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AND THE DARK MATTER CONTENT IN
CLUSTERS OF GALAXIES FROM SPATIALLY
RESOLVED X-RAY SPECTRA**

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**IMPLICATIONS FOR GRAVITATIONAL LENSING AND THE
DARK MATTER CONTENT IN CLUSTERS OF GALAXIES
FROM SPATIALLY RESOLVED X-RAY SPECTRA**

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ABSTRACT

A simple method for deriving well-behaved temperature solutions to the equation of hydrostatic equilibrium for intracluster media with x-ray imaging observations is presented and applied to a series of generalized models as well as observations of the Perseus Cluster and Abell 2256. In these applications the allowed range in the ratio of non-baryons to baryons as a function of radius is derived, taking into account the uncertainties and crude spatial resolution of the x-ray spectra and considering a range of physically reasonable mass models with various scale heights. Particular attention is paid to the central regions of the cluster, and it is found that the dark matter can be sufficiently concentrated to be consistent with the high central mass surface densities derived for moderate redshift clusters from their gravitational lensing properties.

Subject headings: dark matter – galaxies:clustering – intergalactic medium – gravitational lenses

1. INTRODUCTION

Rich clusters of galaxies are the largest known gravitationally bound objects in the universe; and therefore, measuring the distribution of matter in clusters is of primary importance in constraining theories of the formation and development of large-scale structure.

The most straightforward method for estimating the total mass distribution is to apply the equation of hydrostatic equilibrium to the x-ray emitting gas. Direct application of this equation in a model-independent way requires radial density and temperature profiles. Until recent years, the only accurate temperature measurements were emission weighted averages over ~ 1 degree beams from such satellites as *HEAO 1*, *EXOSAT*, and *GINGA*. When conjoined with *Einstein* IPC imaging data, the total amount of mass could be measured to within a factor of ~ 2 -3. Typically, the breakdown of the mass (for $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, as is assumed throughout this paper) was inferred to be the following: $\sim 65\%$ dark matter, $\sim 30\%$ gas, $\sim 5\%$ galaxies (e. g., Cowie, Henriksen, & Mushotzky 1987). Very little information about the *distribution* of dark matter could be derived from these data.

The situation has been improving rapidly with the availability of spectroscopic data with good accuracy *and* some spatial resolution. This has come from such instruments as the *Spartan 1* x-ray detector (Snyder *et al.* 1990), the *Spacelab 2* XRT (Eyles *et al.* 1991, Watt *et al.* 1992), the Broad Band X-ray Telescope (BBXRT; Arnaud *et al.* 1993, Miyaji *et al.* 1993), and the *ROSAT* PSPC (Henry, Briel, & Nulsen 1993). The PSPC also measures densities to greater accuracy, and out to larger radii, than the IPC did. In addition to confirming (with reduced errors) the above conclusions about the amount of dark matter, many of these new measurements seem to indicate that the dark matter is more centrally concentrated than the luminous matter – perhaps an indication that

the dark matter is dissipative, *i. e.* baryonic (but, see Tsai 1993).

However, there are still substantial limitations to the spectral data. In particular, the spatial resolution of temperature profiles is still greatly inferior to that of density profiles, and the temperature profile that is measured is an emission-weighted average over the gas in projection along the line of sight. This means that the mass distribution cannot be derived directly, but instead a parameterized form for the dark matter (or, alternatively, the temperature) must be assumed and then fitted to the data.

In this paper I will examine how the above limitations translate into systematic uncertainties in the dark matter distribution at radii much smaller and larger than the furthest radius where the temperature is measured. For data of the kind and quality only now becoming available, I will evaluate the robustness of the above conclusion about the relative concentrations of dark and luminous matter to assumptions about the shape of the mass profile.

I will investigate just how concentrated the dark matter can be for models consistent with the spatially resolved x-ray spectroscopic data. This has become of great interest because of the discovery of giant luminous arcs and the distortion of faint background galaxies in moderate redshift clusters, both of which seem to be manifestations of gravitational lensing by relatively compact distributions of dark matter associated with the foreground clusters themselves (and not with individual cluster galaxies). The core radius of this dark matter is inferred to be $\sim 0.1 - 0.3$ times the hot gas core radius of nearby clusters (Tyson, Valdes, & Wenk 1990; Hammer 1991; Miralda-Escude 1993), an apparently paradoxical situation considering that dark matter is expected to become more concentrated in clusters as they relax (Merritt 1983). The central mass surface density required for lensing is in excess of $\sim 0.3 \text{ gm cm}^{-2}$. I will address the issue of whether such a compact mass component can be reconciled with the x-ray data in relatively nearby clusters.

This paper is organized as follows. §2 and the Appendix describe the procedure used to construct models for comparison with the x-ray data. This method is a somewhat novel one that is both physically well-founded and mathematically robust. §2 also contains a discussion of the properties of solutions for general, but realistic, mass models, and of the difficulties in ascertaining their mass distributions at small and large radii from x-ray data. §§3 and 4 discuss applications to the Perseus Cluster and to Abell 2256 as observed by BBXRT. §5 is a brief discussion of the sensitivity of the results to assumptions about the luminous matter distribution. §6 contains a summary and discussion of the implications of the results of §§2, 3, and 4, placing particular emphasis on the connection to observations of gravitational lensing by clusters. Conclusions and prospects for the future are presented in §7.

2. GENERAL CONSIDERATIONS AND “GENERIC” MODELS

I will now discuss the limitations in determining detailed total cluster gravitational mass distributions under the assumption of hydrostatic equilibrium, given perfect x-ray imaging and moderate quality (in both accuracy and spatial resolution) x-ray spectral data. This is appropriate for interpretation of *ROSAT* data or *ROSAT* data conjoined with BBXRT or *ASCA* data, from which subarcminute spatial resolution gas density profiles can be very accurately derived from x-ray surface brightness distributions (e.g., Henry *et al.* 1993) and temperatures averaged over the emission along the line of sight (“emission-averaged projected temperature”) can be derived to $\sim 10\text{-}30\%$ accuracy on $\sim 1\text{-}3'$ scales in projection.

In order to be as general as possible, I first consider the equation of hydrostatic equilibrium,

$$\frac{d}{dr} \left(\frac{\rho k T}{m} \right) = - \frac{GM(< r) \rho}{r^2}, \quad (1)$$

in the dimensionless form

$$\frac{d}{dy} (\eta \tau) = \eta \mu. \quad (2)$$

The dimensionless counterparts of the radial coordinate, r , gas density, ρ , total integrated gravitational mass, $M(< r)$, and gas temperature, T , are denoted by y^{-1} , η , μ , and τ , respectively. The gas density is assumed to be representable by the “ β ” model,

$$\rho(r) = \rho_o \left(1 + \frac{r^2}{a^2} \right)^{-3\beta/2} \quad (3)$$

The fiducial dimensional quantities used to define the dimensionless variables are a , ρ_o , $M_o = (4\pi/3)\rho_o a^3$, and $kT_o = GmM_o/a$ for radial coordinate, density, mass, and temperature, respectively, where m is the mean mass per particle. I consider three-component mass models consisting of contributions from gas (μ_1), a component proportional to the light distribution (μ_2) that is assumed to be a modified “King” profile – density = $\rho_g(1 + x_g^2)^{-3/2}$ – and a “pseudo-isothermal” component (μ_3) with density = $\rho_h(1 + x_h^2)^{-1}$. The variables $x_g = x/b_g$ and $x_h = x/b_h$, where $x = r/a$, and b_g and b_h are the core radii for the King (a_g) and pseudo-isothermal (a_h) components in units of a . The total mass $\mu = \mu_1 + \mu_2 + \mu_3$, where

