THE MEASUREMENT PROBLEM IN QUANTUM MECHANICS

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ABSTRACT

We investigate the measurement problem in quantum mechanics by examining the measuring process in terms of the Schroedinger equation and find that it is important to divide the process into three steps: i) Formation of channel, ii) The Schroedinger time development, and iii) Registration. The uncontrollable interaction (we shall call it *Bohr action*) between object and measuring apparatus which had often been emphasized by Bohr, is found to play an indispensable role in the measurement process, to which the substantial attention has scarcely been pointed so far. By referring to some Bohr's sentences in his debate with Einstein, we try to clarify and present a novel view on the measurement problem in quantum mechanics.

1. INTRODUCTION

In this paper we shall offer a summarized and more elaborated version of our previous works⁽¹⁾ on the measurement problem in quantum mechanics, where the measurement process is followed in terms of the Schroedinger equation for the probability amplitude (the wave function), which is assumed to represent the state of an individual quantum object, not of an ensemble. The key problem in the measurement process is how to take into account the uncontrollable interaction between the object and the measurement apparatus (we call this *Bohr action*) whose existence had been often emphasized by Bohr in his lecture of the measurement, and may be regarded as the central subject of the debate between Einstein and Bohr⁽²⁾.

Now, the paper is organized as follows. In Sec.2 we shall take up the Stern-Gerlach type gedanken experiment to show a general outline of an individual measurement process concretely and to introduce the *Bohr action*. In Sec.3 the conditions of the *Bohr action* and its role in the measuring process are given, and in Sec.4 some examples of measurements including the Einstein-Podolsky-Rosen problem are shown for the illustration of the measurement process in terms of the *Bohr action*. In Sec.5 Neumann's theory of measurement is studied in our viewpoint, and the final section will be devoted to the supplementary discussion for the measurement problem.

2. STERN-GERLACH TYPE EXPERIMENT AND THE BOHR ACTION

Before entering the subject, the premises (see also paper I,II)⁽¹⁾ for our argument are:

(1) Any state of a microscopic system is presented by a vector in

the Hilbert space and any linear combination of the states represents also a possible state of the system.

(2) When any measurement of a physical quantity Q of a system indicates definitely a value q_i , the system is regarded to be in the eigenstate $|u_i\rangle$ in which $Q|u_i\rangle = q_i|u_i\rangle$ holds, and vice versa.

(3) Any detector causes some kind of irreversible change, when it interacts with a microscopic system to be measured and the time development of the system is interrupted.

(4) When detectors are arranged so as to correspond to the spectral decomposition of the eigenvalues of a physical quantity to be measured, only one of detectors reacts in the individual measurement, and two or more detectors never act at the same time.

(5) The time development of the state obeys the Schroedinger equation, up to the encounter with a detector which cause an irreversible change.

Now, by taking up such type of gedanken experiment that the eigenvalue of measuring quantity is discrete, we can study a characteristic behavior of the wave function in an individual measurement, from which we will be informed the intervention of a novel interaction, and led to a new concept, *Bohr action*.

Let us measure Z -component of the spin of a quantum mechanical particle with spin $\frac{1}{2}$ by the Stern-Gerlach type gedanken experiment. We, experimentalists set two different detectors, up- and down-detectors, correspondingly to the different eigenvalues of spin S_z up and down, according to classical theory.

If the initial state of the particle is an eigenstate of the measuring quantity S_z , then the measurement apparatus does not

change the eigenstate in the time development (at least up to the interruption by the corresponding detector) and the probability amplitude moves completely corresponding to the movement of the classical particle. (And the object arrives at the corresponding detector, where the irreversible amplification process is activated by the triggering of the object, giving the corresponding eigenvalues). Therefore we can say that the channels leading to the respective detectors (up- or down-channel) are formed by setting the measurement apparatus which corresponds to classical theory. Since we can transfer the Hamiltonian and boundary conditions from classical mechanics to quantum mechanics as it is, so far the situation is completely subjected to the control of experimentalists' ability. (The efficiency of the measurement apparatus is supposed to be 100% for simplicity's sake.)

