-LUTP 94.1
SW 9416

LU TP 94-1
January 1994

CORRELATIONS IN THE HADRONIZATION PROCESS

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Abstract

Pions cannot be fully described as simple $q\bar{q}$ systems. They have a very small mass and may appear as a kind of Goldstone particles. It is demonstrated how this is expected to give correlations in flavour and $p_{\perp}$ in the string model hadronization process. Improved agreement with experimental data is obtained, in particular for η', η production and $\pi^+\pi^-$ mass correlations.

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I. Introduction

The Lund string fragmentation model [1] as it is at present implemented in the JETSET Monte Carlo [2] (from now on called the present Lund model, PLM) has been very successful to describe the experimental data from e^+e^- -annihilation, DIS and hadronic collisions. Accumulating experimental results also exhibit discrepancies, however, and it is necessary to consider some of the properties of the model in more detail. In the first section we consider a set of such discrepancies. We find that they can all of them be related to the assumed rate of $\eta'(\eta)$ and directly produced pions in the model and lead to distortions in the pion spectra compared to the experiments.

In the Lund model a string-like colour force field is assumed to break up into pieces by $q\bar{q}$ -pair production. A q and a $\bar{q}$ from adjacent breakup points (“vertices”) join to form a hadron, cf fig. 1. It is assumed that the hadron comes on mass-shell, but besides this constraint (which leads to longitudinal momentum correlations) there are no other correlations in the PLM. Neither the transverse momentum, $p_\perp$, nor the flavours produced are correlated between the neighbouring vertices.

The space-time structure of the process is illustrated in fig. 2. The vertices are localized around a hyperbola in space-time, which means that the proper times, τ , vary around a mean value, $\langle \tau \rangle$. There is a correlation length, corresponding to the masses in between the vertices and this reflects itself into the above-mentioned longitudinal p_L -correlation. We will advocate below that there should be a corresponding correlation also in $p_\perp$ and in flavour.

II. Problems

1. *Flavour*: The relative production rate of (directly produced) pions and ρ -mesons is in the PLM approximately $\pi/\rho \simeq 1/1$ and is therefore not given by a statistical assumption on the number of spin states. This is due to the assumption that there is a spin-spin interaction affecting the final states as discussed in section III below. The production of η' and η is then linked to the π -production rate. Increasing the number of π :s will then increase also the η' and η .

2. *Transverse momentum*: In the model all directly produced hadrons have the same $p_\perp$ -distribution (in the string rest frame). It was observed in ref. [3] that factorial moments in the PETRA-PEP energy region is very sensitive to the $p_\perp$ of directly produced pions. If the $p_\perp$ of these pions is reduced improved agreement with e.g. TASSO data [4] at PETRA is obtained. This effect disappears in the high energy LEP region because of the increased gluon emission rates.

3. *$\pi\pi$ -correlations*: The correlations between the observed opposite sign pions (i.e. both the directly produced pions and those stemming from the decays) are in the PLM strongly affected both by η' and η production rates. From fig. 3, which depicts the $\pi\pi$ mass correlation from NA22 [5] we find that the PLM predicts too strong peaks around the ρ mass and for very small masses together with a too deep valley in between. The low mass enhancement obtains contributions from η and η' , and we also note that η' has a large branching ratio to ρ (30 %), thus enhancing the ρ -peak.

4. *Bose-Einstein correlations*: It has been pointed out [6] that the model in ref. [7], which is based upon the Lund Model, is strongly affected by the production rate of the

long-lived resonance η' . In case the η' production rate is decreased compared to the PLM, then the model in ref. [7] works very well.

All the problems taken up in this section are evidently related to the production of $\eta'(\eta)$ and pions which in the PLM are linked together, and where the η' -rate strongly affects the observed pion production correlations. Pions are known to be special and in chiral dynamics they behave approximately like Goldstone bosons, i.e. waves in a degenerate vacuum condensate. Therefore it is reasonable to assume that they cannot solely be described as simple $q\bar{q}$ systems, and in particular that the rate of $\eta'(\eta)$ should not be linked to the directly produced pions.

III. Flavour correlation

Tunneling mechanism

We firstly discuss the tunneling mechanism for pair production in a constant force field [8]. When the string breaks by $q\bar{q}$ production, classically the pair is produced at a certain distance so that the string energy between them is transformed into their (transverse) mass. The q and the $\bar{q}$ are then pulled apart by a linear potential given by the string tension κ .