$$\mu_1 \approx (1 - \beta)^{-1} [(1 + x^3)^{1-\beta} - 1], \quad (4)$$

$$\mu_2 = C_2 \left(\log [x_g + (1 + x_g^2)^{1/2}] - x_g(1 + x_g^2)^{-1/2} \right), \quad (5)$$

and

$$\mu_3 = C_3(x_h - \tan^{-1} x_h). \quad (6)$$

The analytic approximation to μ_1 given in equation (4) yields results indistinguishable from those using polynomial fits to the exact numerically integrated gas mass distribution. $C_2 = 9mb_g\sigma^2/kT_o$, where $\sigma^2 = (4\pi G/9)\rho_g a_g^2$ is the square of the one-dimensional velocity dispersion in the King distribution. $C_3 = 2b_h(T_h/T_o)$, where $kT_h = 2\pi Gm\rho_h a_h^2$ is the temperature of the pseudo-isothermal distribution.

Equation 2 is integrated from $y = 0$ to derive solutions for $\tau = [\eta\tau]/\eta$. By expressing Equation 2 in terms of the inverse radial coordinate, y , and integrating inwards from $r = \infty$, solutions that are well-behaved at all radii are automatically obtained. The advantages of this method over the usual outward (from $r = 0$) integrations are the following: (1) solutions where the pressure (but not the density) goes to zero at finite radius are automatically excluded; (2) solutions where the temperature goes to infinity as the radial coordinate goes to infinity that correspond to solutions with asymptotically constant pressure (Hughes 1989) are automatically associated with a particular value of the pressure; (3) “critical solutions” where the pressure goes to zero at infinity are trivially found without resort to iteration; and, (4) the extreme sensitivity to the choice of boundary pressure present in outward solutions is absent. (See the Appendix for more details.) I will consider only critical solutions, both for the sake of simplicity and because solutions that deviate significantly from the critical solutions have rather high pressures at infinity: greater than 10^{-2} in units of the central pressure. Therefore, the solutions for these “generic” models will depend only on the structure parameters of the various mass components – β , b_g , C_2 , b_h , and C_3 – and not on the boundary conditions. In addition, the gravitational mass is cut off at $y = x_t^{-1}$, as discussed in the Appendix.

Analysis for clusters with x-ray imaging and spectroscopic data (e.g., Cowie *et al.* 1987, Hughes 1989, Eyles *et al.* 1991, Miyaji *et al.* 1993) indicates the following: (1) the integrated mass within the radius where both the density profile and the mean temperature are measured is well-constrained (to within $\pm 50\%$ or so), (2) the ratio of mass in gas to luminous mass in galaxies (assuming a mass-to-light ratio of about five in solar units) is $\sim 5 - 10$ within 2-5 gas core radii, and (3) the ratio of total mass to mass in galaxies is $\sim 20 - 50$ within 2-5 core radii. The question I wish to address is how well the mass distribution at radii considerably smaller and larger than the gas core radii are constrained.

That is, I do not consider how formal statistical uncertainties in measured temperatures propagate through to uncertainties in the parameters of particular models of mass profiles – this should be done only within the context of actual fits of model predictions to the total data set (as in Eyles *et al.* 1991). Instead I examine a range of currently observationally indistinguishable models with different mass distributions at small and large radii in order to evaluate the systematic uncertainty associated with the incompleteness of our knowledge of the thermal structure of intracluster media.

As an illustrative example I consider models with $\beta = 0.7$, $b_g = 1$, and $x_t = 10$ (unless otherwise stated). I will assume that emission-averaged temperatures have been measured to an accuracy of $\sim 20 - 30\%$ at several projected radii inside $3a$. Furthermore, I constrain the mass in galaxies to be 0.15 times the mass in gas within $3a$ (corresponding to $C_2 = 0.99$), and the total non-luminous mass to be 30 times the mass in galaxies within $3a$. I assume that the non-luminous matter is dominated by either a mass distribution that follows the light ($C_3 = 0$) or by a pseudo-isothermal mass distribution. In the latter case $C_2 = 0.99$ and C_3 depends on the choice of b_h (see Table 1).

Figure 1 shows the emission-averaged projected dimensionless temperature vs. projected dimensionless radius for four such models (the detailed parameters are shown

in Table 1). The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, dashed, and dot-dashed curves show solutions for models with pseudo-isothermal non-luminous mass distributions with $b_h = 0.1, 0.5$, and 1.0 , respectively. The latter three curves differ only at the 15-20% level at all radii greater than a , and would be difficult to distinguish using current temperature measurements. Moving inward, the curves begin to drastically diverge only in the region where (in a typical cluster) the assumptions of the model start to break down, *i. e.* where cooling becomes important and a simple hydrostatic model may no longer be an accurate representation of the data. Figure 2 shows the same curves as Figure 1 for $x_t = 5$; in this case all four curves are within 10% of each other for all $r > a$. Note that there are temperature inversions in the models with the largest values of b_h ($b_h = 1$). From equation (A8) one can see that the ratio of the temperature at the origin to that at infinity will be less than unity for $b_h > 3^{-1/2}$ provided that $C_3 >> 3b_h^3 + C_2(b_h/b_g)^3$, which is the case for the models under consideration. Although equation (A8) is not strictly applicable to models with cutoffs, it does prove the existence of models where the hydrostatic temperature at the origin is less than that at large radii.

Figure 3 shows the mass distributions for these four models. If $x_t = 10$, the relative spreads in the total mass are on the order of the relative spreads in the temperature profiles (see also Table 1). The distribution of mass in the centers of these models span a much wider range, as can be seen in Figure 4 and Table 1. The spread in allowed central volume densities is a factor of fifty. This corresponds to a factor of five uncertainty in central surface density (Table 1) – the quantity relevant to a consideration of gravitational lensing effects (see below).

The above four models have been chosen to have the same total mass at the maximum radius where a spatially resolved temperature measurement might be made (assumed to be three gas core radii). Alternatively, the models can be tuned to

provide constant emission-averaged projected temperature at some projected radius. The projected temperature distributions for a set of such models that span a factor of two in mass at $3a$ are shown in Figure 5. Choosing the former set for comparison comes closer to minimizing differences between classes of models (*i. e.* mass follows light or pseudo-isothermal) in terms of the overall temperature profile and its average over the entire region. As expected from the virial theorem, the spread in emission-averaged temperature integrated over a large ($> 2a$) beam is an easily distinguished factor of two in the models shown in Figure 5, as compared to a $\sim 20\%$ spread in the models shown in Figure 1.

Figure 5 is useful in illustrating how spatially-resolved temperature measurements can distinguish the pseudo-isothermal models from the mass-proportional-to-light models. For a mass-follows-light model with the same integrated emission-averaged temperature and same projected temperature at some intermediate projected radius as a pseudo-isothermal model, the temperature will be higher at smaller radii and lower at larger radii.

