Abstract

We present a model which supplements the Standard Electroweak Model with three right-handed neutrinos and one extra scalar doublet which does not develop a vacuum expectation value. With the aid of a discrete symmetry the neutrinos are kept strictly massless. This model has several interesting features. It has unsuppressed lepton flavour violating processes, in particular $\nu \rightarrow e\gamma$, hinting at the possibility that these may soon be within experimental reach. The $Z$ and $W$ interactions become non-diagonal at one loop level. In particular, a non-trivial leptonic mixing matrix is seen to arise from the clash between the charged gauge boson and the charged scalar interactions.

1 Introduction

One of the interesting features of the leptonic sector is the fact that neutrino masses are much smaller than the other fermion masses. To date, no experiment has unequivocally detected nonzero neutrino masses. In the Standard Model (SM) [1] one precludes the existence of Dirac masses by not having right-handed neutrino fields in the theory. This is a somewhat peculiar feature of the theory, since all other fermions appear as both right-handed (RH) and left-handed (LH) fields. Therefore we would like to look for a minimal extension of the SM which does not have this shortcoming. However, we must then face the problem of justifying the smallness (or inexistence) of the neutrino Dirac mass terms.
This has been done in several extensions of the SM. In some grand unified theories such as SO(10), a Majorana lepton number violating mass term for the right handed neutrinos yields neutrinos with small masses through the see-saw mechanism [2]. In several other models neutrinos are massless and yet lepton flavour is violated. In some of these [3] [4] this is due to the lack of unitarity of the charged current mixing matrix which translates into non-universal couplings for the neutral current interaction. In others [5], R-parity violating terms in the superpotential generate flavour violating neutral current interactions with the electron and the u and d quarks. Most of these models involve rather speculative assumptions about the field content of the theory.

On the other hand, in the SM there is no justification for the presence of only one Higgs doublet. In fact considerable interest has arisen in multi-Higgs doublet models, ever since T. D. Lee showed [6] that one could have spontaneous CP violation in the two-Higgs doublet model. To avoid flavour changing neutral exchanges, S. L. Glashow and S. Weinberg , and independently E. Paschos, proposed the introduction of a discrete symmetry forcing each fermion to couple to only one Higgs doublet: this is known as natural flavour conservation [7]. In what follows we will use a similar device to show how one can naturally suppress neutrino masses in a two Higgs doublet model with three RH neutrino fields.

In section 2 we present our model. In section 3 we study the experimental constraints arising from the processes \( l_2 \to l_1 \nu \bar{\nu} \), \( l_2 \to l_1 \gamma \) and \( l_3 \to l_2 l_1 \bar{\nu} \). We also discuss briefly the decays \( Z \to l_1 \bar{\nu} \) and \( W \to l_1 \bar{\nu} \). In the last section we draw our conclusions. The appendix contains several derivations needed to establish the experimental bounds on the theory.

2 The Model

Our model has, in addition to the SM fields, three right handed (RH) neutrino fields \(^1\), \( N_R \), and one extra Higgs doublet, \( H_2 \). The Higgs potential is chosen so that \( H_2 \) does not get a vacuum expectation value. In addition, a discrete symmetry is introduced under which

\[ N_R \to -N_R, \quad H_2 \to -H_2, \]

and all other fields remain invariant. The Yukawa and weak lagrangeans are,

\[ -\mathcal{L}_Y = \bar{L}_L H_{1,2} C_R + \bar{L}_L (i\sigma_2 H_2^*) + 2 N_R + h.c., \]

\[ \mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{N}_R \gamma^\mu C_L W^\mu + h.c., \]

where \( \bar{L}_L \) is the left handed (LH) lepton doublet, \( N_R (C_R) \) is the RH neutrino (charged lepton) singlet and \( 1, 2 \) are \( 3 \times 3 \) Yukawa coupling matrices. For later use the Higgs fields will be written as

\[ H_1 = \left( \frac{1}{\sqrt{2}}(v + H^0 + iG^0) \right), \quad H_2 = \left( \frac{H^+}{\sqrt{2}}(R + iI) \right), \]

