Unstable particle mixing and CP violation in heavy particle decays

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We discuss a phenomenological description for unstable particle mixing in CP-violating weak decays. Applications are given to heavy particle decays for the generation of baryon number asymmetry.

1. INTRODUCTION

There are two known methods from which the violation of CP symmetry may be detected. One is to look for interactions forbidden by CP. The other is to compare processes to their mirror processes obtained from CP conjugation. In spite of great efforts, thirty years after the discovery of CP violation we still only have one sighting of CP violation, with all observables generated from the mixing of unstable particles K^0 and $\bar{K}^0$ [1].

In order to have a process differ from its CP mirror, it is often necessary to have a CP-conserving phase in addition to CP-violating phases. Consider the charge asymmetry of K_L decay as an example

$$\delta_L = \frac{\Gamma(K_L \rightarrow \pi^+ \ell \bar{\nu}) - \Gamma(K_L \rightarrow \pi^- \ell \nu)}{\Gamma(K_L \rightarrow \pi^+ \ell \bar{\nu}) + \Gamma(K_L \rightarrow \pi^- \ell \nu)}. \quad (1)$$

δ_L is given by

$$\delta_L = 2\text{Re}\tilde{\epsilon} \approx 4 \frac{m'}{\Delta\Gamma}, \quad (2)$$

where $\tilde{\epsilon}$ is the complex mixing angle of K_1 and K_2 , m' is the CP-violating mixing in the $K_1 - K_2$ mass matrix and $\Delta\Gamma$ is the neutral kaon width difference. Although the choice of $\tilde{\epsilon}$ involves a phase convention, the real part of $\tilde{\epsilon}$ is phase convention independent. In this example, the CP-violating phase is given by m' , and the CP-conserving phase is given by the width difference of the unstable particles, i.e., $\Delta\Gamma$.

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2. AN ILLUSTRATIVE EXAMPLE

Besides being important for the known example in kaon decay, unstable particle mixing may also play an important role in some other processes. Consider the generation of baryon number from heavy particle out-of-equilibrium decay [2]. Suppose that the interaction Lagrangian of a given system is

$$\mathcal{L}_1 = g_a \bar{u}_{R,\alpha} e_R^c S_{a,\alpha} + f_a \bar{u}_{R,\alpha} d_{R,\beta} S_{a,\gamma} \epsilon^{\alpha\beta\gamma} + h.c., \quad (3)$$

where u, d and e are the first generation charged fermions, $S_{a,\alpha}$ are some heavy scalars carrying an $SU(3)$ color with color degrees of freedom specified by α, β and γ , a is a flavor index. This interaction can arise from an $SU(5)$ GUT model with more than one 5-plet of $SU(5)$ Higgs fields. Complex couplings f_a and g_a are the sources of CP violation.

This system has several, say two, heavy scalars $S_{1,2}$, each of which can decay to two different final states

$$|S_{1,2}\rangle \rightarrow |F_1\rangle = |\bar{d}_R u_R^c\rangle, \quad (4)$$

$$|S_{1,2}\rangle \rightarrow |F_2\rangle = |\bar{u}_R e_R^c\rangle. \quad (5)$$

Since the two final states have different baryon number: $B|F_1\rangle = -\frac{2}{3}|F_1\rangle$ while $B|F_2\rangle = \frac{1}{3}|F_2\rangle$, these decays violate baryon number. By exchanging $S_{1,2}$, $|F_{1,2}\rangle$ can scatter to themselves and to each other. Adjoining a t -channel $|F_{1,2}\rangle \rightarrow |F_{1,2}\rangle$ to the decay $|S_{1,2}\rangle \rightarrow |F_{1,2}\rangle$ corresponds to a calculation of the absorptive part of the vertex correction for the required CP-conserving phase [3]. The interference of the vertex correction to the

tree-level amplitude produces an asymmetry

$$\Delta = \Gamma(S_{1,2} \rightarrow \text{all}) - \Gamma(\bar{S}_{1,2} \rightarrow \text{all}). \quad (6)$$