I also show the parameters and results for models with $\beta = 0.55$ and $\beta = 0.85$ in Table 1. Results are qualitatively similar to the $\beta = 0.7$ case. Figure 6a shows the ratio of non-baryonic to baryonic matter in the $\beta = 0.7$ models (assuming that all of the non-luminous matter is non-baryonic). These curves echo the temperature curves of Figure 1: the three pseudo-isothermal models have comparable baryonic/non-baryonic mixes until they start to diverge inside $r = a$; the model where non-luminous matter is proportional to light is more distinctive. Figures 6b and 6c show similar plots for $\beta = 0.55$ and $\beta = 0.85$, respectively (see Table 1 for model parameters and temperatures at $r = 3a$). Note that all models in Figures 6a-6c are normalized to $M_{\text{non-baryonic}}/M_{\text{baryonic}} \approx 4$ at $r = 3a$. In these models, where the non-baryonic component dominates the gravitational potential and the gas dominates the baryonic

mass component,

$$\frac{M_{non-baryonic}}{M_{baryonic}} \propto \frac{T}{\rho r^2} : \quad (7)$$

the variations in β account for most of the variations in slope among Figures 6a-c.

The relative dispersions among the curves in Figures 6a-c are similar in magnitude. The relative spread in model parameters such as total mass, and central mass volume and surface densities are also very similar for all values of β considered (Table 1). Table 1 also shows the central value of $M_{non-baryonic}/M_{baryonic}$; a spread of a factor of 60 is apparent among observationally indistinguishable models. These ratios are calculated for cluster material only, and would decrease with the inclusion of a central galaxy.

These similarities allow me to make the following generalizations. If accurate (emission-averaged, projected) temperature profiles with spatial resolution less than the gas core radius (a) that extend out to $r_{max} \sim 5a$ are available, one can not only derive accurate constraints on the total mass inside r_{max} (which could be derived from a single, averaged temperature), but one can distinguish models where the non-luminous matter is proportional to the light from those with flatter (e.g. r^{-2}) dark matter density profiles because of their distinctive temperature profile slopes. This is true if the cluster extends beyond $5a$ so that the “weight” of the overlying gas varies substantially according to the fall-off of the gravitational potential. This ability to distinguish between models with different dark matter density slopes, an ability that was lacking given only a single averaged temperature, means that an accurate value for the total mass beyond r_{max} can be inferred.

However, as a result of lack of accuracy and spatial resolution as well as projection effects, models where the baryonic and non-baryonic components have the same core radius are not easily distinguished from models where the non-baryonic core radius

is one-tenth the baryonic core radius. Moreover, the observationally allowed range in central volume (surface) mass density is at least a factor of ~ 50 (5). The central surface density is expressed in units of $\rho_o a \approx 10^{-2}(\rho_o/10^{-26} \text{ gm cm}^{-3})(a/300 \text{ kpc})$; therefore the dimensionless surface density must be on the order of 100 to lens background galaxies – a condition that is met by the $b_h = 0.1$ models. These models would seem to be difficult to rule out using current observations, thus opening up the possibility of reconciling x-ray observations of relatively nearby clusters with the presence of gravitational lens induced arcs and mini-arcs in clusters at larger redshift without requiring “negative” evolution of cluster dark matter. Alternatively, the higher values could be ruled out if accurate, spatially resolved imaging and spectroscopic data were extended to $r < a$, *and* the effects of any existing cooling flow could be fully corrected for (see also §6).

3. APPLICATION TO THE PERSEUS CLUSTER

In this and the following section I apply the general method of §2 to the specific cases of the Perseus Cluster and Abell 2256, both of which have spatially resolved spectroscopy from BBXRT (Arnaud *et al.* 1993, Miyaji *et al.* 1993).

Following Eyles *et al.* (1991), I adopt the optically derived parameters (central luminosity density and core radius) of the “standard model” of Kent & Sargent (1983) and $M/L_V = 5$ to describe the mass distribution of galaxies in the Perseus Cluster; and the x-ray derived parameters (central gas density, gas density core radius, and β) of Jones & Forman (1984) to describe the mass distribution of the Perseus intracluster medium. The parameters for the models as described in §2 that are appropriate for the Perseus Cluster include $\beta = 0.573$, $b_g = 1.21$, and $C_2 = 1.68$ (for the galaxy component); x_t is assumed to be equal to twelve. The ratio of mass in gas to mass in galaxies is 6.2 at three gas core radii ($3a = 840 \text{ kpc}$).

I consider five classes of models. The first has dark matter proportional to light ($C_3 = 0$), the other four have pseudo-isothermal dark matter distributions ($C_2 = 1.68$) with $b_h/b_g = 0.1, 0.25, 0.5$, and 1.0 . The dark matter normalizations of the models (C_2 for the first class and C_3/C_2 for the second-through-fifth classes) are then varied until a rough by-eye agreement with both the SL2 and BBXRT temperature profiles (Eyles *et al.* 1991 and Arnaud *et al.* 1993, respectively) are attained. The SL2 data extends further out (1.3 Mpc compared to 0.78 Mpc for the BBXRT data), however the BBXRT data has multiple measurements inside a while the SL2 data has only one. The normalizations that are adopted for the five models described above are $(C_2, C_3/C_2) = (33.5, 0); (1.68, 8); (1.68, 3); (1.68, 1.5);$ and $(1.68, 0.5)$ for the model with dark matter proportional to light and the four pseudo-isothermal models (in order of decreasing b_h). The dimensionless parameters for the Perseus Cluster are similar to those for the $\beta = 0.55$ models presented in §2 (see Tables 1 and 2).

The model parameters for this quintuple of well-fitting models are shown in Table 2, including the total mass and dark-to-luminous matter ratio at 3 and 12 gas core radii. Also shown are the central values of the total mass volume density, surface density, and dark-to-luminous matter ratio. Figures 7a and 7b show the emission-averaged projected temperatures for these five models along with the observed temperatures (and 90% confidence errors) from Eyles *et al.* (1991) – shown as crosses – and Arnaud *et al.* (1993) – shown as boxes. In deriving the temperatures from BBXRT observations, Arnaud *et al.* (1993) have attempted to reduce the effect of the cooling flow component by fitting only the data above 4 keV. Figure 8 shows the integrated mass distributions, Figure 9 the ratios of non-baryonic to baryonic matter vs. radius.

From Figures 7a and 7b, it is clear that these five models are indistinguishable given the present accuracy of the temperature profiles (see also Table 2). (The model emission-averaged temperatures integrated over apertures of 1-3 Mpc also differ by

$\sim 15\%$ or less.) As a result of differences in the values of b_g and β , these models diverge less than the generic models shown in Figure 1. The mass enclosed within the last radius with an accurate temperature measurement spans a very small range for these models: $M(< 1.3 \text{ Mpc}) = 4.4 - 4.8 \cdot 10^{14} M_\odot$ (Figure 8) which is in excellent agreement with the results of Cowie *et al.* (1987) and Eyles *et al.* (1991), even though these authors fit to an assumed functional form for the temperature and derive the mass distribution as opposed to deriving the temperature profile from the assumed mass distribution as is done here. At 3 Mpc, the spread in the integrated masses has widened to $\sim 30\%$, and the ratio of non-baryonic matter to baryons varies between ~ 1 and ~ 2 . What I wish to emphasize is how drastically the observationally acceptable models diverge inside a as a result of the statistical uncertainties and lack of spatial resolution in the temperature profiles as well as projection effects. The ratio of baryonic to non-baryonic matter varies by a factor of 60 among these models at the center, and by more than an order of magnitude at 100 kpc (between models with $b_h/b_g = 1.0$ and 0.1; see Table 2 and Figure 9). Likewise, the central densities in the models span a factor of 40. And the central surface densities in the models range from 0.085 gm cm^{-2} to 0.38 gm cm^{-2} , the latter quantity close to that required for gravitational lensing (Hammer 1991).

