which the global supersymmetry breaking is a consequence of the spontaneous breaking of an underlying N=1 supergravity theory (for reviews see ref. [20]). The locally supersymmetric lagrangian is supposed to undergo a spontaneous breaking in the so called hidden sector, and the effects of this breaking are communicated to the observable sector through gravitational effects. A renormalizable theory is obtained in the limit in which the Planck mass goes to infinity. By doing so we are left with a globally supersymmetric lagrangian and explicit soft breaking terms at some GUT energy scale, which we shall call $M_X$ and for practical purposes identify tout-court with the GUT scale. More specifically we shall consider the following gauge invariant soft breaking Lagrangian:

$$\mathcal{L}_{\text{soft}} = -\mathcal{M}^2 - (\hat{M} + S + h.c.)$$

(6)

where:

1) $\mathcal{M}^2$ is a mass term for all the scalars in the theory

$$\mathcal{M}^2 \equiv \Sigma m_{ij}^2 z_i^* z_j;$$

(7)

2) $\hat{M}$ a mass term for the gauginos $\lambda_\alpha$, considered as Weyl fields

$$\hat{M} \equiv -\frac{M^a}{2} \lambda_\alpha \lambda_\alpha;$$

(8)

3) $S$ is the scalar analogue of the superpotential (notice the explicit massive parameter $m$)

$$S = m \left[ -h^A_i H_2 \tilde{Q} \tilde{U}^c + h^A_D H_1 \tilde{Q} \tilde{D}^c + h^D_E H_1 \tilde{L} \tilde{E}^c + B \mu H_1 H_2 \right]$$

(9)

As we next discuss, the large number of arbitrary parameters present in the soft breaking lagrangian of eqs. (6–9) will be drastically reduced by minimality requirements and the GUT hypothesis.

### 2.2 The low-energy minimal SUSY model

In order to investigate the predictions of the model under consideration for low-energy phenomenology we must consider the renormalization group evolution of the
various parameters from the high energy scale to the scale electroweak interactions. This program, within the supersymmetric context, leads to the successful prediction of $\alpha_s$ in the framework of gauge coupling unification. Similarly, the tau-bottom Yukawa unification in minimal $SU(5)$, which depends on the yet unknown top mass and on the ratio of the two vacuum expectation values, can be realized in a sizable region of the parameter space (we require unification up to 30% correction effects due to GUT scale thresholds and two-loop running).

In supergravity derived models the different renormalization group evolution of the mass parameters in the scalar potential produces the necessary conditions for the spontaneous breaking of the electroweak symmetry. The $m_Z$ mass, seen as a function of the various supersymmetric parameters, allow us to reduce the size of the SUSY parameter space (more on that in the next section). We shall now detail our assumptions for the soft breaking parameters.

The choice the massive parameters $M_a$ and $m_{ij}^2$ is related to the supergravity model under consideration. For instance either models in which the scalar mass terms $m_{ij}^2$ are zero at high scale (no-scale models), or models in which the gaugino masses $M_a$ are very small (models with light gluinos), or models in which these two parameter are related (dilaton dominated supersymmetry breaking), have been considered in the literature.

Similar considerations are valid for the choice of the Yukawa-like parameters $h_{ij}^A$. Our analysis will be carried out assuming the following form of the matrices $m_{ij}^2$ and $h_{ij}^A$:

$$m_{ij}^2 = m^2 \delta_{ij} \quad h_{ij}^A = A h_{ij}$$

(10)

with $A$ a dimensionless constant. This form can be derived by assuming a flat Kähler metric in the supergravity theory, and guarantees the absence of large flavour changing neutral currents in the scalar sector. We will assume also that, analogously to the gauge coupling constants, the three gaugino masses unify at the high energy scale as well:

$$M_a = M \quad \alpha = 1, 2, 3$$

(11)

As a consequence, besides the yet unknown $m_t$, we are left with five “supersymmetric” parameters:

$$A, B, M, m, \mu$$

(12)

As a matter of fact, a relation between $A$ and $B$ holds at $M_X$ for a flat Kähler
metric, namely \( B = A - 1 \). This further reduces the number of free-parameters to four. Recently, Giudice and Roulet have made the interesting observation that the integration of the “heavy” degrees of freedom of any SUSY GUT theory with \( \mu = 0 \) gives an effective theory in which the original relation \( B' = A' - 1 \) translates to \( |B| = 2 \) (the prime indicates the parameters in the complete theory), and in all generality
\[
|B| \geq 2
\] (13)
A calculable model dependent \( \mu \) term is then generated as a function of the original parameters. Assuming this scenario and flat Kähler metric, one should then require \( |B| = 2 \) instead of the commonly used \( B = A - 1 \). Since the corresponding value of \( \mu \) depends on the detailed structure of the SUSY GUT theory, we will analyze the case \( |B| = 2 \) leaving \( \mu \) arbitrary, and compare with the case \( B = A - 1 \).

