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February 1994

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Photoproduction of Jet Pairs at HERA as a Probe of the Perturbative Pomeron

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Abstract

The photoproduction of minijet pairs with small transverse momentum and large relative rapidity at the HERA electron-proton collider is studied. We examine, in particular, the prospects for measuring a rise in the cross section with increasing rapidity gap at fixed transverse momentum, as predicted by the BFKL ‘perturbative pomeron’ of QCD. We show that the distribution in the azimuthal angle difference of the jets is particularly sensitive to the BFKL resummation, and is therefore more likely to provide a distinct signature than the minijet cross section itself.

1 Introduction

Several years ago, Mueller and Navelet [1] showed that the cross section for producing a pair of minijets (i.e. jets with modest transverse momentum) with a large relative rapidity could be used to probe the the Balitsky-Kuraev-Fadin-Lipatov (BFKL) [2] ‘perturbative pomeron’. The cross section is predicted to rise roughly as $e^{\lambda\Delta y}$, where $\lambda \simeq 0.5$, with increasing rapidity difference Δy . This is in exact analogy to the predicted rise $\sim x^{-\lambda}$ at small x of deep inelastic structure functions [3]. In both cases it is the multiple emission of soft gluons which is responsible for the predicted behaviour.

In a recent paper [4] (see also [5]) we performed a quantitative study of the Mueller-Navelet jet cross section for the Fermilab $p\bar{p}$ collider at $\sqrt{s} = 1.8$ TeV. We showed that when the collider energy is fixed, the dependence of the cross section on the rapidity difference is dominated by the fall-off of the parton distributions at large x , rather than by the predicted BFKL behaviour of the parton subprocess cross section. Coupled with theoretical uncertainties concerning the choice of scale, sub-leading corrections etc., this makes the observation of BFKL behaviour from the total minijet cross section alone rather difficult. We proposed instead to study the correlation in azimuthal angle difference between the jets, and showed that while jets with a small rapidity difference are dominantly produced back-to-back in $\Delta\phi$, the correlation weakens as the jets separate in rapidity, until in the limit $\Delta y \rightarrow \infty$ the distribution in $\Delta\phi$ becomes flat. This phenomena is caused by the emission of soft gluons in the rapidity interval between the two jets. It is the leading-logarithm contributions to the cross section from this gluon emission which are resummed by the BFKL equation. Similar conclusions were reached in Ref. [5], where further correlations in the transverse momentum of the jets were also studied.

In this Letter we extend our analysis to the photoproduction of minijet pairs at HERA. We are interested, in particular, in ‘resolved photon’ contributions to the cross section, i.e. those involving quark and gluon constituents of the photon. For the (small) jet transverse momenta that we are interested in, these dominate the ‘direct photon’ contribution, where the photon participates directly in the hard scattering [6]. With the distributions of partons in the electron (obtained by convoluting photon structure functions with the Weizsäcker-Williams distribution) replacing the distribution of partons in the antiproton, as for the $p\bar{p}$ collider, the formalism is essentially the same in the two cases, and many of the results of Ref. [4] can be applied directly to the HERA analysis. There are, however, several important differences. First, the BFKL behaviour becomes more pronounced at large rapidity difference Δy , and so the increased phase space at higher collider energy makes the effect easier to observe. In this respect, the 1.8 TeV of the $p\bar{p}$ collider appears to offer a clear advantage over the 314 GeV of the HERA ep collider. However, this can be partly offset by considering smaller p_T jets at the ep collider, where the cleaner and quieter

underlying event activity should make it easier to distinguish genuine hard-scattering minijets from background hadronic production. A second distinguishing feature at HERA is the asymmetry in the beam energies. At the $p\bar{p}$ collider the largest Δy 's are obtained by considering large equal and opposite jet rapidities, whereas at HERA it is more appropriate to consider one jet produced centrally with another produced at large backward (i.e. proton direction) rapidity.