What sort of improvements in the observations would be required to rule out some subset of these models? If the temperature were known to better than 15% accuracy at several radii between 1 and 2 Mpc, one could distinguish between the model where dark matter is proportional to light, the pseudo-isothermal model with $b_h/b_g = 1$, and the other three pseudo-isothermal models (which are still indistinguishable from each other at this level of accuracy). This would pin down the total cluster mass and the cluster-wide ratio of non-baryons to baryons very accurately. Prospects seem more promising at small radii where the spread in temperature is a factor of two among the models (see Figures 7a and 7b). In particular, the pseudo-isothermal model with

$b_h/b_g = 1$ is only marginally consistent with the BBXRT data; this would seem to rule out models where the ratio of dark-to-luminous matter increases with radius (Figure 9). Also, the model with the most concentrated mass distribution (the pseudo-isothermal model with $b_h/b_g = 0.1$) seems marginally too hot in the center to be consistent with the BBXRT data. However, the presence of the cooling flow in Perseus complicates this interpretation. If the cooling flow is not completely corrected for, the observed temperature may in fact be lower than that predicted by the hydrostatic model. Moreover, the presence of a very concentrated mass profile may manifest itself as a steepening of the density profile and, therefore, a less-pronounced temperature peak. Such a density cusp would be difficult to distinguish from a cooling flow.

To summarize for the Perseus Cluster: x-ray imaging and spectral data marginally favor a mass model with non-baryonic matter more concentrated than baryonic matter (as found by Eyles *et al.* 1991), and an overall ratio of non-baryons to baryons less than 2. An increase in the accuracy and extent of gas temperature measurements could constrain this ratio very accurately and distinguish between various dark matter mass distributions. Models with dark matter core radii of 30 kpc or less and central mass surface densities of 0.38 gm cm^{-2} or more cannot be ruled out at present. To do so would require not only accurate, spatially-resolved temperature and density determinations well inside the core radius, but also a thorough understanding of the thermal characteristics of the cooling flow, the properties of which can mask the effects of a very concentrated dark matter distribution.

4. APPLICATION TO ABELL 2256

Miyaji *et al.* (1993) and Henry *et al.* (1993) have presented spatially resolved x-ray spectroscopy of Abell 2256 using BBXRT and *ROSAT* observations, respectively, and have discussed the implications for the distribution of dark matter in this cluster. Both

sets of authors are careful to exclude what is apparently a merging group from their hydrostatic analysis. The following discussion is an amplification of §4.2.3 of the former work.

I adopt the parameters for the galaxy mass distribution from Henry *et al.* (except that I again assume $M/L_V = 5$), who use the optical data of Fabricant, Kent, & Kurtz (1989), and also their best-fit beta model to the *ROSAT* PSPC x-ray surface brightness data. The resulting model parameters include $\beta = 0.795$, $b_g = 1.46$, and $C_2 = 0.705$ (for no dark matter proportional to light); x_t is assumed to be equal to ten (the PSPC detects emission out to at least eight gas core radii). The ratio of mass in gas to mass in galaxies is 12.0 at three gas core radii ($3a = 1.6$ Mpc).

I again consider five classes of models, the first of which has dark matter proportional to light ($C_3 = 0$). The other four have pseudo-isothermal dark matter distributions ($C_2 = 0.705$) with $b_h/b_g = 0.1, 0.3, 0.5$, and 1.0 . Miyaji *et al.* (1993) evaluated the goodness-of-fit to the BBXRT temperature measurements of these models for various dark matter normalizations (C_2 or C_3/C_2) using the χ^2 statistic. The same parameters shown in Table 2 for the Perseus Cluster are shown in Table 3 for the best-fits for each of the five classes of mass models for Abell 2256. The normalizations for the five best-fit models are $(C_2, C_3/C_2) = (28.2, 0); (0.705, 25); (0.705, 7.5); (0.705, 3.4);$ and $(0.705, 0.7)$ for the model with dark matter proportional to light and the four pseudo-isothermal models (in order of decreasing b_h). All of these models provide acceptable fits to the data. Figure 10 shows the emission-averaged projected temperatures for these five models along with the BBXRT temperature measurements and $1-\sigma$ error bars from Miyaji *et al.* (1993). There is a somewhat larger divergence in the predicted temperature profiles outside the gas core radius among the models here than there is for the Perseus Cluster. As is the case for the Perseus Cluster, the model with the most extended dark matter profile provides the worst fit. The mass enclosed within the outermost radius

observed with BBXRT (0.8 Mpc) ranges from $2.8 - 3.3 \cdot 10^{14} M_{\odot}$ among the best-fit models (Miyaji *et al.* 1993); at 3 Mpc the range is $1.4 - 2.6 \cdot 10^{15} M_{\odot}$ (Figure 11). As noted by Miyaji *et al.* (1993), these numbers are very close to those derived by Henry *et al.* (1993) using somewhat different modeling techniques. The ranges in central volume density and central surface density in these models for Abell 2256 are smaller than those for the Perseus Cluster models considered in the last section, varying over factors of 13 (compared to 42) and 2.3 (compared to 4.5), respectively (see Table 3). Likewise, the central ratio of baryonic to non-baryonic matter ranges over a factor of 29, compared to a factor of 63 for the Perseus Cluster. However, I have found that an additional model with $b_h/b_g = 0.05$ that has a similar dark matter core radius in physical units to that of the most concentrated mass model considered for the Perseus Cluster also provides an acceptable fit to the observed temperature profile. As shown in Table 3, the central mass volume and surface densities in this model are close to those for the $b_h/b_g = 0.1$ model of the Perseus Cluster. The range in the ratio of dark-to-luminous matter at 100 kpc is very similar in both clusters; 1.7-26 for Abell 2256, and 1.9-25 for the Perseus Cluster. As shown in Figure 12, this ratio may increase with radius for Abell 2256. This differs from the case of the Perseus Cluster (Figure 9) because Abell 2256 has a steeper fall-off in gas density (*i. e.* a larger value of β), and the baryonic component is dominated by gas (see equation 7).