We conclude this section with some considerations about the role of the parameter \( \mu \). This parameter appears, let aside interaction terms, in the mass matrix of the scalar particles, of the neutralinos and of the charginos (mixed states of gauginos and higgsinos with assigned charge). It is important to stress that given the form of our eq. (4) and eq. (8) the chargino mass matrix reads:
\[
- (\bar{\tilde{w}}^- \tilde{h}_1^-) \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & -\mu_R \end{pmatrix} \begin{pmatrix} \bar{\tilde{w}}^+ \\ \tilde{h}_2^+ \end{pmatrix} + h.c.
\] (14)
where following ref. [21] we define \( \tilde{w}^\pm \equiv -i\lambda^\pm \equiv -i(\lambda_1 \mp i\lambda_2)/\sqrt{2} \), and all the parameters are taken at the weak scale \( (M \to M_2, \mu \to \mu_R) \). It has to be stressed that abiding by the conventions of ref. [21] (which we closely follow), a minus sign in front of the \( \mu \) entries in the chargino and neutralino mass matrices, with respect to those given in ref. [21], has to be added to be consistent with the Feynman rules and scalar mass matrices there given. Alternatively, one may want to keep the plus sign in the fermion mass matrices and change the sign of \( \mu \) in the scalar mass matrices and Feynman rules. We find more convenient to follow the first prescription. Since the role of the chargino-squark induced amplitude is crucial for the process we are studying it is quite important to make sure that the relevant Feynman rules and mass matrices are derived in a consistent way: for what matters the present analysis, we have found a complete agreement with the results of ref. [21] up to the aforementioned \( \mu \) sign in the chargino and neutralino mass matrices.
2.3 Radiative breaking of the electroweak symmetry

The study of the spontaneous breaking of the electroweak group involve a discussion of the Higgs potential and it renormalization. Let us begin by recalling the structure of the tree-level Higgs potential in the MSSM. Even if we know that the 1-loop corrections are important (and they will be included in our analysis), the analytic formulae that can be found at the tree-level help in the qualitative interpretation of the numerical results.

The part of the scalar potential involving the neutral Higgs fields contains quadratic and quartic terms. The massive parameters in the quadratic part are related via the renormalization group evolution* to the massive parameters discussed in the last section, while those of the quartic terms are directly dictated by supersymmetry:

\[ V_0 = \mu_1^2|H_1^0|^2 + \mu_2^2|H_2^0|^2 - \mu_3^2(H_1^0H_2^0 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2, \]  

(15)

The parameters \( \mu_i^2 \) must be such that the scalar potential is bounded in the direction \( |H_1^0| = |H_2^0| \), where the quartic terms gives no contribution, and that the configuration \(< H_i^0 >= 0 \), corresponding to the unbroken phase, is not a minimum of the potential. These arguments lead to the following relations

\[ \mu_1^2 + \mu_2^2 \geq 2|\mu_3^2| \]  

(16)

and

\[ \mu_1^2 \cdot \mu_2^2 < \mu_3^4 \]  

(17)

which at the \( M_X \) scale cannot be satisfied (\( \mu_1^2 = \mu_2^2 = m^2 + \mu^2 \)). However, what matters for the consistency of the model is that eqs. (16–17) hold at \( \approx m_Z \), a scale much lower than \( M_X \). In fact, due to the heaviness of the top quark the parameters \( \mu_1 \) and \( \mu_2 \) run differently and it is possible to satisfy both eqs. (16–17) at low energy. The solutions depend on the chosen values for the various SUSY parameters and in particular on \( \tan \beta \); when \( \tan \beta \) grows the bottom and top Yukawa coupling become more and more similar and tend to restore the incompatibility of eqs. (16–17) also at the weak scale. From the tree-level potential analysis follows that there exist a