The outline of the paper is as follows. In the next section we define some basic kinematics and discuss the total minijet cross section as a function of Δy . We then consider in Section 3 the distribution in the azimuthal angle difference between the jets. Finally, in Section 4, we present our conclusions. Note that in the present study we concentrate on numerical predictions for HERA. More complete derivations of the cross sections we use can be found, for example, in Refs. [4, 5].

2 Jet Pair Cross Section at HERA

Consider the inclusive two-jet cross section in ep collisions at energy $\sqrt{s}$, where the jets have equal and opposite transverse momentum $p_T \ll \sqrt{s}$, and rapidities y_1 and y_2 . We are interested here in the ‘resolved photon’ contribution, that is, where the hard scattering is mediated by quark and gluon constituents of a quasi-real photon emitted by the electron. The momentum distributions of such partons are obtained by combining the photon structure functions with the standard Weizsäcker-Williams distribution:

$$f_{a/e}(x, \mu^2) = \int_0^1 dy dz f_{\gamma/e}(y) f_{a/\gamma}(z, \mu^2) \delta(x - yz), \quad a = q_i, g. \quad (1)$$

The two-jet inclusive cross section in leading order is given by

$$\frac{d\sigma^{\text{res}}}{dy_1 dy_2 dp_T^2} = \frac{1}{16\pi^2 s^2} \sum_{a,b,c,d=q,g} x_e^{-1} f_{a/e}(x_e, \mu^2) x_p^{-1} f_{b/p}(x_p, \mu^2) \overline{\sum} |\mathcal{M}(ab \rightarrow cd)|^2, \quad (2)$$

where $\overline{\sum}$ denotes the appropriate sums and averages over colours and spins. The momentum fractions are fixed by the final state kinematics:

$$x_e = \frac{p_T}{2E_e} (e^{y_1} + e^{y_2}), \quad x_p = \frac{p_T}{2E_p} (e^{-y_1} + e^{-y_2}). \quad (3)$$

Our convention is that large positive rapidity corresponds to jets moving in the electron direction. At HERA, $E_e \ll E_p$ and so the phase-space limit corresponding to $x_e = 1$ is reached for relatively modest positive jet rapidities. In contrast, for jets with $p_T \ll E_p$ large negative rapidities are accessible. To achieve a large rapidity *difference* between the jets, we therefore require one jet to have small positive rapidity, while the other has large negative rapidity. Introducing $\Delta = y_2 - y_1$ gives

$$x_e = \frac{p_T}{2E_e} e^{y_1} (1 + e^{-\Delta}), \quad x_p = \frac{p_T}{2E_p} e^{y_1} (1 + e^{\Delta}), \quad (4)$$

and we can consider the limit where y_1 is fixed to be $O(1)$ and the difference Δ is large and positive. To be specific, if we take $E_e = 30$ GeV, $E_p = 820$ GeV and $p_T = 5$ GeV/c, then Fig. 1 shows the momentum fractions x_e and x_p for $y_1 = 1$ as a function of Δ . Note that for large Δ , the momentum fraction of the electron partons is relatively constant, while that of the proton partons is an increasing function of Δ , typically in the range $0.01 - 0.1$. This is an important feature of our analysis: we generate a large rapidity difference in a way which corresponds to an approximately constant x_e and a rapidly varying x_p . This means that the poorly known photon structure functions tend to factor out of the cross section, while the proton's structure functions are probed in a region where they are well determined by fixed-target deep inelastic scattering experiments (see for example Ref. [7]).

At this point, it is worth contrasting our analysis with that of Ref. [8], where the two-jet cross section at HERA was also studied. In Ref. [8], the two jets were required to have *equal* and sizable positive rapidity. As can be seen from Eq. (4) with $\Delta = 0$, this forces $x_e \rightarrow 1$ and $x_p \rightarrow 0$, while in the subprocess centre-of-mass frame, the jets are produced at 90° to the beam direction. The subprocess cross section is therefore sampled in a kinematic region where low-order perturbation theory should be a very good approximation, and the behaviour of the cross section as the rapidity increases directly probes the small- x behaviour of the partons in the proton. In contrast, we are more interested here in the BFKL behaviour of the subprocess cross section itself (which is a small momentum-transfer process for our choice of kinematics). With $y_1 = 1$ and $y_2 = 1 - \Delta$ large and negative, both sets of parton distributions are evaluated at medium-to-large x values where they are relatively well known.

