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Relativistic and Gauge Independent Off-shell Corrections to the Toponium Decay Width

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Abstract

Off-shell and relativistic bound-state corrections for the decay $t \rightarrow b + W$ are calculated to $O(\alpha_s^2)$ making full use of the Bethe-Salpeter formalism for weakly bound systems. Thus we are able to take into account all terms to that order in a systematic and straightforward manner. One of the previously not considered contributions cancels precisely gauge dependent terms which appeared in an earlier off-shell calculation. Important cancellations also determine the gauge-independent part.

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1 Introduction

The analysis of radiative corrections in the Standard Model now predicts for the mass of the top quark the range 140-180 GeV . As noted already some time ago, such a superheavy quark possesses unique properties [1, 2]. Due to the fast electroweak decay $t \rightarrow b + W$ with the decay width Γ_0 , the time scale Λ_{QCD}^{-1} necessary to allow strong interactions to become effective and to produce e.g. hadrons with open top cannot compete with the decay time Γ_0^{-1} . In this situation "pure" quantum chromodynamic (QCD) perturbation theory is sufficient for a complete description of a system with strong interactions. This applies as well to Coulombic QCD bound-state effects in toponium although also here the hydrogen-like levels are strongly smeared out by Γ_0 . Nevertheless, the computation of those levels represents a necessary first step in this context [3]. After all, this is the first instance where the application of quantum field theory for weakly bound nonabelian systems is justified because even nonperturbative effects can be taken into account in an equally well defined perturbative manner [4].

As proposed first in ref. [5] the bound state effects for the inclusive reaction $\sigma_{e^+e^- \rightarrow anything}$ near the $t\bar{t}$ threshold could be taken into account by including the width Γ_0 for a freely decaying top quark [2, 6] in the absorptive part of the vacuum polarization, through the replacement of the total energy E by $E + i\Gamma_0$. Subsequently this approach was applied in other detailed numerical studies involving potentials with the running strong coupling constant for the short distance Coulomb term and also including a confining part [7]. However, as noted recently in [8], the introduction of a constant decay rate Γ_0 in the Breit-Wigner resonance part of this formalism leads to problems with unitarity. Therefore a "running" width $\Gamma(p^2)$ for a virtual top was proposed with $p^2 \neq m_t^2$. As expected, a naive off-shell extrapolation of Γ_0 leads to ambiguous and gauge dependent results. $\Gamma(p^2)$ was determined indirectly from the vacuum polarization function making certain approximations [8].

Qualitatively the main physical effects from t decaying in toponium seem to be quite well understood by now [7, 9]: On the one hand, Γ decreases by time dilatation and by reduction of phase space, caused by the decay below the mass shell. Cancellations should occur with an increase of Γ due to the Coulomb-enhancement induced by the Coulomb interaction of the relativistic b -quark, because this overlap effect is known to be important from the qualitatively similar muonium system [10]. The purpose of our present work is to show that existing quantum field theoretical technology, well tested e.g. in the (abelian) positronium case, may be used also here to solve the problem of a gauge-independent "running width", including the qualitative effects listed above. Even the Coulomb enhancement can be taken into account properly in this context, which to the best of our knowledge has not been possible before.

In sect.2 we collect some pertinent formulas from the BS formalism. More details on our conventions can be found in [3]. Sect. 3 describes the calculation of the bound-state correction to the free-toponium width. In the concluding sect. 4 the gauge independent

result is discussed.

2 BS-Perturbation theory

The Bethe-Salpeter (BS) approach for weakly bound systems starts from the BS-equation

$$G = D + DKG \quad (1)$$

for the $t\bar{t}$ Green function $G(P, k, k')$, where D is the product of the two (full) propagators and K represents the 2pi BS-kernel. All four point functions in (1) depend on the total momentum $P = (P_0, \vec{0})$; the ingoing (outgoing) lines carry momentum $P/2 \pm k$ ($P/2 \pm k'$).