\(^1\)We could equally well construct a theory with any other number of right handed neutrinos; for example, one or two. We chose three since we seek a `symmetric' theory with as many right handed singlet fermions as there are left handed fermions.
We can diagonalize the charged lepton mass matrix while keeping the charged current diagonal with transformations,

\[ \mathbf{L}_L \equiv (\mathbf{N}_L, \mathbf{C}_L) = (\nu_l, \bar{\nu}_l)U_L^\dagger , \]

\[ C_R = U_{CR}l_R , \]  

where

\[ D_l \equiv \text{diag}(m_e, m_\mu, m_\tau) = \frac{v}{\sqrt{2}}U_L^\dagger, \]

We can still use the fact that \( \nu \) can be diagonalized by a bi-unitary transformation,

\[ \nu \xrightarrow{X} \text{diag} \nu_R \]

and the freedom to redefine the RH neutrino fields by,

\[ N_R = U_{N_R}^\dagger \nu_R , \]

to write,

\[ -\mathcal{L}_Y = \ldots + (1 + H^0/v)\bar{\nu}_l D_l l_R - H^-\bar{\nu}_l M^{\dagger \nu}_R - H^+\bar{\nu}_R M_l l_l + R/\sqrt{2}[\bar{\nu}_R M\nu_l + \bar{\nu}_l M^\dagger \nu_R ] + iI/\sqrt{2}[\bar{\nu}_R M\nu_l - \bar{\nu}_l M^\dagger \nu_R ] , \]

\[ \mathcal{L}_W = \frac{g}{\sqrt{2}}\bar{\mathbf{N}}_L_\gamma^\nu C L W_L^\nu + h.c. \]

Note that all fields in \( H_2 \) involve the same coupling \( M^l = B^l N_\nu; \) the charged scalars link \( \nu_R \) and \( l_L; \) the neutral scalars link \( \nu_R \) and \( \nu_L. \)

A simple example of lepton flavour violations at low energies occurs in the decay of a LH lepton of flavour \( i \) into a LH lepton of flavour \( j \) and some gauge boson. This arises through a one loop diagram with intermediate \( \nu_R \) and \( H_2 \) fields (Cf., for example, fig. 1), and will be proportional to,

\[ \Omega_{ij} = \sum_{k=1}^{3} M_{ki} M_{jk}^\dagger = \sum_{k=1}^{3} |n_k|^2 B_{ki} B_{kj}^* , \]

showing a suppression in the limit in which the \( n_K \) values are close to each other.

In our model, overall lepton number conservation is imposed forbidding Majorana mass terms and we shall assume that there is no CP violation in the lepton sector (that is, the matrix \( N_\nu \) is real and the matrix \( B \) is orthogonal; but we will keep our formulas general). Moreover, we shall take the new scalar masses to be within an order of magnitude, or so, of the electroweak scale, \( v = 246 GeV. \)

### 3 Experimental Bounds

The first important limit on our theory comes from measurement of \( G_F = 1/\sqrt{2}v \) in the muon decay as compared to that made in the quark sector. For untagged neutrino flavours, we have both charged W-exchange and charged H-exchange tree level diagrams. Due to their different chiral structures, these do not interfere and one can easily find,

\[ (\mu \rightarrow e\nu\bar{\nu}) = \frac{G_F^2 m_\mu^5}{192\pi^2} (1 + \frac{v^4}{16M_H^4} \Omega_{\mu e}^2 \Omega_{ee}^2) , \]

3
where $M_H$ is the mass of the charged scalars $H$.

In our model, the quark mixing matrix is unitary and, therefore, the sum of the magnitudes squared of the elements in its first row must add to one. Any deviation will be a measure of how much the expression within parenthesis in Eq. 12 deviates from one. Using the result \[8\],
\[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9977 \pm 0.0030 \tag{13}\]
we find \footnote{The radiative corrections are dominated by QED effects and are therefore down by $\alpha/4\pi$; almost an order of magnitude below our bound on new physics. However, to improve these constraints, such as by measuring the neutrino spectra in muon decay, these QED radiative corrections must be taken into account. An excellent discussion of this can be found in reference \[9\].},
\[v^4 \frac{\Omega_{\mu\mu}\Omega_{ee}}{M_H^2} < 0.0848 \tag{14}\]
Noting from Eq. 11 that $\Omega_{\mu} \geq 0$, we find that, at tree level, the present sign of the deviation from the mean is consistent with this model. The effect of the one loop amplitudes will be briefly discussed at the end of this section. In any event, it is a good approximation to take the value of $G_F$ as measured from muon decay, in the following analysis.