Quantum mechanically the wave functions are exponentially damped in the classically forbidden region, and then oscillating as illustrated in fig. 4. The value of the wave functions in the centre of the forbidden region can be estimated by the WKB method

$$\psi = \psi_{ctp} \cdot \exp(-\pi m_{\perp q}^2 / 4\kappa) \quad (1)$$

where ψ_{ctp} is the wave function at the classical turning point and $m_{\perp q}^2 = m_q^2 + q_\perp^2$ is the transverse mass of the quark.

The production probability can be estimated from the product of the q and the $\bar{q}$ wave functions and the colour field potential $\bar{A}$.

$$\begin{aligned} \text{Prod. prob.} &\sim (\psi_q \psi_{\bar{q}} \bar{A})^2 \sim \exp(-\pi m_{\perp q}^2 / \kappa) \\ &= \exp(-\pi m_q^2 / \kappa) \cdot \exp(-\pi q_\perp^2 / \kappa) \end{aligned} \quad (2)$$

The result is a Gaussian suppression of large quark masses and a Gaussian distribution in transverse momenta.

Note that in eq. (2) ψ is normalized at the classical turning point. The quark is, however, not pulled off to infinity, but must fit into a hadron wavefunction, which affects the normalization. Assume that the quark joins an antiquark from a neighbouring breakup to form a meson. We assume that their mutual interaction is given by the linear string potential and a local spin interaction

$$V(x) = \kappa |x| + a \bar{S}_1 \bar{S}_2 \delta(x) \quad (3)$$

The wave function will have a discontinuous derivative at $x = 0$. The normalized wave functions for parallel and antiparallel spins (ρ and π) are shown in fig. 5. The normalization at the classical turning point is obviously different in the two cases. We now define the ratio of the absolute squares of the wave functions at the classical turning point

as the relative production probability. In ref. [1] it was estimated that this production probability is approximately proportional to $1/M_{\perp}$, where $M_{\perp}$ is the transverse mass of the meson. For this reason the $\rho:\pi$ ratio was estimated to be approximately 1:1, rather than 3:1 as obtained from pure spin state counting.

Is it possible to have a similar suppression of η' and η ? Their masses are also larger than the pion mass. Thus their wave functions should be more extended, with a smaller normalization. Such a suppression is **not** possible in the PLM with no flavour correlation between neighbouring breakups. The following combinations can give nonstrange systems with spin zero.

$$\begin{aligned} u\bar{d} &: \pi^+ \\ d\bar{u} &: \pi^- \\ u\bar{u} &: \pi^0, \eta \text{ or } \eta' \\ d\bar{d} & \end{aligned}$$

Without flavour correlations these combinations are equally probable. Therefore an enhancement of the pions without isospin breaking (equal numbers of π^0, π^+, π^-) necessarily implies an enhancement of η, η' . A suppression of η, η' implies that $u\bar{d}$ and $d\bar{u}$ should be more common than $u\bar{u}$ and $d\bar{d}$, which implies a flavour correlation between neighbouring breakups.

The η' and η particles also contain an admixture of $s\bar{s}$. As strange quarks are suppressed it is possible to reduce either η' or η by changing the SU(3) mixing angle. This is no solution to the problem, however, because a reduction of e.g. η' is compensated by an increase of η . The sum of η' and η production is independent of the mixing angle.

Pions as Goldstone particles

The pion has a very small mass. Thus the energy from a very small piece of string is used to produce a pion. The pion wave function is much more extended. When a pion is produced the distance between the two neighbouring breakups $\sim m_{\pi\perp}/\kappa$ is in general very small compared to the size of the wave function $\sim 1/m_{\perp}$. It is therefore very natural that these breakups may be correlated.

The fact that the size of the string piece is much smaller than the extension of the pion wave function also implies that it is reasonable that the string picture does not reflect all properties of the pions. The pion can also be regarded as a Goldstone boson, i.e. a wave in a degenerate vacuum condensate.