Perturbation theory starts from an equation similar to eq. (1) with G_0 , D_0 and K_0 chosen in such a way that the exact solution is known. If D_0 consists of a the nonrelativistic top-propagators only and if K_0 is the Coulomb kernel the zero order equation corresponding to (1) (after integrating out k_0 and k'_0) simply reduces to the Schrödinger equation. Another very convenient zero order equation which already includes the relativistic free fermion propagators and still remains solvable has been found by Barbieri and Remiddi (BR) some time ago [11]. It is based on a suitable modification of the Coulomb kernel $K_C = -4\pi\bar{\alpha}(\vec{k} - \vec{k}')^{-2}$ to $(\bar{\alpha} = \frac{4}{3}\alpha_s, m = m_t, E_k = \sqrt{m^2 + \vec{k}^2})$

$$K_0 = K_{BR} \equiv [\gamma_0 \Lambda^+ \lambda^+ \Lambda^+] \otimes [\Lambda^- \lambda^- \Lambda^- \gamma_0] \tilde{K}, \quad (2)$$

with the relativistic and nonrelativistic projection operators

$$\begin{aligned} \Lambda^\pm(\vec{k}) &\equiv \frac{E_k + \vec{\alpha}\vec{k} \pm \beta m}{2E_k} \\ \lambda^\pm &\equiv \frac{1}{2}(1 \pm \gamma_0) \end{aligned} \quad (3)$$

and with $(\mu(k) = 2E_k/(E_k + m), \nu(k) = 2/\sqrt{P_0 + 2E_k})$

$$\tilde{K} \equiv m\mu(k)\nu(k)\mu(k')\nu(k')K_C. \quad (4)$$

The normalized wave functions for energy levels $P_0 = M_{n,0} = 2m\sqrt{1 - \sigma_n^2}$ read ($\omega = E_k - P_0/2 - i\epsilon, \sigma_n = \bar{\alpha}/2n$)

$$\chi(k, \epsilon) = \gamma_0 \bar{\chi}^*(k, -\epsilon) \gamma_0 = -i \frac{\Lambda^+ S \Lambda^-}{(k_0^2 - \omega^2)} \frac{\mu(k)}{\nu(k)} \frac{P_0 - 2E_k}{\sqrt{P_0}} \phi(\vec{k}). \quad (5)$$

The constant 4×4 matrix S describes the spin state of the $t\bar{t}$ system: $S = \gamma_5 \lambda^-$ (singlet), $S = \vec{a} \vec{\gamma} \lambda^-$ (triplet, $|\vec{a}| = 1$). $\phi(\vec{k})$ is the normalized Schrödinger wave function, depending on the usual quantum numbers n, l, m . The nonrelativistic limit of χ is

$$\chi_{nr}(k) = \frac{\sqrt{2}i\omega}{k_0^2 - \omega^2} S \phi(\vec{k}) \quad (6)$$

Near a bound state $M_{n,0}$ the Green function of the unperturbed (BR) equation behaves as

$$G_0(P, k, k') \approx \frac{\chi(k) \otimes \bar{\chi}(k')}{P_0 - M_{n,0}} \quad (7)$$

A generic perturbation H changes G_0 into

$$G = \sum_{\nu=0}^{\infty} G_0 (H G_0)^{\nu}. \quad (8)$$

From now on G, G_0 , etc denote Feynman amplitudes, therefore H differs from such an amplitude by a minus sign. It is straightforward to show that $H = -K + K_0 + iD^{-1} - iD_0^{-1}$ [3]. The l.h.s of eq. (8) looks like (7) with $M_{n,0}$ replaced by M_n and with perturbed wave functions.

Formally expanding both sides of eq. (8) near a certain level $M_{n,0}$

$$\begin{aligned} G_0 &= \sum_{m=0}^{\infty} g_m (P_0 - M_{n,0})^{m-1} \\ H &= \sum_{m=0}^{\infty} h_m (P_0 - M_{n,0})^m \end{aligned} \quad (9)$$

and using $g_0 = \chi \otimes \bar{\chi}$ allows the determination of the mass-shift $\Delta M = M_n - M_{n,0}$ to arbitrary order

$$\Delta M_n = \langle \langle h_0 \rangle \rangle + \langle \langle h_0 g_1 h_0 \rangle \rangle + \langle \langle h_0 \rangle \rangle \langle \langle h_1 \rangle \rangle + O(h^3). \quad (10)$$

A shorthand for BS expectation values has been used which incorporates four-integrations of the relative momenta ($\int dk = (2\pi)^{-4} \int d^4 k$):

$$\langle \langle a \rangle \rangle = \int dk dk' \bar{\chi}_{ij}(k) a_{ii'jj'}(k, k') \chi_{i'j'}(k'), \quad (11)$$

As already in eq. (1) etc., in composite expressions integrations and summations over internal variables are understood. i, j etc. represent spinor as well as color indices. A similar formula as eq. (10) holds for the perturbed wave functions [12].