In terms of the rapidity difference Δ , the subprocess matrix elements in Eq. (2) are evaluated at

$$\frac{\hat{t}}{\hat{s}} = -\frac{1}{2} \left(1 - \tanh\left(\frac{1}{2}\Delta\right) \right), \quad \frac{\hat{u}}{\hat{s}} = -\frac{1}{2} \left(1 + \tanh\left(\frac{1}{2}\Delta\right) \right). \quad (5)$$

Thus $\Delta \rightarrow \infty$ corresponds to $\hat{t}/\hat{s} \rightarrow 0$, $\hat{u}/\hat{s} \rightarrow -1$. In this limit:

- (i) we can use the effective subprocess approximation [9], in which all parton subprocesses are taken into account by using the $gg \rightarrow gg$ matrix element and multiplying by the combination $G \equiv g + (4/9) \sum (q + \bar{q})$ of parton distributions,
- (ii) the subprocess cross section integrated over transverse momenta $p_T > P_T$ exhibits the *scaling* behaviour:

$$P_T^2 \hat{\sigma}(gg \rightarrow gg) \sim \left(\frac{\alpha_s C_A}{\pi} \right)^2 \frac{\pi^3}{2}. \quad (6)$$

When Δ is so large that $\alpha_s \Delta \sim O(1)$, higher-order contributions proportional to $\alpha_s^2 (\alpha_s \Delta)^n$ appear and must be resummed. This is achieved using the BFKL

equation, as described for example in Ref. [4]. Schematically,

$$\hat{\sigma}_{\text{BFKL}}(gg \rightarrow gg + X) \simeq \left(\frac{\alpha_s C_A}{\pi} \right)^2 \frac{\pi^3}{2P_T^2} \left[1 + \sum_{n \geq 1} a_n \left(\frac{\alpha_s C_A}{\pi} \Delta \right)^n + \dots \right], \quad (7)$$

where the $\dots$ on the right-hand side refers to corrections outside the leading logarithm approximation implicit in (7), i.e. terms of order $\alpha_s^n \Delta^{n-1}$, $\alpha_s^n \Delta^{n-2}$, $\dots$ and ‘power-correction’ terms suppressed by powers of $e^{-\Delta}$. The asymptotic behaviour of $\hat{\sigma}_{\text{BFKL}}$ as $\alpha_s \Delta \rightarrow \infty$ is [1, 2]

$$\hat{\sigma}_{\text{BFKL}} \sim \left(\frac{\alpha_s C_A}{\pi} \right)^2 \frac{\pi^2}{4P_T^2} \frac{1}{\sqrt{\frac{1}{2}\pi 7\zeta(3)t}} e^{4 \log 2 t}, \quad t = \alpha_s C_A \Delta / \pi, \quad (8)$$

i.e. approximately $e^{\lambda \Delta}$, with $\lambda = (\alpha_s / \pi) 4 C_A \log 2 \simeq 0.5$.

The soft gluons which generate the higher-order contributions shown in Eq. (7) take up only a small fraction of the total subprocess centre-of-mass energy [1]. The parton distributions are therefore evaluated at the same x values as they would be for the lowest-order $2 \rightarrow 2$ process. We can therefore write [4]

$$\begin{aligned} \left. \frac{d\sigma^{\text{res}}}{dy_1 dy_2} \right|_{y_2=y_1-\Delta} &\equiv \int_{P_T^2} dp_T^2 \left. \frac{d\sigma^{\text{res}}}{dy_1 dy_2 dp_T^2} \right|_{y_2=y_1-\Delta} \\ &\simeq \hat{\sigma}_{\text{BFKL}} \int_1^{X^{-2}} \frac{du^2}{u^4} [x_e f_{G/e}(x_e, \mu^2)]|_{x_e=uX\rho} [x_p f_{G/p}(x_p, \mu^2)]|_{x_p=uX/\rho}. \end{aligned} \quad (9)$$

with $\hat{\sigma}_{\text{BFKL}}$ given in Eq. (7), and

$$X = \frac{2P_T}{\sqrt{s}} \cosh \frac{\Delta}{2}, \quad \rho = \sqrt{\frac{E_p}{E_e}} e^{y_1 - \frac{1}{2}\Delta}. \quad (10)$$