If the pion is treated as a Goldstone particle we can imagine a production mechanism as suggested by Bjorken and coworkers for AA collisions [9]. Consider a colour flux tube surrounded by a degenerate vacuum condensate. This condensate is specified by a particular direction in an internal space (cf fig. 6a). When a $q\bar{q}$ pair is produced it is pulled apart by the field. In the region between the q and the $\bar{q}$ a ground state condensate is restored. This ground state is, however, not necessarily the same as the original vacuum (see fig. 6b). The boundary of the original flux tube is replaced by a domain wall, and the domain wall energy will spread as a wave in the vacuum condensate, i.e. as a Goldstone particle. In this way the q , the $\bar{q}$ and the pion are produced as one unit (see fig. 7a).

It is also conceivable that more than one pion is emitted in the process (cf fig. 7b). In such a case the pions should form a coherent state. Thus we may get either many π^0 or many $\pi^+\pi^-$ -pairs. We note however that the average number of pions produced in such a breakup ought to be small. (In nucleus collisions or more violent hadronic collisions, considered in ref. [9], this number may be large.)

Conclusion: Both if there is a special production mechanism for Goldstone bosons, or if the pions are enhanced just because they have very low masses, we expect string breakups with an intermediate pion to be correlated in flavour. For breakups with intermediate heavier particles we expect this flavour correlation to be much smaller.

IV. Momentum correlations

Longitudinal momenta

The longitudinal momenta of the produced hadrons are determined by the positions of the breakup points in fig. 2. The proper time, τ , for these breakups is varying around an average value $\langle \tau \rangle$, which specifies an hyperbola in the spacetime diagram. If we define the quantity δ by the relation

$$\delta \equiv \ln(\tau / \langle \tau \rangle) \quad (4)$$

then δ has in the Model an almost Gaussian distribution. However, if one breakup is located unusually far away from the average hyperbola, then it takes normally a few steps before this hyperbola is reached again. If τ_1 and τ_2 are the proper times of two neighbouring breakups, then for fixed τ_1 the distribution in τ_2 is a well specified expression, though algebraically somewhat complicated. This distribution depends on the mass of the intermediate hadron, and we note that the same result is obtained if the intermediate system is not a single hadron but a group of hadrons with a specified invariant mass.

The correlation between the breakup points can be expressed in the average value $\langle \delta_1 \delta_2 \rangle$, which falls off with increasing mass, M, for the intermediate system, approximately according to the expression

$$\langle \delta_1 \delta_2 \rangle \sim \frac{1}{1 + M/M_0} \quad (5)$$

with M_0 of the order of 1 GeV. With default values for the fragmentation parameters we find $M_0 \approx 0.9$ GeV.

The hadron formation in fig. 2 can also be expressed as in fig. 8, where the hadron momenta k_i are plotted after each other, forming a curve with an endpoint at $P_{tot} = p_q + p_{\bar{q}}$. With the definition

$$p_i = \sum_{j=1}^i k_j \quad (6)$$

the points p_i are distributed around an hyperbola (see fig. 8) which corresponds directly to the hyperbola for the average proper time in fig. 2. The hyperbola in fig. 8 lies within the $p_L - E$ plane, and the correlations discussed above correspond to fluctuations around the

hyperbola within this plane.

If we include transverse momenta the points p_i do not lie fully within the $p_L - E$ plane, they fluctuate around the hyperbola in fig. 8 both within this plane and transverseley to it. In the PLM all the components $p_{\perp i}$ are assumed to be uncorrelated, but from fig. 8 it would be natural to expect similar correlations in the fluctuations out of the $p_L - E$ plane as within this plane.

We end this subsection by noting that the hyperbola in fig. 8 can be generalized to a situation with one or more gluons. In such cases there is not a well defined longitudinal direction. Nevertheless it is possible to define a curve in 4-momentum space, called the x -curve, such that the points p_i in eq. (6) follow this curve, with limited fluctuations around it [10].