A special attention must be paid when the initial state is not in an eigenstate. Each observation gives also one of eigenvalues because of the perfect efficiency of measuring apparatus. In this case too, we can say that the channels leading to the respective detectors are formed and only one of them is triggered resulting in an irreversible amplification process [premise (4)]. If the observation belongs to the 1st kind, the corresponding eigenstate is realized after the measurement. Since the change of the wave function in this measuring process such as from a superposed state to an eigenstate conserves the probability, this change may be described by an unitary transformation, or guided by the Schroedinger equation (we shall later refer to this point again). Here we shall see a peculiar behavior of the wave function in an individual measurement. One of the

problem is, why it is possible to lead a superposed state to one of the eigenstates by the same measurement apparatus (as the case when the initial state is one of the eigenstate). Is it possible for the different initial states to reach the same final state by the same Schrodinger equation? We think that this is the very point of the uncontrollable interaction foreign to classical physics often described by Bohr⁽²⁾. A superposed state is an eigenstate of another quantity A (e.g., S_x in our case) which does not commute with S_z , $[A, S_z] \neq 0$. Therefore our measurement forces an eigenstate of A to change into one of the eigenstates of S_z and we usually experience that such experiments are often performed. The situation is regarded as just the expression of the uncertainty relation found in the dynamical process. We think that the intervention of the uncontrollable interaction between an object and its measuring apparatus in addition to the Hamiltonian fixed by the apparatus causes just this situation. How is this interaction expressed? Let us cite Bohr.

After investigating an example of a two particle system, which generally would be called EPR-type experiment later, Bohr disagreed with the wording of the criterion of physical reality proposed by Einstein, Podolsky and Rosen, as that there contains an essential ambiguity regarding to the meaning of the expression "without in any way disturbing a system". And he said⁽³⁾ "Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an influence on the very

conditions which define the possible types of predictions regarding the future behavior of the system." (Underlines are due to the present authors.) We understand the sentence as follows: By changing the experimental setting to one object, there is no mechanical disturbance, i.e., no interaction in classical meaning to the other object but there is an influence regarding the future behavior of the system, i.e., an effect upon the development of the probability amplitude. Here we should notice that this influence is not peculiar to the EPR-type experiment but generally results from a setting of a measurement apparatus as shown in detail in paper II. Any mechanical interaction can be expressed by the corresponding Hamiltonian, and we think that the influence in this issue cannot be described by classical theory and therefore by a concrete Hamiltonian in general, but can affect yet the behavior of the probability amplitude (we call this *Bohr action*). By the very existence of this *Bohr action* which cannot be expressed in a concrete Hamiltonian and therefore cannot be controlled, the fluctuations, i.e., the probabilistic character happens in every individual measurement. The readers may imagine here the origin of the noncausal and irreversible change in the Copenhagen interpretation, our view on which will be gradually clear in the followings.

3. THE CONDITIONS FOR THE BOHR ACTION AND THE MEASURING PROCESS

It is well known that the phenomena in the microscopic world are well described by the Schrodinger equation of a concrete Hamiltonian. If the *Bohr action* appears at random, we could not expect the success of quantum mechanics. Then what are the objective conditions for the

appearance of the *Bohr action*? Let us consider a measurement of an observable A with discrete eigenvalues. If the initial state of the object is in an eigenstate of an observable B commutable with A, it is clear that the *Bohr action* does not appear as is known from the argument in the preceding section. The problem arises when B is not commutable with A and in this case the initial state $|\Psi\rangle$ is generally described by a superposition of eigenstates $|u_n\rangle$ of A.

$$|\Psi\rangle = \sum_n c_n |u_n\rangle \quad (1)$$

In the measurement process, $|u_n\rangle$ are the eigenstates in the respective channels which lead the object to the corresponding detectors which distinguish eigenvalues of A each other. But this type of expansion is always possible even if the detectors are absent, and in fact if absent, the state of the object continues the Schrodinger time development (described by the temporal change of the coefficients c_n) by the original Hamiltonian without the detectors.