The present temperature measurements are not sufficient to favor any particular dark matter scale heights; pseudo-isothermal models with core radii of 40-800 kpc (and central mass surface densities of $0.094\text{-}0.42 \text{ g cm}^{-2}$) as well as models where the mass is proportional to the light are consistent with the BBXRT data. However, beyond 1 Mpc models with $b_h < 0.3$ or $b_h > 1.0$ start to diverge drastically from the model with mass proportional to light and pseudo-isothermal models with intermediate values of b_h . Temperature measurements with $\sim 15\%$ accuracy and $1 - 3'$ spatial resolution

in the 0.5-1.5 Mpc (5-15') range of cluster radii could distinguish pseudo-isothermal models with large ($b_h/b_g > 1$) or small ($b_h/b_g < 0.1$) dark matter scale heights from those with intermediate scale heights. A temperature measurement for the inner 100 kpc (1') would also be illuminating (see Figure 10), especially since Abell 2256 is not known to have a cooling flow to dilute the high central temperatures in models with compact dark matter. At this level of accuracy, the mass-follows-light model is not easily distinguished from the $b_h/b_g = 0.5$ pseudo-isothermal model out to 1.5 Mpc. Nevertheless, if the extreme models were ruled out the range in total mass would be quite narrow ($M(5.4 \text{ Mpc}) \sim 3\text{-}4 \cdot 10^{15} M_\odot$), as would the global ratio of non-baryons to baryons ($\sim 2.5\text{-}4$).

5. SENSITIVITY TO CHOICE OF FIXED PARAMETERS

In the models fit to the x-ray spectra of the Perseus Cluster and Abell 2256, the normalization and (for models with pseudo-isothermal dark matter distributions) scale length of the dark matter distribution are varied while the parameters for the luminous matter (gas and galaxies) are held fixed at their observed values. Because the gas and galaxy mass distributions have not been determined with perfect accuracy, I here include a brief discussion of the effects of varying the luminous matter parameters.

The galaxies account for such a small fraction of the total mass ($< 10\%$) that changes in the assumed galactic mass distribution fail to have a significant effect on the temperature solutions. For the models where the dark matter is assumed to be proportional to the optical light the solutions are, of course, sensitive to the choice of galaxy distribution core radius. I have calculated models where this core radius varies by $\pm 30\%$ (see Kent & Sargent 1983 and Henry *et al.* 1993 for the Perseus Cluster and Abell 2256, respectively) from the values used in §§3 and 4. Models that fit the data are found by varying the normalization factor, C_2 ; these models are only distinguishable from each

other at $r < a$. The range of derived parameters (as in Tables 2 and 3) among best-fit models is not widened appreciably – the largest effect of considering these additional models is to allow non-baryonic-to-baryonic mass ratios that are a factor of $\sim 20\%$ lower than the lowest values in Tables 2 and 3.

I have also considered models where the normalization of the gas mass is allowed to vary from the gas distributions considered in §§3 and 4. Since the contribution of the gas to the total gravitating mass is secondary but significant, the effect of these variations are generally small but not negligible. In fact, I found that if the gas mass is reduced by more than $\sim 25\%$ for either Perseus or Abell 2256, the models transit to being totally dark matter dominated and the dark matter parameters must be adjusted to preserve a good fit to the observations. This adjustment is in the sense of reducing the amount of dark matter by up to 30% (for a halving of the gas mass) and more nearly maintaining the ratio of non-baryonic to baryonic matter.

On the other hand, if the gas mass is scaled up by 50% the gas is sufficiently massive to reproduce the observed temperature distribution with a relatively minor contribution from dark matter. This is a separate solution regime from that of the models considered in the previous sections. The total mass distributions are scaled down by about 30%, and the ratio of non-baryonic to baryonic matter at 3 gas core radii can be as low 0.3 in the Perseus Cluster and 0.5 in Abell 2256. High central surface densities are not present in these gas dominated models.

As discussed in the Appendix, varying the cutoff radius, x_t , does not appreciably affect the solutions providing x_t is on the order of ten or greater.

The basic conclusions of the previous three sections are not altered by the considerations of this section, provided that gas masses have not been systematically underestimated to the extent that the intracluster gas is predominantly self-gravitating. The total mass and ratio of non-baryonic to baryonic matter inside radii where multiple

spatially distinct temperatures have been measured are fairly well-determined, and potentially very well determined (e.g., with *ASCA*); but, the total mass distributions near the cluster center are more poorly constrained and may remain problematical (see below).

6. SUMMARY AND DISCUSSION

In the previous three sections of this paper I have attempted to elucidate the uncertainties in the amount and distribution of dark matter in clusters of galaxies that result from the relatively crude spatial resolution of measured temperature profiles and our ignorance about what the dark matter scale height should be. In doing so I have not analyzed in detail the implications of the statistical uncertainties in various observed parameters, nor have I considered a wide range of cluster mass models. Instead I have considered a narrow range of plausible mass models (mass follows light, or pseudo-isothermal with various core radii) that are, in general, presently observationally indistinguishable and examined the resulting systematic uncertainty in the dark matter distribution. Since it has been demonstrated (and is confirmed here) that spatially resolved x-ray spectroscopy can place tight constraints on the total mass within the largest radius where the temperature has been measured, I have focussed on the mass distribution at very large and very small radii.

In §2 I presented physically and mathematically well-defined dimensionless models, and in §§3 and 4 applied these models using the dimensional quantities appropriate to the Perseus Cluster and Abell 2256. From these models one can infer that, in general, accurate ($\delta T/T \sim 10\text{-}15\%$), moderately spatially resolved (resolution less than the gas core radius) x-ray spectroscopy extending from the core radius out to five core radii or so can clearly distinguish models where mass is proportional to light from pseudo-isothermal models (assuming, of course, that accurate gas density profiles out to five core

radii are also available from imaging data). That is, the shape of the radial temperature distribution will be sufficiently well-defined to distinguish between gravitational mass profiles with different slopes. If these criteria are met, the total (out to the limits of the gas distribution) amount of mass and ratio of non-baryons to baryons can be very accurately determined. From §§3 and 4, one can see that the accuracy and extent of BBXRT temperature measurements were not sufficient to make the above distinctions; however, the allowed range in total mass has been considerably narrowed for Abell 2256 and the Perseus Cluster through the use of spatially resolved x-ray spectroscopic data (see Tables 2 and 3). Observations using the *ASCA* satellite will meet all the above requirements for accurate total mass determinations out to redshifts, z , greater than 0.1. Because of the relatively small field of view of the *ASCA* instruments, multiple pointings will generally be required.

I have considered core radii for the pseudo-isothermal dark matter models (a_h) that are 0.1-1.0 times the core radius of the optical light distribution. The hydrostatic temperature distributions among these models are generally most easily distinguished at small radii ($R < 0.5a$): for a given average temperature, the central temperature decreases monotonically with increasing a_h (Figures 1, 7, and 10). (Note, however, that this must be considered on a case by case basis: unlike the “generic” mass models of §2 and the Perseus Cluster mass models, mass models for Abell 2256 –which has an unusually large core radius and steep gas density slope– are as easily distinguished at 1 Mpc as they are at 100 kpc.)