Transverse momenta

In accordance with eq. (6) we define $\bar{p}_{\perp i}$ as the total transverse momentum of the first i hadrons. Due to total $p_{\perp}$ conservation this is also minus the $p_{\perp}$ of the remaining hadrons

$$\bar{p}_{\perp i} = \sum_1^i \bar{k}_{\perp j} = -\sum_{i+1}^n \bar{k}_{\perp j} \quad (7)$$

We make the following assumptions:

1) The distributions in $\bar{p}_{\perp i}$ saturate in the centre. These distributions are thus given by a function f such that

$$d\text{Prob} = f(\bar{p}_{\perp i}) d^2 p_{\perp i} \quad (8)$$

2) There is an iterative structure in the same way as for the longitudinal momentum. Thus there is a given distribution, $g(\bar{k}_{\perp i} | \bar{p}_{\perp i-1})$, in $\bar{k}_{\perp i}$ for fixed $\bar{p}_{\perp i-1}$

$$d\text{Prob} = g(\bar{k}_{\perp i} | \bar{p}_{\perp i-1}) d^2 k_{\perp i} \quad (9)$$

(This also determines the distribution in $\bar{p}_{\perp i} = \bar{p}_{\perp i-1} + \bar{k}_{\perp i}$.)

3) The function $g(\bar{k}_{\perp i} | \bar{p}_{\perp i-1}) \equiv g(\bar{k}_{\perp i} + \gamma \bar{p}_{\perp i-1})$ i.e. it only depends on a particular combination of $\bar{k}_{\perp i}$ and $\bar{p}_{\perp i}$ with γ a parameter discussed below.

4) There is right-left symmetry. The system can be generated from the quark end or from the antiquark end with the same result.

With these assumptions we obtain a unique solution of the following form:

$$f(\bar{p}_{\perp i}) = N \cdot \exp(-\alpha \bar{p}_{\perp i}^2) \\ g(\bar{k}_{\perp i} | \bar{p}_{\perp i-1}) = N' \exp[-\frac{\alpha}{\gamma(2-\gamma)} (\bar{k}_{\perp i} + \gamma \bar{p}_{\perp i-1})^2] \quad (10)$$

This solution has two parameters, α which is equal to $1/\langle p_{\perp i}^2 \rangle$ and γ , which

determines the correlation strength via the relation $\langle \bar{k}_{\perp i} \rangle = -\gamma \bar{p}_{\perp i-1}$.

It is straightforward to calculate the inclusive hadron $k_{\perp}$ -distribution. The result is given by

$$d\text{Prob} = N'' \cdot \exp(-\frac{\alpha}{2\gamma} \bar{k}_{\perp i}^2) d^2 k_{\perp i} \quad (11)$$

Thus the average $k_{\perp}$ is given by

$$\langle k_{\perp i}^2 \rangle = 2\gamma / \alpha \quad (12)$$

Furthermore the correlations can be expressed by the relation

$$\langle \bar{p}_{\perp i} \cdot \bar{p}_{\perp i-1} \rangle = \frac{1}{\alpha} (1 - \gamma) \quad (13)$$

We also find that $\langle \bar{p}_{\perp i} \cdot \bar{p}_{\perp i-k} \rangle = \frac{1}{\alpha} (1 - \gamma)^k$.

We now assume that there is a similar correlation length in transverse as in longitudinal momentum. This is obtained if we assume that (cf eqs. (5) and (13))

$$1 - \gamma = \frac{1}{1 + M/M_0} \quad (14)$$

with M equal to the hadron mass and M_0 around 0.9 GeV. We want to stress that it is consistent with the assumptions above that γ depends on the mass of the hadron i . The powerlike dependence on M corresponds to an exponential fall off in hyperbolic angle or rapidity along the string.

We note that the hadron momentum $k_{\perp}^2$ is proportional to $\gamma = M/(M + M_0)$ (cf eq. (12)). This implies in particular that pions, which take a small piece of string, also get small transverse momenta. For larger masses $\langle k_{\perp}^2 \rangle$ saturates, and thus it is mainly the pions which are noticeably affected.

We note that the PLM corresponds to the case when $\gamma = 1$. The model described above is a generalization of unpublished results obtained previously by us, and used in the UCLA-model [11], in which a constant $\gamma = 1/2$ is assumed. It is of some interest to recognize that the distributions obtained are closely related to the velocity distribution of a particle undergoing Brownian motion. In that case γ is time-dependent, $\gamma \simeq \exp(-t/c)$ with t the typical relaxation time.

V. Results

In this section we will study some consequences of the modifications discussed above. The results presented are obtained with the following set of parameters:

- The $\rho : \rho + \pi$ ratio is 0.35 instead of the default value 0.5. This is in accordance

with experimental results for ρ production, e.g. refs [12].