* For a complete set of RGE’s in explicit matrix form and consistent with the present analysis see Appendix A of ref. [17]. The following typos should be corrected: in eq. (A1), \( 2M_2^2 \rightarrow M_1^2 \); in eqs. (A6,A7), \( \tilde{Y}_{D,U} m_Q^2 \tilde{Y}_{D,U} \rightarrow \tilde{Y}_{D,U} m_Q^2 \tilde{Y}_{D,U} \) and \( \tilde{Y}_{E} m_L^2 \tilde{Y}_{E} \rightarrow \tilde{Y}_{E} m_L^2 \tilde{Y}_{E} \); in eq. (A9), \( \tilde{Y}_e \tilde{Y}_e^A \rightarrow \tilde{Y}_e \tilde{Y}_e^A \).
maximum value of \( \tan \beta \approx m_t/m_b \) beyond which the electroweak breaking does not occur. We will see that this feature is maintained even after the inclusion of the 1-loop corrections to the potential.

It is important to recall that the renormalized parameters are implicit functions of the Yukawa, soft breaking and \( \mu \) parameters at the \( M_X \) scale. We can therefore convert relations like eqs. (16-17) into bounds on allowed regions for the original parameters. For instance, the bilinear soft breaking parameter \( B \) is crucial for the determination of the electroweak vacuum. This parameter, that enters the lagrangian through \( \mu_3^2 \):

\[
\mu_3^2|_{M_X} = -B m_\mu
\]  

(18)

plays a major role in drawing, through eq. (16) and eq. (17) the regions in which the electroweak symmetry breaking can be consistently realized. We will see that fixing its value to \( |B| \geq 2 \) at \( M_X \) drastically reduces the parameter space allowed by the weak breaking.

It has been shown that the 1-loop corrections to the Higgs potential are important in determining the spectrum of the physical Higgs fields. For instance, the MSSM tree-level prediction of a scalar Higgs particle always lighter than the \( Z \) boson is spoiled once we take into account radiative corrections. Furthermore, the inclusion of the 1-loop corrections stabilize the low-energy predictions of the model with respect to variation of the chosen renormalization scale.

The 1-loop corrected scalar potential can be written as

\[
V_1 = V_0 + \Delta V
\]  

(19)

where \( V_0 \) is the tree-level potential and \( \Delta V \) represents the 1-loop correction. In the \( \overline{MS} \) renormalization scheme each particle contributes to the 1-loop potential according to

\[
\Delta V = \frac{1}{64\pi^2} \text{Str} \left[ \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right]
\]  

(20)

where \( \mathcal{M} \) is the generalized mass matrix function of the scalar fields, \( Q \) is the renormalization scale, and Str is the supertrace, that is the sum over the various species of particles of spin \( j \) weighted by \((-)^{2j/(2j + 1)}\). Let us recall that the \( Q \)-dependence of the lagrangian is twofold; the explicit \( Q \)-dependence in the previous formula and the implicit \( Q \)-dependence of the parameters of the lagrangian due to the renormalization group evolution; these two effects combine into a higher order
dependence on $Q$ of the resulting effective lagrangian, and determine the greater stability of the predictions. We will take $Q = m_Z$, and fully include in our numerical analysis the 1-loop contributions of the third family of quarks, leptons, squarks and sleptons.

An alternative way of incorporating the effects of one-loop corrections due for instance to heavy squarks, would be to integrate them out at some intermediate threshold (TeV scale) and consider the resulting renormalization group improved tree-level potential [22]. In this way the effects of large stop (sbottom) contributions would be encoded in the different evolution of the tree-level parameters due to presence of intermediate thresholds, and the task of minimization of the potential would be much simpler. However, we choose not to follow this approach since we do not want to assume any particle to be a priori heavier than the present experimental bounds. On the contrary, our aim is to test whether the constraints coming from $b \to s\gamma$ may push some bounds further up.
3 Predictions for the inclusive $b \to s\gamma$ decay

In this section we analyze the impact of the experimental information on the inclusive $b \to s\gamma$ transition on the MSSM with radiative breaking.