Then it is clear that the key point is the presence of detectors. If the detector efficiency is 100%, the state is forced to converge to one of $|u_n\rangle$. This is due to the very appearance of the *Bohr action*. If the detectors do not cover all the eigenvalues, then not only the channels covered but also the uncovered are respectively distinguished and one of them is forced to be realized by the *Bohr action*. Here we should notice that the selection of an uncovered channel (negative channel) may present a typical example of the negative result measurement, and also offers a means of preparation of the initial object state for an execution of experiment. In any case, the

objective conditions for the appearance of the *Bohr action* seem to become clear. That is, the presence of detectors, which act to stop the Schrodinger time development of the state (we call this process the registration), and that the initial state of the object is not in an eigenstate of an observable to be measured by this detector system.

Now by taking account of the *Bohr action*, we can clarify in more detail the measuring process which was assumed in paper II to consist of the three steps as:

- i) Formation (Selection or preparation) of channel⁴⁾
- ii) The Schrodinger time development
- iii) Registration.

First by setting the measuring equipments which specify the Hamiltonian and boundary condition, we can construct the Hilbert space = the arena for the object whose state is traced by the corresponding Schrodinger equation, and the state is described by a superposition of states of the eigen-channels each subjected to the corresponding detectors. The *Bohr action* does not change the Hamiltonian and the boundary conditions, but within the arena it functions just for the Selection (formation) of a channel and for the modification of the state development in the channel. So, we may expect that a continuous development of the amplitude like as described by the Schrodinger equation still holds in the selected channel even under the intervention of the *Bohr action*. Our reasoning is strengthened by the fact that the Schrodinger equation is found to firmly describe the behavior of the amplitude after a negative channel is selected (by the *Bohr action*), as is well known in the negative result measurement. Therefore we are rather able to know

that the *Bohr action* does not contribute to the Registration process.

About the Registration as the final step of measuring process, Bohr said in 1958⁽⁵⁾ "... it is also essential to remember that all unambiguous information concerning atomic objects is described from the permanent marks --- such as a spot on a photographic plate, caused by the impact of an electron --- left on the bodies which define the experimental conditions. Far from involving any special intricacy, the irreversible amplification effects on which the recording of the presence of atomic objects rests rather remind us of the essential irreversibility inherent in the very concept of observation. The description of atomic phenomena has in these respects a perfectly objective character, in the sense that no explicit reference is made to any individual observer and that therefore, ... no ambiguity is involved in the communication of information". We completely agree with this idea and nothing is necessary to supplement the idea for the Registration process previously proposed by us⁽¹⁾.

We should like to add a few remarks further on the *Bohr action*. Experimentalists can prepare the initial state of an object in an eigenstate of an observable. They can also shift this observable to any extent from the observable to be measured. Therefore it is possible to control the effect of the *Bohr action* in some extent, but essentially probabilistically. To make an outlook for the measuring process, we present some examples in the following section.

4. SOME EXAMPLES

Let us begin with the simple case of a particle passing through a slit in a diaphragm, which make a diffraction pattern on the screen

behind the diaphragm as shown in Fig.1. The diaphragm is considered to be a detector for the particle, because a particle impinging on the diaphragm is absorbed there. Assuming that the initial state $|\Psi\rangle$ of the particle is an eigenstate of momentum P (we take this direction Z axis) perpendicular to the diaphragm, we can expand this state by the spherical waves using polar coordinates whose origin is a proper point before the diaphragm.

$$|\Psi\rangle \sim \int \exp(iPz/\hbar) |r\rangle dr \sim \sum f(\theta_n, \varphi_n) |\Omega_n\rangle \Delta\Omega_n \quad (2)$$

Here for simplicity's sake, we took the solid angle discrete and expanded the state by the eigenstates $|\Omega_n\rangle$ whose eigenvalues are the directions $\Omega_n = \Omega(\theta_n, \varphi_n)$.