However, there are severe difficulties associated with distinguishing models by measuring their temperatures inside $0.5a$. Firstly, there is the breakdown of hydrostatic equilibrium and the simple β -density model. Most clusters of galaxies apparently have cooling flows with central densities rising sharply in excess of the β model (Edge, Stewart, & Fabian 1992). The central temperatures in cooling flow models are, of

course, less than those in the hydrostatic models discussed in this paper. Thus one cannot rule out models with small dark matter core radii without correcting the high projected temperatures in the hydrostatic models for the effects of cooling. Moreover, the presence of a compact dark matter distribution would increase the amount of gravitational compressive heating and could lead to an overestimate of the cooling rate (*i. e.* some of the central density enhancement might be due to the compact potential). Detailed models of intracluster gas flows with cooling in the presence of a compact potential are needed to test the consistency of observed central gas temperatures with a concentrated dark matter distribution. There is also the matter of spatial resolution: the spatial resolution of current x-ray spectroscopic measurements are sufficient to measure the temperature inside $0.5a$ only for $z < 0.05$. Thus the method developed in this paper is strictly applicable to the central parts of clusters only for the rare cases of non-cooling flow clusters with redshifts less than 0.05. And if the speculation by Edge *et al.* (1992) that clusters without cooling flows (such as Abell 2256) have undergone recent mergers is correct, then the hydrostatic assumption may not be strictly applicable to all non-cooling flow clusters, although it may be a fair approximation for the primary component of a merger between two subclusters with a large mass ratio (again, such as Abell 2256). It is (and may continue to be) difficult, from x-ray observations alone, to exclude or detect the presence of concentrations of dark matter in the cores of cluster compact enough to act as gravitational lenses.

Indeed, gravitational lensing of background galaxies by matter in the cores of several moderate redshift clusters has been inferred from observations of giant luminous arcs (e.g., Soucail *et al.* 1987) and “mini-arcs” (e.g., Tyson *et al.* 1990). For a pseudo-isothermal mass distribution the critical mass surface density for lensing is given by the expression

$$\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}}, \quad (8)$$

where D_s , D_d , and D_{ds} are the observer-source, observer-lens, and source-lens distances, respectively (Hinshaw & Krause 1987). For an $\Omega = 1$, $H_o = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ cosmology, and assuming the cluster redshift (z_{lens}) is greater than 0.3, the source redshift (z_{source}) is less than 5, and $z_{source} > 2z_{lens}$,

$$0.32 < \Sigma_{crit} < 0.50 \text{ gm cm}^{-2}.$$

For an $\Omega = 0$ cosmology the limits are not substantially different (0.35-0.65 gm cm⁻²). The surface densities in the most compact mass distributions considered in this paper (pseudo-isothermal mass distributions with core radii $\sim 30 - 40 \text{ kpc}$) that are consistent with x-ray data for the Perseus Cluster and Abell 2256 are $\sim 0.4 \text{ gm cm}^{-2}$. The apparent absence of lensing by $z \leq 0.1$ clusters puts an upper limit on Σ_o of $0.8h_{50} \text{ gm cm}^{-2}$, where h_{50} is Hubble's constant in units of $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, independent of Ω and z_{source} , and implies that the dark matter in some clusters was as concentrated at $z = 0.2 - 0.4$ as it is at the present epoch. As discussed above, the cooling flow in the Perseus Cluster could mask an even higher value of Σ_o . I have also shown in §2 that in general it is difficult to rule out models with $\Sigma_o \sim 1 \text{ gm cm}^{-2}$ using temperature measurements outside of $0.5a$. So it seems that *a dark matter distribution compact enough to lens can be present in nearby clusters and still be consistent with all of the present x-ray data*. Addressing whether such a distribution is consistent with CDM theories of structure formation (Tsai 1993) will require including the contribution of the cD galaxy (for those clusters where one is present) in the accounting of the central mass of baryons.

As an intriguing aside, note that the ratio of Σ_o to Σ_{crit} is proportional to $H_o^{-1.5}$.

Thus, if one could measure Σ_o in a lensing cluster using x-ray observations, the condition $\Sigma_o > \Sigma_{crit}$ could yield an upper limit on H_o . Unfortunately at the redshift of even a relatively nearby lensing cluster ($z \sim 0.2$), $1' \approx 300$ kpc and so spatial resolution effects would limit the accuracy of the Σ_o determination.

7. CONCLUSIONS

A summary of the primary conclusions of this paper follows.

- 1) Solutions to the equation of hydrostatic equilibrium for intracluster temperature profiles that are mathematically and physically sensible at all radii and that are independent of boundary conditions can be constructed, and should form the basis for evaluating the consistency of cluster mass models with present and near-future x-ray observations.
- 2) A range of cluster mass models are consistent with BBXRT observations of the Perseus Cluster and Abell 2256. These models have very similar masses integrated out to the last radius where the temperature was measured, but differ somewhat in their total integrated masses. The ratio of dark matter to luminous baryonic matter is ~ 1 -2 in the Perseus Cluster and ~ 2 -5 in Abell 2256.
- 3) Models where the dark matter core radius is much less than the gas core radius are not observationally distinguishable from models where the two core radii are comparable. Even within the context of the narrow range of cluster mass models considered in this paper, and using the highest quality existing data, central mass volume and surface densities, and baryon fractions are not tightly constrained. Dark matter distributions compact enough to lens background galaxies are allowed (especially if $h_{50} \sim 1$). The accuracy and spatial resolution of currently available x-ray spectra of clusters are not sufficient to rule out such compact mass models, especially when considered in

conjunction with projection effects and complications due to cooling flows.

Future observational and modelling work should significantly narrow the constraints on the distribution of dark matter in clusters of galaxies. Application of the method presented in this paper to observations made with the *ASCA* satellite or, better yet, a refined method that directly compares observed and model emission measures as a function of temperature at various projected radii, will narrow the range of allowed total masses and baryon fraction and distinguish between dark matter distributions with different slopes in some cases. The possible existence of very compact dark matter distributions can be tested in some clusters, especially nearby clusters with weak or non-existent cooling flows. Clusters with central x-ray surface brightnesses in excess of the β model, but with long cooling times (e.g., Abell 1367; Jones & Forman 1984) will be crucial test cases. More general mass models – e.g., hybrid, compact-plus-diffuse dark matter models (hot and cold dark matter or baryonic and non-baryonic dark matter) – can be considered as the data becomes more detailed. Finally, cooling flow models for clusters with compact dark matter distributions should be constructed to see if there are any signatures of such models that are observable with *ASCA* and future x-ray missions.

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APPENDIX

SOLUTION METHODOLOGY

In this paper I have presented solutions to the equation of hydrostatic equilibrium for intracluster atmospheres where the gas temperature distribution is determined for

fixed gas density and gravitational mass distributions. While the equation to be solved is a relatively simple ODE, there are some subtleties regarding boundary conditions and mass distribution cutoffs that should be kept in mind. I discuss these issues in this Appendix.

1. BEHAVIOUR AT INFINITY AND CRITICAL SOLUTIONS

In dimensionless form, the equation of hydrostatic equilibrium is

$$\frac{d}{dx}(\eta\tau) = -\frac{\eta\mu}{x^2}. \quad (A1)$$

The density in units of the central density, η , is assumed to be given by the “ β ” model,

$$\eta(x) = (1 + x^2)^{-3\beta/2}, \quad (A2)$$

where x is the radial coordinate in units of the core radius of the gas. The dimensionless temperature and gravitational mass are denoted by τ and μ , respectively. The mass is assumed to consist of the following three-components: (1) the gas component, (2) a component proportional to the light distribution, and (3) a “pseudo-isothermal” component. The parameters of the mass model are β , the core radii of the second and third components in units of the first – b_g and b_h – and the relative normalizations of the second and third components – C_2 and C_3 . See §2 for more details.