We will first devote some time to a discussion of the individual SM and SUSY contributions the process amplitude. In particular, we will focus our attention on the dependence of the chargino component on the various parameters, and study its large destructive interference with the $W$ and $H^-$ induced amplitudes for large $\tan \beta$.

Finally, the present experimental constraints from collider and $B$-meson physics are used to limit the parameter space allowed for SUSY masses below the $TeV$ scale.

3.1 Amplitude anatomy

For given values of $m_t$, $\tan \beta$, $B$, $M$, and $m$ ($A$ and $\mu$ are then determined from the minimization of the electroweak potential) we numerically compute, after performing the renormalization of the original SUSY lagrangian down to the weak scale, the whole particle spectrum and the interaction terms of the MSSM needed to compute the process at hand. Then, standard QCD renormalization is used to obtain the physical amplitude at its natural $m_t$ scale [16] (see also refs. [17] and [23] for a detailed description of the procedure used).

The $b \to s\gamma$ decay can proceed in the MSSM via five different intermediate particles exchanges:

1. $W^-$ + up-quark
2. $H^-$ + up-quark
3. $\tilde{\chi}^-$ + up-squark
4. $\tilde{g}$ + down-squark
5. $\tilde{\chi}^0$ + down-squark
The total amplitude for the decay is the sum of all these contributions. The complete analytic expressions for the various amplitudes is found in ref. [17].

An effective $b\rightarrow s$ flavour changing transition induced by $W^-$ exchange is the only way through which the process proceeds in the SM. A two-Higgs doublet extension of the SM would include the first two contributions, while the last three are genuinely supersymmetric in nature. Even if one might believe that the gluino exchange could be important, due to the replacement of the weak coupling with the strong one, it is not the case in the model under consideration, given the present bounds on gluino and squark masses.

In Figs. 1 and 2 the relative size of the various amplitudes to the SM one are shown. Although the figures are drawn for given values of $m_t$ and $\tan \beta$ ($m_t = 160 \text{ GeV}$ and $\tan \beta = 8$) they exhibit the general features of the various contributions for a wide range of parameters. In particular, we observe that gluino and neutralino exchange can be neglected in comparison with charged Higgs and chargino amplitudes (however all the contributions are included in our final numerical results). A thorough discussion for a qualitative understanding of the relevance of the various component of the amplitude can be found in ref. [17].

The dotted area in the figures corresponds to spanning the soft breaking parameters $m$ and $M$ in the ranges $[0, 800]$ GeV, and $[-100, 400]$ GeV respectively. Both the assignments $B = A - 1$ and $B = 2$ are shown. In the latter case, we show the results only for positive values of $B$ since the patterns for the corresponding negative range turn out to be quite similar. This can be understood by noticing that for the minimization of the tree level potential $B \rightarrow -B$ is equivalent to $\mu \rightarrow -\mu$ (the one-loop corrections to the potential do not sensibly spoil this feature) and that for given $B$ we always span a symmetric range in $\mu$. We have shown the case $B = 2$ since it corresponds directly to $B = A - 1$ (flat Kähler metric) in the original SUSY-GUT lagrangian. In addition, it turns out to be the least restrictive choice for the model considered in the range $|B| \geq 2$. A discussion on the relevance of this parameter will follow.

Since gluino and neutralino contributions are numerically irrelevant in the next figures we focus our attention on charged Higgs and charginos by studying more in detail the dependence of the corresponding amplitudes on their masses and the value of $\tan \beta$. In Figs. 3, 4 and 5 the ratios of charged Higgs and chargino induced
amplitudes with the SM contribution are shown for $\tan \beta = 2, 20, 40$, while the top mass is kept at 160 GeV. The Higgs amplitude is shown as a function of the charged Higgs mass, while the chargino amplitude is shown both as a function of the lightest chargino mass and the lightest stop mass. No substantially different features appear by varying $m_t$ in a few ten GeV interval from our preferred value. It should be recalled however that the maximum value of $\tan \beta$ allowed by the radiative breaking depends on the top mass. In the case under consideration we roughly have $\tan \beta_{max} \approx 45$, while for $m_t = 140 \tan \beta_{max} \approx 40$ and for $m_t = 180 \tan \beta_{max} \approx 50$. This statement is clearly dependent on the window of values of $(m, M)$ we have considered, and on the value of $B$. We think we have tested wide enough ranges to make the above statements indicative for phenomenological considerations.