Note that the corresponding $p\bar{p}$ collider result of Ref. [4] is obtained by setting $E_e = E_p$ and $y_1 = \frac{1}{2}\Delta$, i.e. $\rho = 1$. In the limit where $p_T \ll E_e, E_p$, one can obtain a further approximation by noting that the u integral in Eq.(9) is dominated by the contribution near $u \simeq 1$, and so

$$\begin{aligned} \int_1^{X^{-2}} \frac{du^2}{u^4} [x_e f_{G/e}(x_e, \mu^2)]|_{x_e=uX\rho} [x_p f_{G/p}(x_p, \mu^2)]|_{x_p=uX/\rho} \\ \simeq X_e f_{G/e}(X_e, \mu^2) X_p f_{G/p}(X_p, \mu^2), \end{aligned} \quad (11)$$

with

$$X_e = \frac{P_T}{2E_e} e^{y_1} (1 + e^{-\Delta}), \quad X_p = \frac{P_T}{2E_p} e^{y_1} (1 + e^{\Delta}). \quad (12)$$

For HERA, however, this approximation is not particularly good, and so in what follows we will use the expression in Eq. (9).

Figure 2 shows the two-jet inclusive cross section as a function of the rapidity difference Δ . For our standard set of parameters we take $E_e = 30$ GeV, $E_p = 820$ GeV, $y_1 = 1$ and $p_T > P_T = 5$ GeV/c. These are illustrative of the kinematical region in which our analysis is valid: in practice, the precise choice of jet cuts will be largely determined by the capabilities of the detectors. The proton parton distributions are the latest MRS(H) set [7], while for the photon we use the GRV (NLO) distributions of Ref. [10]. As we have already discussed, our results are essentially unchanged for other choices of photon distributions, apart from possible overall slight variations in normalization. Our scale choice is $\mu = P_T$ in both the parton distributions and strong coupling constants. The three curves in Fig. 2 correspond to:

- (i) (dotted line) the exact lowest-order $2 \rightarrow 2$ cross section defined in Eq. (2), integrated over $p_T > P_T$, with all the different subprocesses correctly included,
- (ii) (dashed line) the approximate scaling form of the lowest-order cross section, i.e. Eq. (9) with the subprocess cross section of Eq. (7) without the $n \geq 1$ terms in the series,
- (iii) (solid line) Eq. (9) with the all-orders cross section defined in Eq. (7), which incorporates the full BFKL behaviour.

We see that at large Δ the lowest-order cross section is indeed well-approximated in shape by the scaling form. The BFKL behaviour of the subprocess cross section is then reflected in an increase in normalization and a slower fall-off in rapidity, as the $e^{\lambda\Delta}$ behaviour partially compensates the decrease in the proton's parton distributions at large x_p . Figure 2 also shows the predictions for $P_T = 10$ GeV/c. The cross sections are now smaller, and the kinematic boundary is reached for smaller values of Δ . Nevertheless, the effect may still be observable.

There are two important questions which arise from the above discussion. First, in the range of Δ values accessible at HERA, does the BFKL leading-logarithm formalism accurately reflect the true behaviour of the theory? The answer to this depends on the size of the sub-leading corrections, which are largely unknown at present. There are preliminary indications from structure function measurements at HERA [11] that the effective value of λ is smaller than the canonical 0.5 estimate. The MRS(H) distributions, for example, fit the F_2 data with $\lambda = 0.3$ [7]. The difference could easily result from sub-leading effects, see for example [12]. In any case, we should regard the solid curve in Fig. 2 as a 'naive' BFKL prediction which serves as a useful starting point for comparisons with data.