Consider the integration of equation (A1) outwards from $x = 0$ for various values of $\tau_o \equiv \tau(x = 0) = \eta(x = 0)\tau(x = 0)$. The gravitational term, $-\mu x^{-2}$, is to be balanced by the pressure gradient term, $\tau(\tau' + \eta')$, where the “prime” denotes differentiation with respect to $\log x$. This balance can occur in solutions where $\tau \ll 1$ and $\tau' \gg 1$, or $\tau \gg 1$ and $(\tau' + \eta') \ll 1$ ($d\tau/dx \gg 1$), as well as for “better behaved” temperature distributions. As a result, the detailed solution is quite sensitive to the choice of τ_o , as

can be seen in Figure A1 which shows nine solutions of equation (A1) for one of the “generic” models of §2 ($\beta = 0.7$, $C_2 = 0.99$, $b_g = 1.0$, $b_h = 0.5$, $C_3 = 5.64$) for values of τ_o ranging from 5.9375 – 6.0938. A similar sensitivity can be seen in the plots of Hughes (1989) and Henry *et al.* (1993). Apparently, either $\tau \rightarrow 0$ or $\tau \rightarrow \infty$ at finite x (this is not necessarily the case, as I shall show shortly). Hughes (1989) identifies the $\tau \rightarrow 0$ case with gravitationally bound solutions; however, while the pressure goes to zero at finite radius the density does not. In consideration of this and the likelihood of a finite pressure intercluster medium and/or continued infall of gas from large radius, I reject these solutions as unphysical. The $\tau \rightarrow \infty$ solutions correspond to those where the pressure, $\eta\tau$, goes to a constant value at large radius (Hughes 1989); this asymptotic pressure increases rapidly with τ_o . There is also a unique “critical” solution where $\eta\tau \rightarrow 0$ asymptotically as $x \rightarrow \infty$.

The most straightforward method for finding critical solutions without using a complicated iterative procedure is to integrate *inward* from $x = \infty$. We can rewrite equation (A1) as

$$\frac{d}{dy}(\eta\tau) = \eta\mu, \quad (\text{A3})$$

where $y = x^{-1}$, and integrate from $y = 0$ with the boundary condition $\eta(y = 0)\tau(y = 0) = \psi_\infty$. I consider $0.5 < \beta < 1$ that corresponds to the observed range. At small y , either the gas (equation 4) or pseudo-isothermal (equation 6) mass component dominates, depending on the value of β . If $C_3 = 0$ or $\beta < 2/3$, $\mu \rightarrow (1 - \beta)^{-1}y^{3\beta-3}$ as $y \rightarrow 0$. If $\beta \geq 2/3$ (assuming $C_3 \neq 0$ for $\beta > 2/3$), $\mu \rightarrow By^{-1}$ as $y \rightarrow 0$, where $B = C_3b_h^{-1}$ if $\beta > 2/3$ and $B = 3 + C_3b_h^{-1}$ if $\beta = 2/3$. In all cases considered, $\eta\mu \rightarrow 0$ as $y \rightarrow 0$, so that $d(\eta\tau)/dy \rightarrow 0$ and $\eta\tau \rightarrow \psi_\infty$, where ψ_∞ is the finite or zero constant pressure at infinity. If $\psi_\infty > 0$, $\tau \rightarrow \psi_\infty y^{-3\beta} \rightarrow \infty$ as $y \rightarrow 0$. These solutions correspond precisely to the $\tau \rightarrow \infty$ outward-solutions ($\tau_o \geq 6.0681$) shown in Figure A1; we can

now identify these with particular values of the asymptotic value of the pressure. In the critical solutions where $\psi_\infty = 0$, $\tau \rightarrow \tau_\infty$ as $y \rightarrow 0$ if $\beta \geq 2/3$ (mass increasing no faster than y^{-1} as $y \rightarrow 0$), where τ_∞ is a constant. The value of this constant dimensionless temperature

$$\tau_\infty = \begin{cases} \frac{1}{2}(3 + C_3 b_h^{-1}), & \beta = \frac{2}{3}; \\ \frac{1}{3\beta} C_3 b_h^{-1}, & \beta > \frac{2}{3}. \end{cases} \quad (A4)$$

If $\beta < 2/3$, the asymptotic solution for the temperature is

$$\tau \approx \frac{y^{3\beta-2}}{(6\beta-2)(1-\beta)}, \quad (A5)$$

and $\tau \rightarrow \infty$ as $y \rightarrow 0$, but not as rapidly as in non-critical solutions.

One can also derive the solutions near the origin ($x = 0$) straightforwardly since all of the mass components approach 0 as x^3 . As $x \rightarrow 0$,

$$\frac{d\tau}{dx} \rightarrow 0, \quad (A6)$$

and

$$\tau \rightarrow \tau_o \equiv \frac{1}{3\beta} \left(1 + \frac{C_2}{3b_g^3} + \frac{C_3}{3b_h^3} \right). \quad (A7)$$

Therefore for $\beta > 2/3$,

$$\frac{\tau_o}{\tau_\infty} = \left(\frac{C_3}{b_h} \right)^{-1} \left(1 + \frac{C_2}{3b_g^3} + \frac{C_3}{3b_h^3} \right), \quad (A8)$$

which can be either less than or greater than one depending on the relative temperatures and scale heights of the three mass components.

Figure A2 shows the solutions for various values of ψ_∞ for the generic model described above. As ψ_∞ increases, the solutions break away from the $\psi_\infty = 0$ solution and follow curves of nearly constant pressure ($\tau \approx \psi_\infty y^{-3\beta}$) at increasingly smaller radii. As in Figure A1, the horizontal scale goes out to 100 (gas) core radii. On the more observationally relevant scale of $10a$, the curves for $\psi_\infty < 10^{-2}$ are practically indistinguishable from each other (and from the dotted curve in Figure A1), as shown in Figure A3. This is true for all values of β in the considered range. We will assume that the intercluster pressure is less than 10^{-2} times the central cluster pressure, so that we need only consider critical solutions. This simplifies matters since the solutions will depend only on the structure parameters of the various mass components and not on the boundary conditions.

2. EFFECT OF MASS CUTOFF

An unrealistic aspect of the infinite hydrostatic intracluster atmospheres is the divergence of the integrated mass profiles as $x \rightarrow \infty$. I now consider solutions where all three mass components are truncated at a common radius, x_t , but (non-self-consistently) the gas density extends to infinity. One could, instead, integrate inward from $x = x_t$, but as is the case for the outward integrations the solutions will be hypersensitive to the choice of boundary pressure ($\tau \rightarrow 0$ or ∞ as $x \rightarrow 0$ for most choices). The integration from $x = \infty$ picks out a pressure at $x = x_t$ that assures a well-behaved solution both for $x \rightarrow 0$ and $x \rightarrow \infty$. As before, there are two types of solutions. For $\psi_\infty > 0$, $\tau \rightarrow \psi_\infty x^{3\beta} \rightarrow \infty$ as $x \rightarrow \infty$. For critical solutions ($\psi_\infty = 0$), $\tau \rightarrow \mu(x_t)(3\beta + 1)^{-1} x^{-1} \rightarrow 0$ as $x \rightarrow \infty$.