Inspection of Figs. 3-5 shows that, while the charged Higgs amplitude interferes always constructively with the SM one (this is at the root of the sharp constraints on the charged Higgs mass found in Higgs extensions of the SM [3–5]), the chargino amplitude can give rise to substantial destructive interference with the SM and $H^+$ amplitudes, becoming for large $\tan \beta$ the dominant contribution. This effect clearly renders the CLEO upper limit in eq. (2) a less severe constraint for the MSSM than for models in which the negative interference is absent (see for instance non-supersymmetric two Higgs doublet models). In spite of that, we shall see that the full inclusion of the present experimental limits is already enough to exclude a large portion of the tested parameter space.

### 3.1.1 The chargino exchange contribution

Due to the relevance of the chargino amplitude for the present discussion, it is worth trying to have a better understanding of the nature of the features exhibited by this amplitude in the previous figures. We shall look closely to the formula of the chargino amplitude for $b \rightarrow s \gamma$ derived in ref. [17], to which we refer the reader for all definitions and details:

$$A_{\tilde{\chi}^-} = -\frac{\alpha_w \sqrt{\alpha}}{2\sqrt{\pi}} \sum_{j=1}^{2} \sum_{k=1}^{6} \frac{1}{m_{\tilde{\chi}^-}^2}$$

$$\times \left\{ (G_{UL}^{jk} - H_{UR}^{jk})(G_{UL}^{jk} - H_{UR}^{jk}) \left[ F_1(x_{\tilde{\chi}^-_j \tilde{\chi}^-_{j_k}}) + e_U F_2(x_{\tilde{\chi}^-_{j_k}}) \right] \right. $$

$$- H_{UL}^{jk} G_{UL}^{jk} \left. H_{UR}^{jk} \right) \frac{m_{\tilde{\chi}^-}}{m_b} \left[ F_3(x_{\tilde{\chi}^-_j \tilde{\chi}^-_{j_k}}) + e_U F_4(x_{\tilde{\chi}^-_{j_k}}) \right] \right\}$$

(21)
where \( j = 1, 2 \) is the label of the chargino mass eigenstates (from light to heavy) and
\( k = 1, ..., 6 \) is the analogous label for the up-squarks; the matricial couplings \( G_{UL} \)
arise from charged gaugino-squark-quark vertices, whereas \( H_{UL} \) and \( H_{UR} \) are related
to the charged higgsino-squark-quark vertices. These couplings contain among else
the unitary rotations \( U \) and \( V \) which diagonalize the chargino mass matrices. All
the quantities in eq. (21) are defined in ref. [17].

An explicit \( \tan \beta \) dependence is found in \( H_{UL} \) and \( H_{UR} \) where quark Yukawa
couplings are present; more precisely, \( H_{UL} \) is proportional to the down-quark Yukawa
coupling, which grows with \( \tan \beta \) as \( 1/\cos \beta \), whereas \( H_{UR} \) contains the up-quark
Yukawa coupling, that approaches in the large \( \tan \beta \) limit a constant value (\( \alpha \)
\( 1/\sin \beta \)). It is in fact the contribution of the third line in eq. (21) that determines
the behaviour of the amplitude in the large \( \tan \beta \) regime (specifically the component
\( H_{UL}H_{UR}^* \)). Graphically it corresponds to the Feynman diagram depicted in
Fig. 6, which exhibits in terms of squark and chargino interaction eigenstates the
structure of this component of the amplitude. For \( \tan \beta \to \infty \) the amplitude
diverges, but since we work in a perturbative scheme we are bounded by the request
of perturbativity of the down Yukawa couplings, say \( \tan \beta < 60 \).