For S_1 decay the result is

$$\Delta_{\text{vertex}} = - \frac{M_1 \text{Im}(g_1^* g_2 f_1 f_2^*)}{32\pi^2} \times \left[1 - \frac{M_2^2}{M_1^2} \ln \left(1 + \frac{M_1^2}{M_2^2} \right) \right], \quad (7)$$

where $M_{1,2}$ are the masses of the heavy scalars and $\text{Im}(g_1^* g_2 f_1 f_2^*)$ gives the required CP-violating phase. This contribution has been considered by every one. It has an interesting asymptotic behavior: in the limit in which the external scalar mass is much greater than the virtual one, i.e., $M_1 \gg M_2$, the size of the asymmetry is maximal. By contrast, in the opposite limit $M_1 \ll M_2$ the asymmetry goes to zero.

There is another contribution, often neglected in the literature [4]. It arises from final-states s -channel scattering. Adjoining the s -channel scattering to the tree-level amplitude corresponds to a calculation of the absorptive part of wave function correction. This correction introduces an additional mixing between S_1 and S_2 . The interference of the wave function correction to the tree-level amplitude provides an additional contribution. If $S_{1,2}$ are not degenerate, one finds [5] from a simple Feynman diagram calculation

$$\Delta_{\text{mixing}} = \frac{M_1 \text{Im}(g_1^* g_2 f_1 f_2^*)}{32\pi^2} \left(\frac{M_1^2}{M_1^2 - M_2^2} \right). \quad (8)$$

Compared to Δ_{vertex} , Δ_{mixing} has the opposite asymptotic behavior. Adding (8) to (7) one sees that no matter which one of the scalars is much heavier than the other, the final result is always zero in leading order of their mass ratio.

Although the complete cancellation of the leading terms in Δ_{vertex} and Δ_{mixing} is a special feature of the present theoretical model, in general contributions from mixing and vertex corrections are equally important.

The result given by (8) is invalid if the unstable particles $S_{1,2}$ are degenerate in mass. Based upon a naive extrapolation of (8) one might expect that contributions from mixing is enhanced as $M_1 - M_2 \rightarrow 0$. Such resonance enhancement in particle mixing is crucial for some mechanisms [6] of baryogenesis.

3. A REMARK ON B-VIOLATING FINAL STATE INTERACTION

In order to have a baryon number asymmetry, i.e., $\Delta \neq 0$, the final-state interactions (either from the s - or t -channel) must also violate baryon number B . It has long been known [7] that in the absence of B -violating final-state interaction, B -violating particle out-of-equilibrium decay cannot generate a B . Processes involving B -conserving final-state interactions may generate a CP-violating partial rate difference. But from CPT invariance and unitarity such asymmetries cancel once one adds them up to obtain B . It is therefore necessary that baryon number be violated at least twice: once in the decay and again in the final-state interaction.

This statement can be formulated more precisely. Consider the decay of a heavy particle P to some final states F_α , i.e., $P \rightarrow F_\alpha$, where the index α specifies the final state, and compare it to its CP mirror process $\bar{P} \rightarrow \bar{F}_\alpha$. CPT invariance and unitarity imply a relationship [8] between their amplitudes

$$\langle \bar{F}_\alpha | T | \bar{P} \rangle^* = \sum_{\alpha'} \langle F_\alpha | S^\dagger | F_{\alpha'} \rangle \langle F_{\alpha'} | T | P \rangle. \quad (9)$$

It is convenient to write the S -matrix for the final-state interaction as

$$S = S_0 + 2i\sqrt{S_0} S' \sqrt{S_0}, \quad (10)$$

where S_0 is the S -matrix that connects final states with the same baryon number. The matrix $i\sqrt{S_0} S' \sqrt{S_0}$ connects states with different baryon number and hence its elements give the B -violating final-state interaction. Suppose that the B -violating final-state interaction can be treated as a perturbation, we find [9] from (9) and (10)

$$\Delta \sim \sum_{\alpha, \beta} (B_\alpha - B_\beta) \text{Im}[T_\alpha T_\beta^* S'_{\beta\alpha}], \quad (11)$$

where B_α is the baryon number of the state labeled by α , T_α is the decay amplitude calculated to the first order of B -violating but all orders of B -conserving interactions.