The situation concerning the tau branching ratios, is still unclear \[10\]. This is particularly so for the decay into electron and two neutrinos where the relation
\[\tau_\tau = \tau_\mu \left( \frac{G_\mu}{G_\tau} \right)^2 \left( \frac{m_\mu}{m_\tau} \right)^5 B(\tau \rightarrow e\nu\bar{\nu}) \tag{15}\]
has only recently become consistent with the Standard Model ($G_\mu = G_\tau$) \[11\]. It is widely believed that this problem will continue to disappear with further experiments. Therefore, we shall not derive any constraints from this decay.

As can be seen from Eq. 11, we will have lepton flavour violation in processes such as $l_2 \rightarrow l_1 \gamma$ or $l_3 \rightarrow l_1 l_2$. The first of these processes has one-loop contributions from the diagrams in fig. 1, involving an intermediate charged scalar and RH neutrinos. Clearly no such diagram exists with an intermediate gauge boson.

The most general form of the invariant amplitude for the process in fig.1 is,
\[\mathcal{M} = e_\mu T^\mu_\gamma \nonumber\]
\[= e_\mu \bar{u}_1(p_1) [i m_2 \gamma^\nu k^\nu (A_{\gamma} + B_{\gamma} \gamma_5) \nonumber\]
\[+ k^2 \gamma^\mu (C_{\gamma} - D_{\gamma} \gamma_5) - k^\mu (E_\gamma + F_{\gamma} \gamma_5) ] u_2(p) \tag{16}\]
where $k = p - p_1$, gauge invariance guarantees that,
\[C_{\gamma} = \frac{E_\gamma}{m_2 - m_1} \nonumber, \quad D_{\gamma} = \frac{F_\gamma}{m_2 + m_1} \tag{17}\]
and the factor of $k^2$ in the $\gamma^\mu$ terms reflects the fact that this contribution vanishes in the $k^2 = 0$ limit.

\[\text{\frac{\varepsilon}{\text{Vol}}}=\frac{1}{2}\text{\sum}\text{\frac{\varepsilon}{\text{Vol}}=\frac{1}{2}\sum}\]
The diagrams in figs. 1b and 1c only contribute to the $\gamma^\mu$ terms. So, it is easier to use the diagram in fig. 1a to find $A_\gamma$, $B_\gamma$, $E_\gamma$ and $F_\gamma$ and then use Eq. 17 to get $C_\gamma$ and $D_\gamma$. We relegate the full expressions to the appendix. To lowest order in $k^2$, and using $M_H^2 >> m_2^2 >> m_1^2$, we find

$$A_\gamma \approx B_\gamma \approx \frac{e\Omega_2}{32\pi^2} \frac{1}{12M_H^2}, \quad (18)$$

$$C_\gamma \approx D_\gamma \approx \frac{e\Omega_2}{32\pi^2} \frac{1}{18M_H^2}. \quad (19)$$

For the physical process $l_2 \rightarrow l_1\gamma$, the photon is on mass shell and only the $\sigma^{\mu\nu}$ terms contribute,

$$, (l_2 \rightarrow l_1\gamma) = \frac{m_\mu^5}{8\pi}(|A_\gamma|^2 + |B_\gamma|^2). \quad (20)$$

In particular, for the muon decay we can use,

$$, (\mu \rightarrow e\bar{\nu}_e\nu_e) \approx \frac{G_F^2 m_\mu^5}{192\pi^3} \approx , (\mu \rightarrow all). \quad (21)$$