- η' and η are suppressed compared to pions by factors 0.2 and 0.7 respectively.

Without this suppression the reduced $\rho:\pi$ ratio would give too many η :s and further enhance the already too high η' rate in the old model.

- The parameter M_0 in eq. (14) is chosen to be 0.9 GeV, and α^{-1} (i.e. $\langle p_{\perp}^2 \rangle$) is 0.25 GeV². With these values the $\langle k_{\perp} \rangle$ for all pions (including resonance decay products) is unchanged equal to 0.25 GeV, while $\langle k_{\perp} \rangle$ for the directly produced pions is reduced from 0.40 GeV to 0.23 GeV.

With these changes of the PLM we get the following results:

1) The η' **and η production** rates agree well with experimental data [13], as is illustrated in fig. 9.

2) $\pi^+\pi^-$ **mass correlations** are shown in fig. 10. To more clearly illustrate the difference between the old and the modified model, the simpler e^+e^- -annihilation system is chosen, rather than a hadronic collision. We note that the ρ -peak is significantly reduced, while the valley between 400 and 650 GeV is partly filled in (cf fig. 3).

3) **Factorial moments**. In ref. [4] it was shown that reducing the $\rho:\rho+\pi$ ratio to 0.35 and decreasing the $\langle k_{\perp} \rangle$ for directly produced π improved the agreement with TASSO data on factorial moments at 36 GeV [5]. The model presented here gives results for factorial moments which are very close to those of ref. [4].

4) **Baryon production**. Baryons can be produced if the string breaks by production of a diquark and an antidiquark system. In the “pop-corn mechanism” [14] it is possible to produce a meson in between the baryon and the antibaryon, as shown in fig. 11. If this meson is a pion we may expect, from the arguments above, that the $qq-\pi-\bar{q}\bar{q}$ system is produced as a correlated system. Thus it is e.g. conceivable that total isospin 0 is favoured for such a system, which will affect baryon production and $B\bar{B}$ correlations. These possible effects on baryon production are still under investigation.

VI. Conclusions

Pions are very special. They have a small mass and are a kind of Goldstone bosons. In the string fragmentation process we expect correlations in flavour and $p_{\perp}$ between neighbouring string breakups, which in particular affects the pion production. The modifications compared to the old Lund model imply improved agreement with data for η' production and $\pi\pi$ -correlations. The effects for baryon production are still under investigation.

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Fig. 1. In the old Lund string fragmentation model p_L is locally conserved in each string breakup, and there is no correlation in p_L or flavour between neighbouring breakups.

Fig. 2. Space-time picture of the string breakup. The $q\bar{q}$ production points are located around a hyperbola given by $\tau = \langle \tau \rangle$, where τ is the proper time.

Fig. 3. $\pi^+ \pi^-$ mass correlations. Data for $K_2(M_{inv})$ from NA22 [5] compared with results from the FRITOF model.

Fig. 4. Wave functions for a quark and an antiquark pulled apart when the string breaks.

Fig. 5. Wave functions for a pion and a ρ , showing the different normalization at the classical turning point, which is marked with a dash.

Fig. 6a) The string can be regarded as a colour fluxtube imbedded in a degenerate vacuum condensated, which is specified by a direction in an internal space. b) When a $q\bar{q}$ -pair is pulled apart, the ground state formed between them is not necessarily identical to the vacuum state.

Fig. 7a) A quark, an antiquark and an intermediate pion are produced as one unit. b) If more than one Goldstone pion is produced, they should form a coherent state.

Fig. 8. If the hadron momenta k_i are plotted after each other they form a curve with points p_i approximately following a hyperbola in energy-momentum space.

Fig. 9. Production spectra for η' and η in e^+e^- -annihilation at 91 GeV. Results from the old (dashed line) and the modified (solid line) Lund model are compared with data from ALEPH [13].

Fig. 10. $\pi^+\pi^-$ mass correlations at LEP. Predictions from the old (dotted line) and the modified (solid line) Lund model. For comparison is shown the result after changing the flavour correlation, but using the transverse momentum distribution from the old Lund Model (dashed line).

Fig. 11. In the "pop-corn mechanism" a meson is formed between a baryon and an antibaryon.