Eq. (11) has all the required features for the generation of B [10]. First, B must be violated, i.e., not all final-states have the same baryon

number. Second, CP must be violated i.e., the amplitudes of the decay and the final-state interaction cannot be relatively real. The violation of CP can occur either in the decay amplitudes or in the final-state interaction or in both. To have a sufficiently large Δ , it is necessary that there be a large CP violation and that the B -violating final-state interaction S' is large. However, S' cannot be arbitrary large, since a too large S' would wash out the asymmetry generated at first place. This conflict can only be resolved if, third, the process is out of equilibrium. It has been shown recently [9] that the strength of B -violating interaction is controlled by the couplings of the interaction and the dimensionality of the effective B -violating operator.

The out-of-equilibrium constraint is a constraint on the B -violating final-state interaction. Heavy particle decay in equilibrium cannot produce any baryon number because of the B -violating final-state interaction, which would equilibrate all the asymmetries generated by the decay, not because of the reversed processes, i.e., $F_\alpha \rightarrow P$. The reversed processes actually help the production of baryon number [2,11].

Eq. (11) has a practical application. In calculating baryon number asymmetry, one only needs to consider diagrams in which the final-state interaction violates baryon number. Diagrams with final-state interaction violating flavor but conserving B may contribute to some CP-violating partial rate differences, but they do not contribute to baryogenesis.

4. FORMALISM FOR UNSTABLE PARTICLE MIXING

In the rest part of this paper we will develop a phenomenological description which goes beyond the direct Feynman diagram calculation for unstable particle mixing discussed above.

Earlier studies of unstable particle mixing followed two physically equivalent paths. One is due to Weisskopf and Wigner [12], in which one introduces an effective complex mass matrix. The evolution of the system is determined by the standard time-dependent Hamiltonian formalism [13]. The other is due to Sachs [14], in which one

studies the dynamics of the complex pole of the kaon field propagator. Both approaches are phenomenological, having difficulties handling ultra-violet divergences arising from higher order corrections. In spite of these fundamental difficulties, the phenomenological formalisms have been very successful. They provide the standard descriptions for the study of unstable particle mixing.

Consider the weak decay of a set of unstable particles ϕ_a produced at $t = 0$, where the index $a = 1, 2, \dots$ labels different flavor of ϕ . Suppose the lowest order amplitude of ϕ_a decaying into a final state $|F_f\rangle$ is given by

$$T_{fa} = \langle F_f | H_{\text{weak}} | \phi_a \rangle. \quad (12)$$

By CPT invariance, to first order of H_{weak} , a replacement of F_f and ϕ_a by their antiparticles $\bar{F}_f$ and $\bar{\phi}_a$ corresponds to changing T_{fa} to T_{fa}^* . By the superposition principle, the weak amplitudes at a later time t are

$$T_{fa}(t) = \sum_{b,c} T_{fb} V_{bc}^{-1} V_{ca} e^{-i\omega_c t}, \quad (13)$$

$$\bar{T}_{fa}(t) = \sum_{b,c} T_{fb}^* \bar{V}_{bc}^{-1} \bar{V}_{ca} e^{-i\omega_c t}, \quad (14)$$

where V and $\bar{V}$ are the mixing matrices

$$|\phi_a\rangle = V_{ca} |\phi'_c\rangle, \quad (15)$$

$$|\bar{\phi}_a\rangle = \bar{V}_{ca} |\bar{\phi}'_c\rangle, \quad (16)$$

and $|\phi'_c\rangle$, $|\bar{\phi}'_c\rangle$ are the eigenstates of an effective Hamiltonian, H_{eff} , which is correct up to second order in H_{weak} . The eigenvalue of $|\phi'_c\rangle$ and $|\bar{\phi}'_c\rangle$ is $\omega_c = m_c - i\gamma_c/2$ in the rest frame ϕ'_c , where m_c and γ_c may be interpreted, respectively, as the mass and width of ϕ'_c and $\bar{\phi}'_c$. It follows from CPT that [15]

$$\bar{V}_{ba} = V_{ab}^{-1}. \quad (17)$$

The time-integrated rate difference is

$$\Delta_{fa} = \int d\Omega \int_0^\infty \frac{dt}{\tau_a} \left[|T_{fa}(t)|^2 - |\bar{T}_{fa}(t)|^2 \right], \quad (18)$$

where $1/\tau_a = -2\text{Im}\omega_a$ is the width of the particle and $\int d\Omega$ represents a phase space sum.