An analytic approximation of the \( H_{UL}H_{UR}^\ast \) component of the chargino amplitude
can be derived which shows explicitly a number of interesting features. Let us consider
the possibility that the chargino mass matrix in eq. (14) might be
approximately diagonal:

\[
M_X \approx \text{diag}(M_2, -\mu_R) \quad (22)
\]

One can show that this approximation holds effectively when \( |M_2^2 - \mu_R^2| = \)
\( O(\max(M_2^2, \mu_R^2)) \gg m_W^2 \) and \( M_2^2, \mu_R^2 \sim m_W^2 \). It is important to notice that these
requirements, and therefore the approximation of eq. (22) are consistent with one
of the eigenvalues, say \( |\mu_R| \), being of the order of \( m_W \), while the other remains much
heavier. Being the chargino mass matrix already diagonal the approximate mass
eigenvalues are simply given by the absolute values of the parameters \( M_2 \) and \( \mu_R \),
and the two unitary rotations which “diagonalize” the chargino mass matrix can be
written as:

\[
U \approx \text{diag}(\text{sign}\{M_2\}, -\text{sign}\{\mu_R\}),
V \approx 1 \quad (23)
\]

Using eqs. (22–23) and the definitions of \( H_{UL,R} \) given in ref. [17], we can find
a simple and instructive expression for the part of the chargino amplitude relevant
for large $\tan \beta$:

$$A_{\chi^-} \approx G_F \sqrt{\frac{\alpha}{(2\pi)^3}} K_{1a} K_{1b} \left\{ \frac{1}{\sin 2\beta} \frac{m_{\tilde{t}_1}}{\mu_R} \left[ \mathcal{F} \left( \frac{m_{\tilde{t}_1}^2}{\mu_R^2} \right) - \mathcal{F} \left( \frac{m_{\tilde{t}_2}^2}{\mu_R^2} \right) \right] \right\}$$  \hspace{1cm} (24)

where $m_{\tilde{t}_1}^2$ (resp. $m_{\tilde{t}_2}^2$) is the mass of the lighter (resp. heavier) stop, $m_{\chi^-} = |\mu_R|$ is taken to be the lightest chargino eigenvalue and the function $\mathcal{F}$ is defined to be

$$\mathcal{F}(x) = \frac{1}{x} \left[ F_3 \left( \frac{1}{x} \right) + e_U F_4 \left( \frac{1}{x} \right) \right]$$

$$= \frac{1}{6(1 - x)^3} \left( 5 - 12x + 7x^2 + 2x(2 - 3x) \log x \right)$$  \hspace{1cm} (25)

The curly bracketed term exhibit the main features of the chargino induced amplitude for large $\tan \beta$. Let us emphasize three important features of this contribution.

1. For $\tan \beta \gg 1$ we can write $1/ \sin 2\beta \approx \tan \beta/2$, showing explicitly the leading linear behaviour of this part of the amplitude in the parameter $\tan \beta$.

2. The value of the amplitude is crucially dependent on the splitting of the two stop mass eigenstates $m_{\tilde{t}_{1,2}}$, $\tilde{t}_1$ being the lighter stop quark. The splitting of the two mass eigenstates depends on the size of the L-R entry in the stop mass matrix, and corresponds in fact to the L-R mass insertion on the squark line in the interaction representation of Fig. 6. Since $\mathcal{F}(x)$ is a positive, monotonically decreasing function of $x$ in the interval $x \in [0, \infty]$ ($\mathcal{F}(0) = 5/6$, $\mathcal{F}(\infty) = 0$) the term in square brackets is maximized when one of the two stop eigenstates is light.

3. The sign of the amplitude depends directly on the sign of the parameter $\mu_R$, that is the sign of $\mu$, the parameter introduced at the GUT scale (the renormalization for $\mu$ is in fact multiplicative); this fact means that the region in which the chargino amplitude gives rise to a destructive interference effect with the other amplitudes corresponds to the region in which $\mu$ is negative. This behaviour was observed also in ref. [12] but there not understood. It is also important to notice that $\mu_R$ should be as light as possible ($\mu_R \approx m_W$) in order not to suppress the contribution (for large $\mu_R$ the amplitude decreases linearly with $\mu_R$). Notice also that the amplitude vanishes for $\mu_R \to 0$, as it should be since from the interaction state diagram of Fig. 6 it appears that this component of the chargino amplitude is proportional to the $\hat{h}_1 - \hat{h}_2$ higgsino mixing, namely $\mu_R$.  

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In passing it is worthwhile remarking that this component of the chargino amplitude for the effective $b \to s \gamma$ vertex is exactly the analogue of the chargino dipole component that in the flavour diagonal case might dominate the electric dipole of the elementary quarks in the MSSM, as recently discussed in ref. [21].