Fig.1

Now, the states whose paths impinge to the diaphragm are absorbed at the diaphragm, i.e., the diaphragm plays the role of detector. Therefore the Bohr action appears and in the case that the channel leading to the slit is accidentally selected, the state (2) of the object will be developed going through the slit. The state after going out of the slit can be again expanded like (2). Then each point on the screen is also a detector and the probability amplitude will reduce on a point under the *Bohr action*.

Next, we consider the photo-electric effect experiment. As shown in Fig.2, the radiated light from the left hand side enters the metal target and an emitted electron is measured by the detector D. The light is assumed to enter through the slit S whose section is far larger than the dimension of an atom. There are a huge number of channels leading to the various atoms in the metal target, including the channels in which the light is simply reflected or absorbed by an

atom resulting in emission of an electron (the photo-electric effect). Furthermore, in other cases, the energy of the absorbed light may diffuse and change to thermal motion in the metal. Let us investigate these circumstances in more detail.

Fig.2

We have no super selection rule for a photon like an electron (e.g., the particle number conservation), but we may imagine the path of a photon (particle) in classical picture. We also suppose that there is a set of observables characterizing these paths and that the state of the photon can be expressed by the expansion of the eigen-waves corresponding to the set of the observables. If the metal target has a function that irreversibly transforms the photon energy to thermal motion, the Schroedinger development is interrupted in the channel. Therefore the metal target plays a role of detector and it causes an intervention of the *Bohr action* and the Selection process follows. If a channel leading to a thermal motion in the metal target is selected, the Schroedinger development of the state is stopped there. Otherwise it will continue as a superposition of states with regard to all the other channels (see the argument of the negative result measurement in ref.(1)). Now, if there is no more channel interrupting this progress, it terminates at an electron detector D and settles the matter of the photo-electric effect.

Finally we shall discuss the EPR-problem by taking into consideration the decay of π^0 meson into $e^+ e^-$ pair. The scenario is the same as described in paper II other than a reference to the *Bohr action*. The state of $e^+ e^-$ pair (its spatial wave function are

$u(r_1)_+ u(r_2)_-$ before the measurement is

$$(|\uparrow_+ \downarrow_- \rangle - |\downarrow_+ \uparrow_- \rangle) u(r_1)_+ u(r_2)_- \quad (3)$$

where + or - means e^+ or e^- and $\uparrow$ or $\downarrow$ means spin up or down which we want to measure. By setting the apparatus with detectors recording the electron's spin at r_2 , either up- or down-channel discriminated, and chosen by the *Bohr action*. If the up-channel is chosen, the amplitude $|\uparrow_+ \downarrow_- \rangle$ gradually disappears, enhancing $|\downarrow_+ \uparrow_- \rangle$ to unity also due to the *Bohr action*. When the electron is guided to the detector and activates an irreversible change in it, the condition of the state $|\uparrow_- \rangle$ will be lost. But the very progress of affair confirms that the electron state was of up spin, and then the state correlation between the electron and positron such as $|\downarrow_+ \uparrow_- \rangle$ is lost, unless any device to hold the correlation is combined.

Dissimilarly to our scenario, the prevailed understanding may be as follows: if we get the electron's spin up, then the spin of the positron at the place r_1 , far from r_2 , is suddenly decided to be 'down' just at the moment. In our case there is no sudden change because of the Schroedinger time development until the activation of irreversible process in the detector. At this final stage (Registratjion), the condition of the electron's spin state $|\uparrow_- \rangle$ is lost because of a phase transition (stopping of the Schroedinger equation). Notice that the correlation $|\downarrow_+ \uparrow_- \rangle$ is unilaterally violated without the response of the other side (no faster-than-light communication).