The second important question is whether the parton-level predictions shown in Fig. 2, in particular the differences between the dashed and solid curves, survive a full hadronization and detector simulation. At sufficiently large Δ , the backward

moving jet overlaps with the spectator jet from the incoming proton, and is undetectable. The ability of the detectors to identify and measure forward and backward jets will therefore be a crucial factor. It will therefore be important to perform a more complete Monte Carlo simulation incorporating the BFKL behaviour.

Note, finally, that the scaling behaviour of the $gg \rightarrow gg$ subprocess cross section at large Δ shown in Eq. (6) is characteristic of massless spin-one t -channel exchange. The corresponding direct photon subprocess $\gamma g \rightarrow q\bar{q}$, with t -channel quark exchange, is heavily suppressed in this limit:

$$\frac{\hat{\sigma}(\gamma g \rightarrow q\bar{q})}{\hat{\sigma}(gg \rightarrow gg)} \sim e_q^2 \frac{\alpha}{\alpha_s} \frac{2}{9} e^{-\Delta}. \quad (13)$$

3 Correlations in Azimuthal Angle

We turn now to the azimuthal angle difference $\Delta\phi = \phi_1 - \phi_2$ between the two jets. For convenience, we define $\phi = \pi - \Delta\phi$, so that $\phi = 0$ corresponds to jets which are back-to-back in the transverse plane. It is straightforward to extend the BFKL analysis to include the ϕ distribution [4, 5]. The result is

$$\frac{1}{\sigma^{\text{res}}} \frac{d\sigma^{\text{res}}}{d\phi} \simeq F(\phi, \Delta), \quad (14)$$

where σ^{res} denotes the cross section of Eq. (9), and

$$F(\phi, \Delta) = \delta(\phi) + \sum_{n \geq 1} f_n(\phi) \left(\frac{\alpha_s C_A}{\pi} \Delta \right)^n. \quad (15)$$

Expressions for the functions $f_n(\phi)$ can be found in Ref. [4]. Note that in the absence of gluon emission, the two jets are back-to-back as expected. In the limit $\alpha_s \Delta \rightarrow \infty$, we have [4]

$$F(\phi, \Delta) \sim \frac{1}{2\pi} \left[\frac{1}{\sqrt{\frac{1}{2}\pi 7\zeta(3)t}} e^{4\log 2 t} + \frac{2 \cos \phi}{\sqrt{\frac{1}{2}\pi \zeta(3)t}} + \frac{2 \cos 2\phi}{\sqrt{\frac{1}{2}\pi (7\zeta(3) - 8)t}} e^{4(\log 2 - 1)t} + \dots \right], \quad (16)$$

with $t = \alpha_s C_A \Delta / \pi$. Asymptotically, the ϕ distribution becomes *flat*: the emission of an infinite number of soft gluons completely smears out the back-to-back correlation exhibited by the lowest contributions to the perturbation series.

Figure 3 shows the ϕ distributions for various Δ , with the same parameters as Fig. 2. The distributions are normalized to have unit area for each value of Δ . The transition from a distribution strongly peaked at $\phi = 0^\circ$ for small Δ , to a flat

distribution at large Δ , is evident. If, as seems possible from Fig. 2, Δ values as high as 4 are accessible at HERA, then this should be large enough to allow the flattening effect to be observed. Larger values of Δ can of course be probed at the 1.8 TeV $p\bar{p}$ collider.

The weakening of the azimuthal correlation is encapsulated in the behaviour of the average $\langle \cos \phi \rangle$ as a function of Δ , shown in Fig. 4. In this figure we have superimposed (dashed line) the result obtained when the perturbation series in Eq. (15) is truncated at next-to-leading order ($n = 1$), i.e.

$$\langle \cos \phi \rangle_{\text{NLO}} = 1 + \frac{\alpha_s C_A}{\pi} \Delta \int_{-\pi}^{+\pi} d\phi \cos \phi f_1(\phi) = 1 + \frac{\alpha_s C_A}{\pi} 4(\log 2 - 1) \Delta, \quad (17)$$

taken from Ref. [4]. This shows that the higher-order perturbative contributions are already becoming important at $\Delta \approx 3$. Note that since $a_1 = \int d\phi f_1(\phi) = 0$ [4], the effects of multigluon emission are felt earlier in the ϕ distribution than in the total ϕ -integrated cross section.