The correction to the decay width is obtained from the imaginary part of the expectation values, receiving contributions from the graphs in fig.1 and fig.2. . It is important to note that eq. (10) is not a power series in α_s . In (10) it will be essential that $O(\alpha_s^2)$ corrections arise from all three terms denoted here if the BR wave-functions are used in lowest order. It should be emphasized that starting from the Schrödinger equation instead, the inclusion of the proper relativistic corrections to the free fermion propagators would have necessitated the inclusion of complicated higher order terms in h . We also anticipate that in the present application of this formalism for the imaginary part of (10), $\text{Im}\Delta M = -\Gamma/2$, renormalization effects need not be considered.

3 Bound state corrections

The graph fig.1 gives rise to the BS perturbation kernel

$$H^\Sigma = i[\Sigma(p) \otimes (\not{p}' - m) + (\not{p} - m) \otimes \Sigma(p')](2\pi)^4 \delta(k - k') \quad (12)$$

where $p = \frac{P_0}{2} + k$ and $-p' = \frac{P_0}{2} - k'$ denote the four momentum of the quark and the antiquark, respectively. The direct product refers to the $t \otimes \bar{t}$ spinor space. The factors $(\not{p} - m)$ and $(\not{p}' - m)$ compensate the superfluous propagator on the line without Σ .

Since we are calculating higher order effects we neglect the mass of the bottom quark although it may be included in principle. For the electroweak theory we use the R_ξ -gauge with the gauge fixing Lagrangian (M denotes the mass of the W-Boson)

$$\mathcal{L}_{gf} = -\xi |\partial^\mu W_\mu^+ - i \frac{M}{\xi} \phi^+|^2$$

because it eliminates mixed $W - \phi$ propagators. The gauge parameter ξ will not be fixed in the following. We obtain for the imaginary parts of Σ in (12) ($s = \sin \theta_W$, $\mathcal{P}_\pm = (1 \pm \gamma_5)/2$):

$$\text{Im}\Sigma_W = \frac{e^2}{s^2} \frac{\not{p}}{16\pi} \Theta(p^2 - M^2) [\mathcal{P}_+ A^{(W)} + \mathcal{P}_- A^{(\phi)}] \quad (13)$$

where

$$\begin{aligned} A^{(W)} &= \tau(p^2) + \rho(p^2, \xi) \\ A^{(\phi)} &= m^2 \left(\frac{\tau(p^2)}{2M^2} - \frac{\rho(p^2, \xi)}{p^2} \right) \\ \tau(p^2) &= \frac{(p^2 - M^2)^2}{2p^4}, \quad \rho(p^2, \xi) = \frac{1 - \xi}{2\xi} \left(1 - \frac{M^2}{2p^2} \frac{1 + \xi}{\xi} \right) \end{aligned} \quad (14)$$

In evaluating the expectation value according to the rule (11) we encounter for both terms the same trace (cf (3) and (5))

$$\begin{aligned} T &:= \frac{1}{2} \text{tr}[\gamma_0 \Lambda^- S^\dagger \Lambda^+ \gamma_0 \mathcal{P}_\pm \not{p} \Lambda^+ S \Lambda^- (\not{p} - m)] = \\ &= \frac{1}{2} (p_0 + \omega) \left[m \left(1 - \frac{\vec{p}^2}{m^2} \right) + (p_0 - \omega) \right] + O(\alpha_s^4). \end{aligned} \quad (15)$$

It is seen to gives rise to (spin independent) relativistic corrections of the same order as the off-shell ones from Σ . Inserting (14) into (13) we may expand in powers of $(p^2 - m^2)$, because terms of order $(p^2 - m^2)^2$ already yield negligible orders in α_s :

$$\begin{aligned} \text{Im}\Sigma^{(W)} &= \frac{e^2 m}{32\pi s^2} \left[\left(2 + \frac{m^2}{M^2} \right) \tau(m^2) + C \frac{p^2 - m^2}{m^2} \right] + O((p^2 - m^2)^2) \\ C &= \left(2 + \frac{m^2}{M^2} \right) \sqrt{2\tau(m^2)} \frac{M^2}{m^2} + 2\rho(m^2, \xi) \end{aligned} \quad (16)$$