In order for Δ_{fa} be nonzero it is necessary that CP be violated and to have significant non-trivial

CP-conserving phases. According to Eqs. (13) and (14), a CP-conserving phase may arise either from the mixing matrix V or $\bar{V}$ or/and from the evolution phase $e^{-i\omega_c t}$. For a completely degenerate system, i.e., $\omega_a = \omega$, the evolution phase factors

$$T_{fa}(t) = e^{-i\omega t} T_{fa}, \quad (19)$$

$$\bar{T}_{fa}(t) = e^{-i\omega t} T_{fa}^*. \quad (20)$$

Although a nonreal T_{fa} implies CP violation, the contribution to the rate difference from mixing is seen to vanish because $|T_{fa}(t)|^2 = |\bar{T}_{fa}(t)|^2$.

This result provides an answer to the question raised in the baryogenesis example discussed earlier. It shows that contributions to the generation of baryon number asymmetry from unstable particle mixing is highly suppressed whenever the particles in question have nearly equal mass and lifetime, and is zero in the complete degenerate limit.

Another important feature of Eqs. (13) and (14) is that, when the mixings are small, the diagonal mixings and phases are irrelevant for Δ_{fa} . Indeed, for small mixings, we expand V and $\bar{V}$ as

$$V_{ca} = e^{i\alpha_c} [\delta_{ca} + \Delta V_{ca}], \quad (21)$$

$$\bar{V}_{ca} = e^{i\bar{\alpha}_c} [\delta_{ca} + \Delta \bar{V}_{ca}], \quad (22)$$

where the diagonal phases α_c and $\bar{\alpha}_c$ are real. The elements in ΔV and $\Delta \bar{V}$ are assumed to be much smaller than unity. To first order in ΔV and $\Delta \bar{V}$ we have

$$T_{fa}(t) = e^{-i\omega_a t} T_{fa} + \sum_b T_{fb} \Delta V_{ba} [e^{-i\omega_b t} - e^{-i\omega_a t}], \quad (23)$$

$$\bar{T}_{fa}(t) = e^{-i\omega_a t} T_{fa}^* + \sum_b T_{fb}^* \Delta \bar{V}_{ba} [e^{-i\omega_b t} - e^{-i\omega_a t}]. \quad (24)$$

The second terms in Eqs. (23) and (24) vanish for diagonal mixings. Therefore, neglecting off-diagonal mixings the CP-conserving phase $e^{-i\omega_a t}$ factors and $|T_{fa}(t)|^2 - |\bar{T}_{fa}(t)|^2 = 0$. Hence, for small mixings the diagonal phases α_a , $\bar{\alpha}_a$ and $\omega_a t$ do not enter into the determination of Δ_{fa} ; only the off-diagonal mixings, i.e., $b \neq a$, are relevant.

Thus, for the calculation of Δ_{fa} one does not need to consider flavor-conserving one-particle-reducible diagrams.

Let us return to the heavy scalar decay example. Suppose that the mass difference of the scalars is much smaller than their width difference

$$(M_1 - M_2)^2 \ll (\Gamma_1 - \Gamma_2)^2, \quad (25)$$

and that the mixing in the decay matrix is small. By employing the formalism presented above one can show that the asymmetry from mixing is now changed to

$$\Delta_{\text{mixing}} \rightarrow \Delta_{\text{mixing}} \frac{(M_1^2 - M_2^2)^2}{M^2(\Gamma_1 - \Gamma_2)^2}, \quad (26)$$

where M is either M_1 or M_2 . Compared to (8), (26) is further suppressed by a small ratio. In particular, all the CP-violating partial rare differences vanish in the limit $M_1 = M_2$.

5. SOME TECHNICAL DETAILS

There are two important steps in carrying out the present prescription for unstable particle mixing. The first one is to construct the effective Hamiltonian H_{eff} for a given unstable particle system.