As pointed out above, the first sign is not a strict equality since, in this model, there is also an intermediate charged scalar diagram. Thus,

$$B(\mu \rightarrow e\gamma) = \frac{\alpha}{32\pi^2} \frac{1}{12} \sum_{k=1}^{3} \left( \frac{\nu n_k}{M_H} \right)^2 B_{k\mu} B_{k\gamma}^* \quad (22)$$

and the model is already constrained by experiment. This should be compared with what would happen if only RH neutrinos and Dirac masses $m_k$ were added to the SM. In that case, to lowest order in an expansion in powers of $m_k/M_W$ one obtains [12],

$$B(\mu \rightarrow e\gamma) = \frac{\alpha}{32\pi^2} \left( \frac{m_k}{M_W} \right)^2 \sum_{k=1}^{3} \left( \frac{U_{k\mu}}{M_W} \right)^2 \quad (23)$$

where $U$ is the lepton mixing matrix [13] and there is no term of order zero due to the GIM mechanism [14]. Therefore, even if the neutrino masses saturate the cosmological bound, we get a branching ratio of the order of $10^{-40}$! Furthermore, even if one includes both Dirac and Majorana masses, only considerable fine tuning would lead to experimentally relevant values for this branching ratio [15]. By contrast, our model is already constrained by experiment.

From the experimental result [8] of $B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$ we find,

$$\frac{v^2}{M_H^2} |\Omega_{\mu e}| < 2.8 \times 10^{-3}. \quad (21)$$

It seems natural to suspect that the $n_k$ aren’t much smaller than one, $M_H$ is within an order of magnitude of $v$ and that the $B$ matrix might have some off-diagonal elements of order 0.1, say. This would raise the exciting prospect of a positive result with further
experiments. For the tau decays, \(B(\tau \rightarrow \mu \gamma) < 5.5 \times 10^{-4}\) and \(B(\tau \rightarrow e \gamma) < 2.0 \times 10^{-4}\) lead to the considerably weaker bounds,
\[
\frac{v^2}{M_H^2} |\Omega_{\tau \mu}| < 22 , \quad \frac{v^2}{M_H^2} |\Omega_{\tau \ell}| < 13 . \tag{25}
\]

Similarly, we will have flavour violation in processes such as \(l_2 \rightarrow 3l_1\) due to the diagrams in fig. 2. Clearly, the \(Z\) diagram is very suppressed due to the \(Z\) propagator and we can ignore it. The general expressions have been derived in the appendix. For our model,
\[
, \quad (l_2 \rightarrow 3l_1) = \frac{G_F^2 m_2^5}{192 \pi^3} \frac{1}{(32 \pi^2)^2} \frac{v^4}{M_H^4} |\Omega_{21}|^2 \left[ \frac{8 \pi^2 \alpha^2}{9} \left(2 \ln \left(\frac{m_2}{2m_1}\right) - \frac{23}{12}\right) + \frac{1}{8} |\Omega_{11}|^2 + \frac{4 \pi \alpha}{9} Re \{\Omega_{11}\} \right] , \tag{26}
\]
and, using the experimental constraints we find,
\[
\frac{v^2}{M_H^2} |\Omega_{\mu e}| \sqrt{1 + 36.4 |\Omega_{ee}|^2 + 2.96 Re \{\Omega_{ee}\}} \leq 5.4 \times 10^{-3} , \\
\frac{v^2}{M_H^2} |\Omega_{\tau \mu}| \sqrt{1 + 114 |\Omega_{\mu \mu}|^2 + 9.28 Re \{\Omega_{\mu \mu}\}} \leq 90 , \\
\frac{v^2}{M_H^2} |\Omega_{\tau e}| \sqrt{1 + 20.6 |\Omega_{ee}|^2 + 1.68 Re \{\Omega_{ee}\}} \leq 48 . \tag{27}
\]

These are limits on different quantities than the ones showing up above, but point to roughly the same order of magnitude.

In this model, one also has lepton flavour violating \(Z\) decays but these have far worse experimental constraints than the flavour violating lepton decays. In addition, the theory already predicts that, for example, \(B(Z \rightarrow \mu^+ e^-)\) should be much smaller than \(B(\mu \rightarrow e \gamma)\). This arises from the fact that the dominating \(W\) exchange decay mode of the muon has a three body final state phase space suppression, together with the fact that the \(Z\) has many decay modes.