Thus the total contribution of fig.1 to the decay width becomes ($h_0^\Sigma = H^\Sigma|_{P_0=M_{n,0}}$)

$$\begin{aligned}\Gamma_1 &= -2\text{Im}\langle\langle h_0^\Sigma \rangle\rangle = 2\langle(1 - \frac{5\vec{p}^2}{4m^2} - \frac{3}{4}\sigma_n^2)\text{Im}\Sigma\rangle = \\ &= \Gamma_0 - 2\Gamma_0\sigma_n^2 + \frac{e^2m}{16\pi s^2}C\langle\frac{p^2 - m^2}{m^2}\rangle,\end{aligned}\quad (17)$$

where Γ_0 is the well known zero order result which is twice the decay width of a free top quark [2, 5]. A factor 2 arises here and in the following, counting both self energy contributions of the t and $\bar{t}$ line:

$$\Gamma_0 = 2\text{Im}\Sigma^{(W)}(m) = \frac{e^2m}{16\pi s^2}(1 + \frac{m^2}{2M^2})\frac{(m^2 - M^2)^2}{m^4}\quad (18)$$

The expectation value $\langle\frac{p^2 - m^2}{m^2}\rangle$, left over after the k_0 integration is taken in the ordinary sense, i.e. between the (normalized) Coulomb wave functions $\phi_{nlm}(\vec{k})$ in (5).

In the graphs shown in fig. 2 to the required order it proves sufficient to evaluate the leading contribution by applying the nonrelativistic wave functions (6). After a straightforward but tedious calculation we obtain for the imaginary part of the vertex function $\Lambda_0^{(W)}$ ($\xi > M^2/m^2$ and $1 > M^2/m^2$):

$$\begin{aligned}\text{Im}\Lambda_0^{(W)} &= \frac{e^2}{32\pi s^2}\gamma_0 F \\ F &= (1 + \frac{m^2}{2M^2})(1 - \frac{M^2}{m^2})(1 + 3\frac{M^2}{m^2}) + 4\rho(m^2, \xi).\end{aligned}\quad (19)$$

In the BS expectation value corresponding to the first term on the r.h.s of (10) the k_0 integration to this order allows the simple replacement $k_0 \rightarrow -\omega$. With the spinor trace $(-\frac{1}{2})$ after including the nonrelativistic projectors in (6) we arrive with $\text{Im}\Lambda_0^{(W)}$ at $(\vec{q} = \vec{k} - \vec{k}')$

$$\Gamma_2 = \frac{e^2}{16\pi s^2}\langle\frac{4\pi\bar{\alpha}}{\vec{q}^2}\rangle F. \quad (20)$$

Remembering that $\vec{p} = O(\bar{\alpha})$, the last expectation value of eq. (17) can be rewritten

$$\langle\frac{p^2 - m^2}{m^2}\rangle \approx -2\langle\sigma^2 + \frac{\vec{p}^2}{m^2}\rangle = -\langle\frac{2}{m}\frac{4\pi\alpha}{\vec{q}^2}\rangle, \quad (21)$$

where the Schrödinger equation with reduced mass $m/2$ has been used in the last equality. We note already here that with eq. (21) the gauge dependent terms proportional $\rho(m^2, \xi)$ exactly cancel in (17) plus (20)!

So far only the first term in (10) has been considered. The second order terms in h for the leading correction $O(\alpha_s^2\alpha)$ can be summarized as

$$\Gamma_3 = -2\text{Im}[\langle\langle h_0^{QCD} g_1 h_0^\Sigma \rangle\rangle + \langle\langle h_0^\Sigma g_1 h_0^{QCD} \rangle\rangle] \quad (22)$$

$$\Gamma_4 = -2\text{Im}[\langle\langle h_0^{QCD} \rangle\rangle\langle\langle h_1^\Sigma \rangle\rangle + \langle\langle h_0^\Sigma \rangle\rangle\langle\langle h_1^{QCD} \rangle\rangle] \quad (23)$$

For g_1 (cf. eq. (9)) in (22) we find it convenient to write

$$g_1 := \left[G_0 - \frac{\bar{\chi} \otimes \chi}{P_0 - M_{n,0}} \right]_{P_0=M_{n,0}} \quad (24)$$