If we want to measure the spin correlation, we have to set some proper apparatuses at both places (r_1 and r_2) including a coincidence

circuit⁶. In this case the channels are more clear, one is so to speak, {e⁺up ; e⁻down}-channel and the other {e⁻down ; e⁺up}-channel. The object system (e⁺ and e⁻) develops by the Schroedinger equation (until the corresponding detector's trigger) in one of the channels under the *Bohr action*. This circumstance of the correlation is just the same as the momentum correlation between two classical particles imposed beforehand by its conservation law and so in principle we also have no means of faster-than-light communication by the EPR-type experiment.

5. NEUMANN'S THEORY OF MEASUREMENT

In 1937 Bohr visited Japan. Sakata⁷ attended Bohr's lecture and sent a post card to Taketani informing him " . . . as Bohr told that Neumann's argument of measurement is very dangerous, the debate has been raised by Tomiyama and others. But the discussion resulted in a fade-out." Y.Fujioka also wrote⁸ about this issue "Bohr told that Neumann's mathematical description of measurement contains some anxiety of danger because the phenomena and the measurement apparatus are difficult to separate". In any case Bohr had firmly complained of Neumann's argument of measurement. What is it concretely ?

Neumann says⁹ that we must always divide the world into two parts, the observed system and the observer, and that the measurement is a related process of the subjective perception of the observer. What Neumann intended to show in his theory, is that the boundary between the observed system and the observer can be displaced arbitrarily for the so-called principle of the psycho-physical

parallelism, which is considered a fundamental requirement of the scientific viewpoint by Neumann. However we think that the measurement process concerned should be defined independently of the subjective perception (see Bohr's quotation in Sec.3).

Now let us see Neumann's treatment in more detail. He describes the interaction between the object and the apparatus as follows. If an object (system I) in a superposed state $|\Psi\rangle$ of some eigenstates $|u_n\rangle$ of an observable to be measured

$$|\Psi\rangle = \sum c_n |u_n\rangle \quad (4)$$

is combined with the apparatus (system II) in a state $|\Phi_0\rangle$, then the state of the total system (I + II) changes like

$$\sum c_n |u_n\rangle |\Phi_0\rangle \longrightarrow \sum c_n |u_n\rangle |\Phi_n\rangle \quad (5)$$

by the Schroedinger equation. This change is considered to be required because the information of the object (system I) must be faithfully transferred to the apparatus (system II). Thus if we combine the second apparatus (system III) further, the information of the object is successively transferred to these apparatuses in a superposed state like (5). From our point of view, the story is rather opposite. By setting an apparatus, the Schroedinger equation (the Hamiltonian and boundary condition) is fixed and the behaviour of the object is essentially regulated by the apparatus. And in the measuring process, the object merely acts as triggering an irreversible process of the detector corresponding to a special eigenvalue of the observable to be measured (registration). Neumann's change (5) is equivalent to a special case of quantum mechanical combination of two microscopic objects and therefore according to our idea, an actual measurement is never attained no matter how many such

combinations are repeated.

Now where does the discrimination between the object and the apparatus lie? In our argument the spatial discrimination is clear, and the temporal boundary is the point. In fact the state of the object is described by the wave function, while the apparatus fixes the Hamiltonian and boundary conditions in the Schroedinger equation. We think the temporal boundary is the time when a detector is activated (an irreversible change happens). You might think it is the time when the *Bohr action* appears (of course maybe non-local time), but it is not appropriate. Because, as shown in the negative result measurement, there are cases in which the probability waves are continued under the Schroedinger equation even after the *Bohr actions* are activated. And also because if the initial state of the object is an eigenstate of the observable to be measured, then the object is directly brought to the detector (corresponding to the eigenvalue) without the *Bohr action*. It is the temporal boundary when the registration process happens and when the phenomena becomes objective in the sense of Bohr. About the spatial boundary, Bohr said⁽¹⁰⁾ after explaining the discrimination and the individuality between the object and the apparatus (Sec.2) "... in each experimental arrangement and measuring procedure we have only a free choice of this place (i.e., this boundary) within a region where the quantum-mechanical description of the process concerned is effectively equivalent with the classical description". We understand this as follows. If the quantum mechanical description of an object is equivalent to the classical mechanical one, i.e., the measurement is carried out in a chain of eigenstates (because a