Using the results derived in the appendix we find that since \(M_Z^2 \gg m_2^2 \gg m_1^2\), the dominant contribution comes from the \(\gamma \mu\) terms yielding,
\[
, \quad (Z \rightarrow l_1 \tilde{l}_2) = \frac{M_Z^2}{12\pi} |C_Z(k^2 = M_Z^2)|^2 + |D_Z(k^2 = M_Z^2)|^2 , \tag{28}
\]
where, for \(M_Z^2 \gg M_Z^2\),
\[
D_Z(k^2 = M_Z^2) \approx C_Z(k^2 = M_Z^2) \approx \frac{e \Omega_{21}}{32 \pi^2} \cot(2\theta_W) \frac{M_Z^2}{18M_H^2} . \tag{29}
\]
From this one can easily derive a relation between the two branching ratios, namely,
\[
B(Z \rightarrow e^- \mu^+) \approx 3.8 \times 10^{-5} B(\mu^- \rightarrow e^- \gamma) , \tag{30}
\]
showing that even for the tau (for which the coefficient is around 5.6 times larger) this rate is unmeasurably small.

A similar analysis may be performed for the charged gauge interactions. The results can again be found in the appendix. The most important feature is the appearance at one loop level of off-diagonal vertices with \( W \), leptons and neutrinos. In particular, the left-handed vector interaction acquires a non-diagonal leptonic mixing matrix,

\[
V_{12} = \frac{\Omega_{21}}{32\pi^2} f[M_R, M_I, M_H],
\]

where the function

\[
f[M_R, M_I, M_H] = \left[ \frac{1}{2} \frac{M_R^2 + M_I^2}{M_M^2 - M_R^2} \ln \left( \frac{M_R}{M_H} \right) \right] + [R \rightarrow I] + 1
\]

vanishes for \( M_R = M_I = M_H \), is negative elsewhere and remains of order \((-1)\) for any reasonable ratios of scalar masses. Note that this mixing matrix is not unitary and therefore cannot be absorbed by a redefinition of the left-handed neutrinos. Thus we find the interesting feature of a non trivial leptonic mixing matrix even with massless neutrinos [3const] with its possible implications for neutrino propagation in matter [4const].

To be exhaustive we should now go back and reassess our calculation for the muon decay. Indeed, the one loop corrections to the \( W \)-exchange amplitude might be comparable to the \( H \)-exchange amplitude if

\[
\frac{\Omega_{ee}}{32\pi^2} f[M_R, M_I, M_H] \approx \frac{v^2}{4M_R^2} \sqrt{\Omega_{\mu\mu} \Omega_{ee}}
\]

This will depend on the masses of the new scalars and on the \( \Omega_{ij} \). It is also clear that, for example, the analysis of Beta decay must take this into account modifying the extraction of the CKM matrix elements.

4 Conclusions

We have developed a model with massless neutrinos inspired by a minimal ‘democracy’ assumption: there should exist a right-handed singlet partner for every left-handed particle; the fundamental scalars might also exist in several families. This was achieved at the expense of a ‘discriminatory’ discrete symmetry.

This model has several interesting characteristics. Contrary to what happens if one adds massive Dirac neutrinos to the SM, in which case the cosmological limit imposes minute lepton flavour violations, in this model, such lepton flavour violating processes might be within experimental reach, especially \( \mu \rightarrow e\gamma \). Although one also gets non-diagonal \( Z \) decays we found that these have levels beyond experimental verification. Finally, one also finds non-diagonal \( W \) interactions, and this despite the fact that the neutrinos are massless. A novel feature is that this comes about as the result of a clash between the \( W \) and the \( H \) interactions with leptons.