3.1.2 The role of the parameter B

We conclude this section by remarking that if $|B| = 2$ instead of $B = A - 1$ is imposed, the region of parameter space for which the radiative breaking of the electroweak symmetry can be realized diminishes considerably. We have also found that for $|B| > 2$ the allowed regions become even smaller. As we have already remarked this observation is in agreement with the results of ref. [10], where the authors noticed that $B$ lying in the neighbourhood of zero represents the most favourable case. The constraint $B = A - 1$, to be imposed if one does not require the existence of a grand unified scenario, includes naturally this region. A detailed and general analysis of the parameter space and low energy particle spectrum of the model under consideration will be matter of a forthcoming paper.

3.2 Inclusive branching ratio: numerical results

In Figs. 7–10 we present our numerical results for the branching ratio of the process under consideration. We have chosen to show them for our preferred value of the top mass, namely $m_t = 160$ GeV, and $\tan \beta = 2, 8, 20, 40$ (figures 7, 8, 9, 10 respectively). In each figure we plot the total branching ratio (which includes all SUSY contributions) versus the three relevant SUSY masses: that of the charged Higgs, of the lightest chargino and of the lightest top squark (which is generally the lightest squark in the model). The cases $B = A - 1$ and $B = 2$ are compared. The horizontal solid line represents the SM result, which depends only on the top quark mass and the value of the strong coupling which enters through the important QCD corrections. For the purpose of comparison with the SUSY outcomes we show our results for a given value of $\alpha_s$, namely $\alpha_s(m_Z\overline{MS}) = 0.12$. As it is widely discussed in the literature the present experimental error on $\alpha_s$ implies an uncertainty in the predicted $BR(b \to s \gamma)$ of about 15–20%. Even larger might be the error due to the neglect of next-to-leading effects in the renormalization of the dipole operator, which
may be as large as 30% (for a recent and complete discussion on these issues see A.J. Buras et al. in ref. [16]). Here the problem is that the calculation of the next-to-leading Hamiltonian for the $b \to s\gamma$ operator involves three-loop mixings and their evaluation is a true formidable task. A better understanding on the uncertainties related to the strong renormalization of the $b \to s\gamma$ amplitude could imply an upward or downward shift of the shaded areas in the figures (the largest renormalization effect is additive due to operator mixing, and depends only on the strong coupling). This effect becomes less relevant for large $\tan \beta$ due to the dominance of the chargino amplitude. Keeping this in mind, let us analyze the main features of our results.

When considering values of $\tan \beta$ of $O(1)$ (Fig. 7) the destructive interference effect, discussed in the previous subsection, is quite small. In this case, already the present CLEO inclusive upper bound restricts in a sizeable way the area in parameter space allowed by the model. For instance, a lower bound of about 200 GeV on the charged Higgs mass is obtainable for both choices of $B$ (recall that the corresponding LEP1 lower bound is 45 GeV). However, as soon as $\tan \beta$ is of $O(10)$ and larger the destructive interference effect of the chargino amplitude becomes substantial and the CLEO upper bound would not be by itself very effective in constraining the low energy SUSY spectrum. It has to be noticed however that for large $\tan \beta$ values and $B = 2$ the allowed parameter space for the model becomes very small and the model becomes correspondingly quite predictive.

At any rate, a positive evidence for the inclusive decay could be crucial in excluding large portions of the available parameter space for the model. This can be already seen by overlapping both the upper and lower bounds of eq. (2) to Figs. 7–10. In a more suggestive way the outcomes of imposing the full constraint of eq. (2) are shown in Figs. 11 and 12. In Fig. 11 the shaded areas show the implementation of the CLEO inclusive limits in the plane of the mass of the lightest Higgs boson $H_1^0$ and the mass of the CP odd scalar $H_3^0$. For a wide range of $\tan \beta$ the masses of $H_1^0$ and $H_3^0$ allowed by internal consistency of the model and the experimental $b \to s\gamma$ bounds are higher than present LEP limits ($m_{H_3^0} \gtrsim 50$ GeV, for a recent review see for instance ref. [25]). Notice that the effect of radiative corrections on the Higgs masses allows for the lightest Higgs boson to be heavier than the $Z$ mass. In the present model the allowed range for $m_{H_3^0}$ is bounded from above by about 120 GeV, for both choices of $B$. In Fig. 12 the corresponding regions in the gluino mass – $\mu_R$ plane are shown.
In conclusion, we have presented an updated study of the implications of the recent CLEO results on $BR(b \rightarrow s\gamma)$ for the minimal supersymmetric extension of the SM with radiative breaking of the electroweak group. We have fully included in the analysis of the model the leading squark (and slepton) contributions to the one-loop effective potential in order to make the results stable against variations of the low-energy renormalization scale. We have imposed two loop-gauge coupling unification and approximate tau-bottom Yukawa unification (within 30%). Input pole masses ($m_\tau, m_\tilde{b}, m_\tilde{t}$) have been related to $\overline{\text{MS}}$ running masses and approximate Yukawa matrices have been constructed at the $m_Z$ scale using present central values for the Kobayashi-Maskawa matrix entries [26], following the approach described in ref. [17].