We first evaluate the contribution $\Gamma_3^{(1)}$ of the Green function G_0 expanded around $P_0 - M_n = 2m\sqrt{1-\sigma^2} - 2m\sqrt{1-\sigma_n^2} \approx m(\sigma_n - \sigma^2)$. For our purpose it suffices to calculate the leading nonrelativistic contribution since relativistic corrections are suppressed by an additional factor $O(\alpha_s^2)$. Therefore we can use the nonrelativistic Green function $G_{nr}(k'', k', \sigma)$, i.e. essentially Schwinger's solution $G_{nr}(\vec{k}'', \vec{k}', \sigma)$ to the Coulomb problem [13], supplemented by appropriate factors $(k_0^2 - \omega^2)^{-1}$ so as to include also the zero component of relative momentum [14, 15]. For $P_0 \neq M_{n,0}$ and thus $\sigma \neq \sigma_n$ the first term of eq. (24) yields a contribution (cf. (6))

$$\Gamma_3^{(1)} = -4 \int dk dk' dk'' \bar{\chi}_{nr}(k'')(k_0'' + \omega_n'')(i\text{Im}\Sigma) G_{nr}(k'', k', \sigma) h_0^{QCD}(k', k) \chi_{nr}(k) \quad (25)$$

where again $-(k_0'' + \omega_n'')$ compensates a superfluous nonrelativistic propagator in fig. 1. Carrying out the k_0 integrations leads to $(d\vec{k} := d^3k/(2\pi)^3 \text{ etc. })$

$$\Gamma_3^{(1)} = -2\Gamma_0 \int d\vec{k} d\vec{k}' d\vec{k}'' \Phi^*(\vec{k}'') [1 - \frac{m(\sigma^2 - \sigma_n^2)}{4\omega''}] G_{nr}(\vec{k}'', \vec{k}', \sigma) h_0^{QCD}(\vec{k}', \vec{k}) \Phi(\vec{k}), \quad (26)$$

if we use the fact that for h_0^{QCD} to $O(\bar{\alpha}^2)$ the QCD corrections in H from $K_C - K_{BR}$, Coulomb gluon exchange etc. are real and independent of k_0 .

The factor proportional to 1 from the square bracket in (26) exactly reproduces the second part of (24), to be subtracted out before the limit $\sigma \rightarrow \sigma_n$ is taken. The limit $(\sigma^2 - \sigma_n^2) \rightarrow 0$ just selects the corresponding pole term $\phi_n^* \phi_n [\frac{m}{2}(\sigma^2 - \sigma_n^2)]^{-1}$ in $G_{nr}(\vec{k}'', \vec{k}', \sigma)$. Thus from (22) only

$$\Gamma_3 = \frac{\Gamma_0}{2} \langle h_0^{QCD} \rangle \langle \frac{1}{\omega} \rangle \quad (27)$$

remains. $\langle h_0^{QCD} \rangle$ could be taken from refs. [3, 16]. Its explicit form will turn out not to be relevant here.

For the first term of (23) h_1^Σ , i.e. the derivative of (12) with respect to P_0 at $P_0 = M_{n,0}$ is needed. Again the factor $\langle h_0^{QCD} \rangle$ provides additional powers of α_s and hence allows using the mass shell condition $P_0 \rightarrow 2m$ in

$$h_1^\Sigma = (2\pi)^4 i \delta(k - k') [\gamma_0 \Sigma \otimes (\frac{1}{2} \gamma_0) + (\frac{1}{2} \gamma_0) \otimes \gamma_0 \Sigma]. \quad (28)$$

Thus from this term

$$\Gamma_4^{(1)} = -2\langle\langle h_0^{QCD}\rangle\rangle\text{Im}\langle\langle h_1^\Sigma\rangle\rangle \approx -\frac{\Gamma_0}{2}\langle h_0^{QCD}\rangle\langle\frac{1}{\omega}\rangle \quad (29)$$

follows.

In the second part of eq. (23) h_1^{QCD} , i.e. the first derivative of H^{QCD} , will only receive contributions from $H = -K_C + K_{BR}$ to $O(\alpha_s^2)$. The instantaneous Coulomb kernel K_C does not depend on P_0 , nor does the vacuum polarization [16] correction which had provided the leading contribution to $\langle h_0^{QCD}\rangle$ above:

$$\Gamma_4^{(2)} = -2[\text{Im}\langle\langle h_0^\Sigma\rangle\rangle]\langle\langle\frac{\partial}{\partial P_0}K_{BR}|_{P_0=M_{n,0}}\rangle\rangle. \quad (30)$$