This new sector of the theory will also have implications for Cosmology. In particular, the new interactions must be weak enough to decouple the new particles early enough not to affect significantly primordial nucleosynthesis. This work is currently under way.
A Derivation of the invariant amplitudes

In this appendix we derive the expressions for the $l_2 \rightarrow l_1 \gamma$, $l_2 \rightarrow l_1 Z$ and $l_3 \rightarrow l_1 \bar{l}_1 l_2$ decays, in our model. The invariant amplitude for $l_2 \rightarrow l_1 \gamma$, $\mathcal{M} = c_\mu T_\gamma^\mu$ with

$$T_\gamma^\mu = \bar{u}_1(p_1)[i m_2 \sigma^{\mu\nu} k_\nu (A_\gamma + B_\gamma \gamma_5) + k^2 \gamma^\mu (C_\gamma - D_\gamma \gamma_5) - k^\mu (E_\gamma + F_\gamma \gamma_5)]u_2(p) , \quad (34)$$

may be found by computing solely the Feynman diagram of fig. 1a. This is due to the fact that figs. 1b and 1c only contribute to the vector and axial vector parameters which are easier to find with the help of the Ward identity. We find

$$A_\gamma(k^2) = \frac{e\Omega_{21}}{32\pi^2 m_2} \int_0^1 dx_1 \int_0^{x_1} dx_2 \frac{m_2(x_1 - x_2) + m_1(1 - x_1)}{\Delta(x_1, x_2)}(x_2) , \quad (35)$$

$$E_\gamma(k^2) = \frac{e\Omega_{21}}{32\pi^2} \int_0^1 dx_1 \int_0^{x_1} dx_2 \frac{m_2(x_1 - x_2) + m_1(1 - x_1)}{\Delta(x_1, x_2)}(2x_1 - 1 - x_2) , \quad (36)$$

where,

$$\Delta(x_1, x_2) = M_H^2(1 - x_2) - m_2^2(x_1 - x_2)x_2 - m_1^2(1 - x_1)x_2 - k^2(x_1 - x_2)(1 - x_1) . \quad (37)$$

The parameter $B_\gamma$ ($F_\gamma$) has the same expression as $A_\gamma$ ($E_\gamma$), except for a minus sign for the $m_1$ term in Eq. 35 (Eq. 36). Taking $M_H^2 >> m_2^2 >> m_1^2$, we find the result in Eq. 18, to lowest order in $k^2$. The parameters $E_\gamma$ and $F_\gamma$ do not enter in either process but, as mentioned above, we can use them to find $C_\gamma$ and $D_\gamma$ through Eq. 17. The result, with the same approximation, is in Eq. 19.

The invariant amplitude for $l_2 \rightarrow l_1 Z$ has the same structure,

$$T_Z^\mu = \bar{u}_1(p_1)[i m_2 \sigma^{\mu\nu} k_\nu (A_Z + B_Z \gamma_5) + \gamma^\mu (C_Z - D_Z \gamma_5) - k^\mu (E_Z + F_Z \gamma_5)]u_2(p) , \quad (38)$$

except that, due to the mass of the Z, the $\gamma^\mu$ terms do not vanish in the $k^2 = 0$ limit. The calculation is just a repetition of that for the photon. The difference in the diagram of fig. 1a (and therefore in the $\sigma^{\mu\nu}$ and $k^\mu$ terms) is just due to the $\xi_Z = \cot(2\theta_W)$ ratio between the $HHZ$ and $HH\gamma$ vertices. This is also the relevant ratio for the $\gamma^\mu \gamma_L$ term of figs. 1b and 1c while the $\gamma^\mu \gamma_R$ term involves $-\tan \theta_W$ (and will be proportional to $m_1 m_2$ since one chirality flip is needed in each external leg to reproduce this structure). It is then easy to see that $\alpha_Z = \xi_Z \alpha_\gamma$ for $\alpha = A, B, E, F$ while,

$$C_Z(k^2) = \xi_Z[\eta_Z C_0 + k^2 C_\gamma(k^2)] ,$$

$$D_Z(k^2) = \xi_Z[-\eta_Z C_0 + k^2 D_\gamma(k^2)] , \quad (39)$$

where,

$$C_0 = \frac{e\Omega_{21} m_1 m_2}{32\pi^2 m_2^2 - m_1^2} \int_0^1 t dt \ln \frac{M_H^2 + m_1^2(t - 1)}{M_H^2 + m_2^2(t - 1)} , \quad (40)$$