Finally, it is important to consider also *non-perturbative* mechanisms which could give rise to a decorrelation in the jets' momenta. In Ref. [4], we considered a multi-peripheral model in which a large number of soft clusters or particles, ordered in rapidity and with limited transverse momenta, are emitted in the rapidity gap between the two primary jets. Such a model could be considered as the small P_T limit of the perturbative effects discussed above. We showed how the rate of decrease in azimuthal correlation depended on the density of clusters emitted per unit rapidity interval, and was in general much more rapid than the perturbative BFKL decorrelation.

4 Conclusions

The HERA ep collider can probe the QCD ‘perturbative pomeron’ in several ways. The paradigm is the rise at small x of deep inelastic structure functions. However the inclusive nature of such quantities means that perturbative and non-perturbative effects are very likely intertwined in a non-trivial way, and therefore obtaining information on the former requires some sort of model-dependent subtraction of the latter. This problem can be resolved in the context of deep inelastic scattering, at least in principle, by requiring an associated jet, with longitudinal momentum fraction $x' \gg x_{\text{Bj}}$ and comparable transverse momentum to the virtual photon momentum Q [13]. The BFKL behaviour is then reflected in the growth of the structure function with $\log(x'/x_{\text{Bj}})$.

A more direct method is to study the two-jet inclusive cross section in parton-parton scattering directly, as advocated in Ref. [1]. In this study we have used the ‘resolved photon’ contribution to the photoproduction of jet pairs at HERA to probe perturbative pomeron physics. The particular asymmetric kinematics at HERA

suggest a configuration where one minijet is produced at moderate forward rapidity, while a second is produced at large backward rapidity. We have shown how the dependence of the cross section on the rapidity difference and, especially, on the jet azimuthal angle difference is sensitive to the multigluon emission characteristic of the perturbative pomeron.

While we have demonstrated that the effects corresponding to the leading behaviour are almost certainly measureable at the parton level, it will be vitally important to confirm that these survive the inclusion of sub-leading contributions, and also to perform more realistic Monte Carlo simulations to quantify the effects of hadronization, jet reconstruction, and detector effects.

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Figure Captions

- [1] Parton momentum fractions in the proton (x_p) and in the electron (x_e) for the $2 \rightarrow 2$ jet cross section. The jets have $p_T = 5$ GeV/c and rapidities $y_1 = 1$ and $y_2 = 1 - \Delta$.
- [2] The two-jet inclusive cross section $d\sigma/dy_1 dy_2(y_1 = 1, y_2 = 1 - \Delta, p_{T1}, p_{T2} > 5$ GeV/c) for ep collisions at $\sqrt{s} = 314$ GeV as a function of Δ : (i) (dotted line) the lowest-order $2 \rightarrow 2$ cross section defined in Eq. (2), (ii) (dashed line) the approximate scaling form of the lowest-order cross section, i.e. Eq. (7) without the $n \geq 1$ terms in the series, and (iii) (solid line) the approximate all-orders cross section defined in Eq. (7). Parameter values and parton distributions as described in the text.
- [3] The $\phi = \pi - \Delta\phi$ distributions normalized to unit area, at fixed Δ with $y_1 = 1$, $y_2 = 1 - \Delta$ and $p_T > 5$ GeV/c, for ep collisions at $\sqrt{s} = 314$ GeV. The parton distributions are the same as in Fig. 2.
- [4] The all-orders BFKL prediction for the average azimuthal angle difference $\langle \cos \phi \rangle = -\langle \cos \Delta\phi \rangle$ as a function of Δ , for the same parameters and parton distributions as in Fig. 3. Also shown (dashed line) is the next-to-leading order approximation.

Fig. 1

Fig. 2

Fig. 3

Fig. 4