The advent of renormalizable gauge theory provides a connection between the parameters of a phenomenological theory discussed above and the parameters of a given fundamental theory. The effective Hamiltonian H_{eff} is most conveniently obtained by evaluating loop corrections to the particle propagator [16]. H_{eff} is generally non-hermitian. For convenience, one can separate H_{eff} into a hermitian and an antihermitian part

$$H_{\text{eff}} = M - i\Gamma, \quad (27)$$

where

$$M = M^\dagger = \frac{H_{\text{eff}} + H_{\text{eff}}^\dagger}{2}, \quad (28)$$

$$\Gamma = \Gamma^\dagger = i \frac{H_{\text{eff}} - H_{\text{eff}}^\dagger}{2}, \quad (29)$$

where M and Γ play the roles of mass and decay matrices, respectively. The parameters in the mass matrix are related to the parameters of the

given fundamental theory. They are renormalizable and determined experimentally at some energy scale. The parameters in the decay matrix are perturbatively calculable.

Once H_{eff} is determined, the next important step is to diagonalize it. Since H_{eff} is non-hermitian, the diagonalization of H_{eff} is highly non-trivial. Although H_{eff} can always be diagonalized by a bi-unitary transformation, i.e., $[V_L H_{\text{eff}} V_R^\dagger]_{ab} = \lambda_a \delta_{ab}$, where V_L and V_R are unitary and the eigenvalues λ_a are real, this procedure is not useful for us unless $V_L V_R^\dagger = 1$; otherwise the kinetic energy part of the lagrangian will not remain diagonal under such a transformation. Methods for diagonalizing a 2×2 complex non-hermitian matrix are known [13]. Here we will generalize them to arbitrary cases [15].

Let us start from simple cases involving real M and Γ . These correspond to situations in which the mixed unstable-particle system conserves CP. In that case, H_{eff} is symmetric with $n(n+1)$ independent parameters. We can diagonalize such an H_{eff} by an orthogonal transformation with complex mixing angles

$$O H_{\text{eff}} O^T = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}, \quad (30)$$

where

$$O O^T = 1. \quad (31)$$

O differs from a usual orthogonal matrix by having complex mixing angles, i.e., $\theta_i = \text{Re}\theta_i + i\text{Im}\theta_i$ ($i = 1, \dots, n(n-1)/2$). For a 2×2 system the result is

$$O = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (32)$$

It should be emphasized that a complex mixing angle is not equivalent to a real mixing angle times a real phase. Indeed, $|\cos \theta|$ could be bigger than 1 if θ is complex, whereas $|\cos \theta e^{i\alpha}| \leq 1$ if both θ and α are real. Nevertheless, trigonometric functions with complex angles have very similar properties, such as $\sin^2 \theta + \cos^2 \theta = 1$ etc., as the elementary trigonometric functions. This

is the advantage of introducing complex mixing angles.

The complex mixing angle of an unstable system has a simple physical meaning if the mixing is small: The real part of the mixing angle measures the mixing in the mass matrix, whereas the imaginary part measures the mixing in the decay matrix.

The $n(n+1)$ parameters of H_{eff} determine the n complex eigenvalues, which have $2n$ parameters, and the $n(n-1)/2$ complex angles involving $n(n-1)$ parameters.

The above formalism is useful for describing the so called ‘mix-and-decay’ phenomena in unstable neutrino oscillation [17]. For each mixing angle there are two independent parameters corresponding to the real and imaginary part. It should be pointed out that

$$|\sin \theta|^2 + |\cos \theta|^2 \neq 1. \quad (33)$$

The idea of introducing complex mixing angle can be extended to situations in which both M and Γ are complex, provided the phases involved in the diagonalization are regarded as complex as well. In dealing with phases, one must specify the phase convention involved, even though physical observables are phase convention independent. Let us first review the diagonalization of a hermitian matrix H

$$U H U^{-1} = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}, \quad (34)$$

where λ_i are real. $U^{-1} = U^\dagger$ is unitary with $n(n-1)/2$ real angles and $n(n+1)/2$ real phases. However, not all phases of U are determined by (34). To be specific, we write

$$U = K U', \quad (35)$$

where U' is a *reduced* unitary matrix with $n(n-1)/2$ angles and $n(n-1)/2$ phase, K is diagonal with n phases

$$K = \begin{pmatrix} e^{i\alpha_1} & & 0 \\ & \ddots & \\ 0 & & e^{i\alpha_n} \end{pmatrix}. \quad (36)$$

Clearly, if U diagonalizes H so does its reduced matrix U' .