and,

$$\eta_Z = \tan(\theta_W) \tan(2\theta_W) + 1 . \quad (41)$$

In the same limit, $M_H^2 >> m_2^2 >> m_1^2$, we get

$$C_0 \approx \frac{e\Omega_{21} m_1 m_2}{32\pi^2} \frac{6 M_H^2}{M_H^2} . \quad (42)$$
A straightforward calculation leads to
\[ (Z \rightarrow l_1 \overline{l}_2) = \frac{M_Z}{12\pi} |(|C_Z|^2 + |D_Z|^2) - 3m_Z^2\Re\left\{ A_Z C_Z^* + B_Z D_Z^* \right\} + m_Z^2 M_Z^2 / 2(|A_Z|^2 + |B_Z|^2) |, \]
with the parameters evaluated at \( k^2 = M_Z^2 \). For \( M_{H_d}^2 \gg M_Z^2 \) we can use the approximate expressions derived for the photon parameters in lowest order of \( k^2 / M_{H_d}^2 \) to get,
\[ C_Z(k^2 = M_Z^2) = \frac{\epsilon \Omega_{21}}{32\pi^2} \eta_Z \{ \frac{m_1 m_2}{6} M_{H_d}^2 + \frac{1}{18} M_{H_d}^2 + O(M_Z / M_{H_d})^4 \} \approx \frac{\epsilon \Omega_{21}}{32\pi^2} \frac{M_Z^2}{18 M_{H_d}^2}, \]
and similarly for \( D_Z \).

A similar calculation shows that the W interactions also become off diagonal at one loop level. For the Z and \( \gamma \) interactions, the off-diagonal couplings are excluded by symmetry at tree level. Therefore, they are protected against one loop divergences. For the W interactions the situation is different since the lack of such off-diagonal couplings corresponds to a basis choice and is not dictated by any symmetry. In this case, the infinities are cancelled by the contributions from the counterterms, which we calculated in the on-mass-shell renormalization scheme [16]. For the process \( l_2 \rightarrow W^- \nu_1 \), we find,
\[ T_W^\mu = \bar{u}_{\nu_1}(p_1) [im_2 \sigma^{\mu\nu} k_\nu A_W (1 + \gamma_5) + \gamma^\mu C_W (1 - \gamma_5) - k^\mu E_W (1 + \gamma_5)] u_2(p) , \]
where, in the limit that the square of the new scalar masses (\( M_{H_d}^2 \) for the charged scalars, \( M_{H_d}^2 \) for the neutral scalar and \( M_{H_d}^2 \) for the pseudoscalar) are much larger than \( M_{H_d}^2 \), and to first order in \( k^2 \), we get,
\[ A_W = \frac{g}{\sqrt{2}} \frac{\Omega_{21}}{32\pi^2} \left\{ \frac{1}{6(M_{H_d}^2 - M_{H_d}^2)} (M_{H_d}^2 - M_{H_d}^2 + M_{H_d}^2 \ln (M_{H_d}^2 / M_{H_d}^2)) + [R \rightarrow I] \right\} , \]
\[ C_W = \frac{g}{\sqrt{2}} \frac{\Omega_{21}}{32\pi^2} \left\{ \frac{1}{2} \frac{M_{H_d}^2 + M_{H_d}^2}{M_{H_d}^2 - M_{H_d}^2} \ln (M_{H_d}^2 / M_{H_d}^2) + [R \rightarrow I] + 1 \right\} , \]
\[ E_W = \frac{g}{\sqrt{2}} \frac{\Omega_{21}}{32\pi^2} m_2 \left\{ \frac{M_{H_d}^2}{3(M_{H_d}^2 - M_{H_d}^2)} \right\} \left\{ 2(M_{H_d}^2 - M_{H_d}^2) + (M_{H_d}^2 + M_{H_d}^2) \ln (M_{H_d}^2 / M_{H_d}^2) \right\} \right\} + [R \rightarrow I] \right\} . \]
As an easy check, we note that, except for a factor of \( g / \sqrt{2} \) instead of \( \epsilon \), we recover the results for \( l_2 \rightarrow l_1 \gamma \) when \( M_{H_d}^2 = M_{H_d}^2 = M_{H_d}^2 \). In particular, in that limit, we obtain \( C_W = 0 \) reflecting the fact that the vector coefficients for \( l_2 \rightarrow l_1 \gamma \) vanish at zero momentum transfer.