superposed state cannot exist in classical mechanics and has no equivalent description), then just within this region, we can freely move this boundary (i.e., the place inserting the detector).

Finally let us examine the question of "hidden parameters" argued by Neumann⁽¹¹⁾ who insisted that it is not because of "hidden parameters" that quantum phenomena are described probabilistically. We again consider the Stern-Gerlach type experiment. Assuming that the initial state of an object is the eigenstate $\sigma_x = +1$ of the Pauli spin matrix, we measure the value of σ_z . In this case, the *Bohr action* appears because of $[\sigma_x, \sigma_z] \neq 0$ and even if the measurements are repeated under the same condition, the measurement value fluctuates each time. The ensemble of the objects regarding the measuring value for the repeated measurements is divided into two ensembles according to $\sigma_z = +1$ or -1 . From the theory of "hidden parameter", each measurement is completely deterministically but it looks probabilistic because we have overlooked some physical quantities ("hidden parameters"). Next, take the ensemble of $\sigma_z = +1$, and measure σ_x . In this ensemble all members should have $\sigma_x = +1$, because such classical observation does not change the measuring value that the "hidden parameter" theory presumes. However again, by the *Bohr action*, the measurement values fluctuate and the existence of "hidden parameters" results in the contradiction. Thus our conclusion is essentially the same as Neumann.

6. CONCLUDING REMARKS

We think that our argument of measurement in quantum mechanics follows Bohr's interpretation in the essential points, while Bohr's

argument itself seems to be not always obvious. One of the reasons for obscurity in his lecture may be thought to be stemmed mainly from the non-use of the concept "state" and the Schroedinger equation. In fact he had tried to explain the novel feature of behavior in quantum objects by only classical language and to make clear the uncertainty relation foreign to classical physics in the measurement of two quantities conjugate each other, where he could not but introduce the uncontrollable interaction (Bohr action) between the object and the measuring apparatus in the observation.

He had shown the consistency of the uncertainty relation imposed on measured values of conjugate variables succeeding under a canonical transformation⁽²⁾, but he had not made clear how the *Bohr action* functions dynamically in the measurement process. The circumstances are naturally expected because of the lack of the concept "state" and its Schroedinger development in his lecture. The *Bohr action* appears very peculiar and may look even arbitrary in its function at the measurement. In fact Einstein had not accepted the existence of the *Bohr action* to the last as "It seems hard to look in God's cards. But I cannot for a moment believe that He plays dice and makes use of 'telepathic' means (as the current quantum theory alleges He does)"⁽¹²⁾. But when we follow the development of the state in the measurement concretely under the premise that the wave (state) function reflects the behavior of an object objectively and faithfully, we cannot disregard a certain (*Bohr*) *action* besides the conventional interaction working in the measurement process, as described in the argument in Sec.2 and 3, however strange it looks and may need a further examination.

In our argument we have not assumed the probability law in quantum mechanics. Hartle⁽¹³⁾ derived it on the basis of the premises (1) and (2) in Sec.2 of this paper, which was already referred to in papers I and II. However there is a great difference in the idea of the state function between us and Hartle who regards the wave function not to be an objective property for an object. We think a further examination is needed on the probabilistic nature in quantum mechanics together with the registration process as the irreversible one which we have not entered in detail in this paper. And behind the registration process, the same problem seems to lie in the transformation from the Newtonian mechanics to the statistical mechanics.

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Fig.1

Fig.2