Phenomenologically interesting regions for the SUSY soft breaking parameters have been analyzed. In particular we have discussed the impact on the model of the constraint $|B| \geq 2$. We have discussed in detail the chargino contribution to the amplitude, its specific $\tan \beta$ dependence, which is responsible for the large interference effects in the large $\tan \beta$ region, and the further dependence of this interference effect on the relevant SUSY parameters. We have found that the combination of the upper bound on the inclusive decay and the measurement of the lightest exclusive channel (which leads to bounding from below the inclusive transition) already constrains in a sizeable way the low energy structure of the model. As a consequence, a positive experimental evidence of the inclusive $b \rightarrow s\gamma$ decay, hopefully together with the top discovery, remains certainly among the most relevant (indirect) tests for this minimal supersymmetric scenario.
Figure Captions

Figure 1. Ratios of SUSY induced over SM amplitudes, for $m_t = 160 \text{ GeV}$, $\tan \beta = 8$ and $B = A - 1$. The charged Higgs $(a)$, chargino $(b)$, gluino $(c)$ and neutralino $(d)$ components of the total amplitude are plotted versus the masses of the charged Higgs, lightest chargino, gluino and lightest neutralino respectively.

Figure 2. Same as in Fig. 1 for $B = 2$.

Figure 3. Ratios of SUSY induced over SM amplitudes, for $m_t = 160 \text{ GeV}$, $\tan \beta = 2$. Both choices $B = A - 1 \ (a, b, c)$ and $B = 2 \ (d, e, f)$ are shown. The charged Higgs $(a, d)$ component of the total amplitude is plotted versus the mass of the charged Higgs boson which runs in the loop together with up-type quarks, whereas the chargino induced amplitude is plotted both versus the mass of the lightest chargino $(b, e)$ and the lightest top squark $(c, f)$.

Figure 4. Same as in Fig. 3 for $\tan \beta = 20$.

Figure 5. Same as in Fig. 3 for $\tan \beta = 40$.

Figure 6. The leading component of the chargino amplitude in the limit of large $\tan \beta$ is shown in the higgsino-squark interaction basis. The photon is attached in all possible ways. The crosses indicate the presence of higgsino and stop mass insertions.

Figure 7. The total inclusive branching ratio in the MSSM is shown for $m_t = 160 \text{ GeV}$, $\tan \beta = 2$, $B = A - 1 \ (a, b, c)$ and $B = 2 \ (d, e, f)$, as a function of the masses of the charged Higgs boson $(a, d)$, the lightest chargino $(b, e)$, and the lightest top squark $(c, f)$. The SM prediction for $m_t = 160 \text{ GeV}$ and $\alpha_s(m_Z)\overline{\mathrm{MS}} = 0.12$ is also shown (horizontal solid line) for comparison.

Figure 8. Same as in Fig. 7 for $\tan \beta = 8$.

Figure 9. Same as in Fig. 7 for $\tan \beta = 20$.

Figure 10. Same as in Fig. 7 for $\tan \beta = 40$.

Figure 11. The scattered dot areas represent the allowed MSSM regions in the plane of the lightest Higgs boson $H_1^0$ and the $CP$ odd scalar $H_3^0$ masses after inclusion of
the $b \to s\gamma$ bounds in eq. (2). Different values of $\tan \beta$ are shown.

**Figure 12.** Same as in Fig. 11, in the gluino mass $- \mu_R$ plane (the latter is the $\tilde{h}_1 - \tilde{h}_2$ supersymmetric mixing parameter renormalized at the weak scale).
References


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