An arbitrary H_{eff} has $2n^2$ parameters: among them, n^2 in M and the rest in Γ . The transformation that leaves the unity matrix invariant and diagonalizes H_{eff} is

$$V H_{\text{eff}} V^{-1} = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}, \quad (37)$$

where λ_i are complex. In practice we may parameterize V in terms of a reduced unitary matrix, such as U' discussed above, but change all the mixing angles and phases into complex variables. Now, the $2n^2$ parameters determine the n complex eigenvalues which involve $2n$ parameters, the $n(n-1)/2$ complex mixing angles (with $n(n-1)$ parameters) and the $n(n-1)/2$ complex phases, which also have $n(n-1)$ parameters.

For example, for a 2×2 system, H_{eff} can be most generally parameterized as

$$H_{\text{eff}} = \begin{pmatrix} \tilde{M}_{11}^2 & \tilde{M}_{12}^2 \\ \tilde{M}_{12}^{2*} & \tilde{M}_{22}^2 \end{pmatrix} - i \begin{pmatrix} \tilde{\Gamma}_{11}^2 & \tilde{\Gamma}_{12}^2 \\ \tilde{\Gamma}_{12}^{2*} & \tilde{\Gamma}_{22}^2 \end{pmatrix}. \quad (38)$$

The mixing matrix has one complex angle and one complex phase

$$V = \begin{pmatrix} \cos \theta & -\sin \theta e^{i\phi} \\ \sin \theta e^{-i\phi} & \cos \theta \end{pmatrix}, \quad (39)$$

where the parameters in the mixing matrix are determined by the parameters of H_{eff} by the following relations:

$$\tan^2 2\theta = 4 \frac{[\tilde{M}_{12}^2 - i\tilde{\Gamma}_{12}^2][\tilde{M}_{12}^{2*} - i\tilde{\Gamma}_{12}^{2*}]}{[\tilde{M}_{11}^2 - \tilde{M}_{22}^2 - i(\tilde{\Gamma}_{11}^2 - \tilde{\Gamma}_{22}^2)]^2}, \quad (40)$$

$$e^{-i2\phi} = \frac{\tilde{M}_{12}^{2*} - i\tilde{\Gamma}_{12}^{2*}}{\tilde{M}_{12}^2 - i\tilde{\Gamma}_{12}^2}. \quad (41)$$

For a 3×3 H_{eff} , V has three complex mixing angles and three complex mixing phases. It may be parameterized as

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 e^{i\alpha_2} \\ 0 & s_2 e^{-i\alpha_2} & -c_2 \end{pmatrix} \quad (42)$$

$$\times \begin{pmatrix} c_1 & s_1 e^{i\alpha_1} & 0 \\ -s_1 e^{-i\alpha_1} & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (43)$$

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 e^{i\alpha_3} \\ 0 & -s_3 e^{-i\alpha_3} & c_3 \end{pmatrix}, \quad (44)$$

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$ ($i = 1, 2, 3$). θ_i and α_i are complex.

The CP-conserving phase required for the generation of a CP-violating partial rate difference is given by the imaginary part of the mixing angle.

6. CONCLUSION

Through much of the content of this paper consists of combining and rephrasing known arguments, we believe that we have provided a clearer discussion than was previously available of many of the features for unstable particle mixing in CP-violating weak decays. We discussed the importance of final-state interactions for the generation of CP-violating partial rate difference. In particular, we provided a formalism for unstable particle mixing based upon one-loop renormalization of field theory. By introducing complex mixing angle and phase, we presented a general method for diagonalizing an arbitrary $n \times n$ matrix. The crucial role played by B -violating final-state interaction in creating baryon number is also emphasized.

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