Inserting the explicit (real) expression of K_{BR} , eq. (2), we obtain

$$\langle\langle\frac{\partial}{\partial P_0}K_{BR}\rangle\rangle = \int \frac{d^3k}{(2\pi)^3}|\Phi(\vec{k})|^2\frac{P_0 - 2E_k}{2P_0} \approx -\frac{\sigma_n^2}{2} \quad (31)$$

and thus

$$\Gamma_4^{(2)} = -\frac{1}{2}\Gamma_0\sigma_n^2. \quad (32)$$

4 Results and Conclusion

Before adding all contributions we note that precisely the contribution Γ_1 in (17) has been quoted already in ref. [8] as a result as a specific off-shell extrapolation of the free decay width. It was obtained there by the intuitive replacement $\not{p} \rightarrow \sqrt{p^2}$ in the on-shell expression. However adding Γ_2 of (20), with (21) *all* gauge dependent terms are found to cancel. Moreover a striking feature of the sum $\Gamma_1 + \Gamma_2$ is that even the *gauge-independent* terms precisely sum to zero as well, which is by no means "natural" in view of the explicit form of F and C , as compared to Γ_0 (18) occurring in the second term of (17). In a similar manner also $\Gamma_3 + \Gamma_4^{(1)}$ cancel each other (cf. eq. (27) and (29)), leaving only $\Gamma_4^{(2)}$ in the final result

$$\Gamma_{\text{boundstate}} = \Gamma_0(1 - \frac{1}{2}\sigma_n^2 + O(\alpha_s^3)). \quad (33)$$

Recalling that $1 - \sigma_n^2/2 \approx (1 - \langle\vec{k}^2/m^2\rangle)^{1/2}$, the residual effect may be interpreted as an approximation of the γ -factor for the time dilatation for the weakly bound top. The origin of that term, the relativistically generalized Coulomb-kernel K_{BR} gives some credit to this interpretation. For the complete cancellation of the other terms we confess to have not found as yet a convincing explanation. It maybe truly an "accident" to this order because in the abelian case of muonium only a partial cancellation is known to occur, with the (higher order) vacuum polarization with an fermion loop replacing the W-propagator [10].

Among the cancelled terms the other expected effects can be more or less isolated: Γ_1 (fig.1) includes the off-shell corrections and the decrease due to the phase space reduction. But this is here completely cancelled by the "Coulomb enhancement" (fig.2). It should be emphasized, though, that bound-state effects of the $b\bar{t}$ or $b\bar{b}$ systems are, of course, irrelevant for total energies near the $t\bar{t}$ threshold. Our systematic perturbative approach also correctly describes the toponium decay to $O(\alpha_{weak})$ as a sum of the decays widths $t \rightarrow b + W^+$ and $\bar{t} \rightarrow \bar{b} + W^-$ (cf. the sum of the terms in fig.1).

A reader used to phenomenological calculations may wonder why no running coupling in α_s or even a more "realistic" potential involving a confining piece has been used. The simple answer is that any generalization of this type would completely ruin the strictly perturbative approach (and also the built-in gauge-independence) advocated here, where e.g. the gluon vacuum polarization (cf. h_0^{QCD} above) must be accounted for as a separate perturbation and must under no circumstances be mixed with higher order leading logs in perturbation theory, as summarized in a running α_s . In fact, the toponium system with a superheavy top-quark probably for the first time represents a realistic situation where the gluonic Coulomb-interaction is sufficiently precise to provide a proper starting point for a reliable perturbation theory in a weakly bound system¹, although the basic philosophy of such an approach has a long history [15].

Here we have presented a field theoretically consistent picture of the "smearing" of the individual levels in toponium below threshold. It must be stressed that the treatment of the entire threshold region cross section also requires an extension of our study to the "continuum" $P_0 > 2m_t$ for which the existing BS formalism must be adapted. Related work is in progress [17]. In any case, since the real part of the weak bound-state corrections can be estimated to be small, the net effect of our result (33) for the discrete levels of toponium allows a simple interpretation: It consists of the (level-independent) free decay width Γ_0 and a very small imaginary correction $1 + i\Gamma_0/2m$ to the Rydberg energy.

Of course, a clean and direct experimental study of the $t\bar{t}$ threshold must wait for the next generation of e^+e^- -colliders. But as implied by a recent discussion regarding the importance of those effects in virtual photon lines for electroweak parameters [18, 19, 20] we believe that the strict field theoretical approach for which [3] and our present work are first steps, may even sooner acquire more immediate relevance.

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¹As argued recently [18] under certain conditions the inclusion of a linearly rising part in a phenomenological potential may even lead to large errors.

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Fig. 1: Weak off-shell corrections (Σ)

Fig. 2: Coulomb enhancement (Λ)