For the process \( l_2 \rightarrow l_1 \overline{l}_1 \), we get contributions from fig. 2. Note that there is no WW-box contribution since, at tree level, there is no mixing matrix in the lepton sector. Similarly, chirality considerations exclude the WH-box diagram.

The amplitude for this process can be written as,
\[ \mathcal{M}(l_2 \rightarrow l_1 \overline{l}_1) = \mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1) \]
\[ (49) \]
with,
\[ \mathcal{M}(p_1, p_2) = \mathcal{M}^\gamma(p_1, p_2) + \mathcal{M}^H(p_1, p_2) + \mathcal{M}^Z(p_1, p_2) \]  
(50)

where \( \mathcal{M}^\gamma \) is the one photon exchange amplitude and \( \mathcal{M}^H \) is the HH-box contribution. \( \mathcal{M}^Z \) is the Z-exchange amplitude which is very suppressed by the Z propagator since the momentum transfer is bound by \( m_2^2 \). Therefore we will ignore this contribution. Using the expression found above, it is easy to get,
\[ \mathcal{M}^\gamma(p_1, p_2) = \frac{[T_\mu][\bar{u}_1(p_2)c\gamma_\mu v_1(p_3)]/k_1^2}{}, \]
(51)

with \( k_1 = p - p_1 \). Similarly,
\[ \mathcal{M}^H(p_1, p_2) = S[\bar{u}_1(p_1)\gamma^\mu(1 - \gamma_5)u_2(p)][\bar{u}_1(p_2)\gamma_\mu(1 - \gamma_5)v_1(p_3)] \],
(52)

where, in our model,
\[ S = -\frac{\Omega_{21}\Omega_{11}}{256\pi^2M_H^2}. \]
(53)

In the limit \( m_2 >> m_1 \), this leads to a decay rate,
\[ (l_2 \to 3l_1) = \frac{m_2^5}{768\pi^2}(, \gamma + , H + , \gamma^H), \]
(54)

where
\[ , \gamma = 4\left[4\ln \left(\frac{m_2}{2m_1}\right) - \frac{13}{6}\right]|A|^2 + |B|^2 - 12Re\{AC^* + BD^*\} + 3(|C|^2 + |D|^2), \]
\[ , H = 16|S|^2, \]
\[ , \gamma^H = 8Re\{(C + D - 2A - 2B)S^*\} \],
(55)

and the subscript \( \gamma \) is implied.

A process such as \( l_3 \to l_1\bar{l}_1l_2 \) has a similar expression in the limit \( m_3 >> m_2, m_1 \), the only difference showing up in the \( \Omega \) factors and in the logarithm. The processes with no identical particles in the final state are, of course, easier to calculate. They lead to limits on different combinations of \( \Omega \) factors but give no qualitatively new information.

We thank R. Holman, L. Lavoura, L.-F. Li and H. Vogel for useful discussions. We are also indebted to M. Savage and L. Wolfenstein for useful suggestions and for reading and criticizing the manuscript. The work of J. P. S. was partially supported by the Portuguese JNICT, under CIÊNCIA grant # BD/374/90-RM, and partially supported by the United States Department of Energy, under the contract # DE-FG02-91ER-40682. J. P. S. is also indebted to the Santa Barbara Institute for Theoretical Physics where portions of this work were done.

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FIGURE CAPTIONS

Figure 1: One-loop contributions to the lepton flavour violating $l_2 \rightarrow l_1 \gamma$ vertex. The diagrams in figures 1b and 1c only contribute to the vector and axial-vector parameters.

Figure 2: One-loop contributions to the process $l_2 \rightarrow 3l_1$. In our model, there are contributions from a penguin diagram (fig. 2a) and from a HH-box diagram (fig. 2b).