Again, we consider critical solutions – differences between these and solutions for reasonably small values of ψ_∞ are even less significant than for the solutions with no cutoff. Figure A4 shows solutions for the generic model described above for $x_t = 3, 5, 10$,

and 100 (the latter is essentially indistinguishable from $x_t = \infty$). It can be seen that as x_t increases, the pressure, and therefore the temperature, must increase at each radius in order for the increasing weight of the overlying gas to be supported. There is some resemblance between solutions for models with cutoffs and those outward integrations that have $\tau \rightarrow 0$ at finite radius; however temperatures in the inward integrations with cutoffs are strictly finite except precisely at infinity. Since *ROSAT* observations indicate that intracluster atmospheres can extend beyond 3 Mpc (e. g., Briel *et al.* 1991), I generally assume $x_t = 10$: the temperature solutions inside of $r = 5a$ are not sensitive to the precise value of x_t for x_t on the order of ten or more.

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Table 1
Dimensionless Generic Model Parameters

β	C_2	b_h	C_3	$\mu(3)$	$\mu(x_t)$	$\tau(3)$	ρ_o	Σ_o	M_{nb}/M_b
0.7	31	...	0.0	32	85	5.0	11	23	7.4
0.7	0.99	1.0	15	33	150	6.2	6.3	19	3.7
0.7	0.99	0.5	5.6	33	130	5.8	16	27	11
0.7	0.99	0.1	0.92	33	120	5.5	310	100	230
0.55	40	...	0.0	43	130	8.4	14	31	9.1
0.55	1.3	1.0	20	43	220	11	7.9	26	4.5
0.55	1.3	0.5	7.4	43	190	9.8	21	36	14
0.55	1.3	0.1	1.2	43	170	9.3	400	130	280
0.85	23	...	0.0	25	59	3.2	8.8	18	6.0
0.85	0.75	1.0	11	25	110	3.8	5.0	15	3.0
0.85	0.75	0.5	4.3	25	93	3.6	13	21	9.1
0.85	0.75	0.1	0.70	25	82	3.4	230	76	190

Table 2
Perseus Cluster Model Parameters

b_h	C_3	$M(3a)$ ($10^{14} M_\odot$)	$M(12a)$ ($10^{15} M_\odot$)	$kT(3a)$ (keV)	ρ_o ($10^{-3} M_\odot pc^{-3}$)	Σ_o $gm cm^{-2}$	$\frac{M_{nb}}{M_b}(0)$	$\frac{M_{nb}}{M_b}(3a)$	$\frac{M_{nb}}{M_b}(12a)$
...	0.0	3.0	1.2	5.7	0.72	0.11	4.6	2.7	1.1
1.2	13	2.5	1.6	6.2	0.38	0.084	1.9	2.0	2.0
0.60	5.0	2.5	1.4	5.8	0.88	0.11	5.8	2.1	1.6
0.30	2.5	2.8	1.5	6.1	3.1	0.20	23	2.5	1.7
0.12	0.84	2.7	1.3	5.6	16	0.38	120	2.3	1.4

Table 3
Abell 2256 Model Parameters

b_h	C_3	$M(3a)$ ($10^{14} M_\odot$)	$M(10a)$ ($10^{15} M_\odot$)	$kT(3a)$ (keV)	ρ_o ($10^{-3} M_\odot pc^{-3}$)	Σ_o $gm cm^{-2}$	$\frac{M_{nh}(0)}{M_b}$	$\frac{M_{nh}(3a)}{M_b}$	$\frac{M_{nh}(10a)}{M_b}$
...	0.0	10	3.0	7.5	0.30	0.094	2.7	3.0	2.7
1.5	18	11	5.4	9.9	0.22	0.095	1.7	3.2	5.8
0.73	5.4	9.7	3.9	8.0	0.41	0.11	4.2	2.8	3.9
0.44	2.4	8.8	3.3	7.0	0.77	0.13	8.7	2.5	3.1
0.15	0.48	7.1	2.4	5.4	4.0	0.22	49	1.8	2.0
0.075	0.25	7.3	2.4	5.5	16	0.42	195	1.9	2.0

FIGURE CAPTIONS

Fig. 1.— Emission-averaged, projected dimensionless temperature vs. projected dimensionless radius for four models with $\beta = 0.7$ and $x_t = 10$. The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, dashed, and dot-dashed curves show solutions for the models with pseudo-isothermal non-luminous mass distributions for $b_h = 0.1, 0.5$, and 1.0 , respectively. The models are normalized to have the same total mass at $x = 3$.

Fig. 2.— Same as Fig. 1 for $x_t = 5$.

Fig. 3.— Dimensionless total mass vs. dimensionless radius for the four models detailed in Fig. 1.

Fig. 4.— Dimensionless total mass density vs. dimensionless radius for the four models detailed in Fig. 1.

Fig. 5.— Emission-averaged, projected dimensionless temperature vs. projected dimensionless radius for four models with $\beta = 0.7$ and $x_t = 10$. The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, dashed, and dot-dashed curves show solutions for the models with pseudo-isothermal non-luminous mass distributions for $b_h = 0.1, 0.5$, and 1.0 , respectively. The models are normalized to have similar temperatures for $x > 7$.

Fig. 6a.— Ratio of integrated mass in dark matter to integrated mass in baryons vs. projected dimensionless radius for four models with $\beta = 0.7$ and $x_t = 10$. The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, dashed, and dot-dashed curves show solutions for the models with pseudo-isothermal non-luminous mass distributions for $b_h = 0.1, 0.5$, and 1.0 , respectively.

Fig. 6b. — Same as Fig. 6a for $\beta = 0.55$.

Fig. 6c. – Same as Fig. 6a for $\beta = 0.85$.

Fig. 7a.– Emission-averaged, projected temperature vs. projected radius for five mass models of the Perseus Cluster. The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, long-dashed, short-dashed, and dot-dashed curves show solutions for the models with pseudo-isothermal non-luminous mass distributions for $b_h/b_g = 0.1, 0.25, 0.5$, and 1.0 , respectively. The models have similar total masses at $r \sim 1.3$ Mpc. Also shown are the temperature measurements and 90% confidence limits from Eyles *et al.* (1991).

Fig. 7b.– The inner 500 kpc of the emission-averaged, projected temperature distributions for the models detailed in Fig. 7a with the temperature measurements and 90% confidence limits from Arnaud *et al.* (1993).

Fig. 8.– Total mass vs. radius for the five models detailed in Fig. 7a.

Fig. 9.– Ratio of integrated mass in dark matter to integrated mass in baryons vs. radius for the five models detailed in Fig. 7a.

Fig. 10.– Emission-averaged, projected temperature vs. projected radius for five mass models of Abell 2256. The solid curve shows the solution for the model where the non-luminous mass is proportional to light; the dotted, long-dashed, short-dashed, and dot-dashed curves show solutions for the models with pseudo-isothermal non-luminous mass distributions for $b_h/b_g = 0.1, 0.30, 0.5$, and 1.0 , respectively. The models have similar total masses at $r \sim 0.8$ Mpc.

Fig. 11.– Total mass vs. radius for the five models detailed in Fig. 10.

Fig. 12.– Ratio of integrated mass in dark matter to integrated mass in baryons vs. radius for the five models detailed in Fig. 10.

Fig. A1.– Nine outward integrations of equation (A1) for a model with $\beta = 0.7$,

$C_2 = 0.99$, $b_g = 1.0$, $b_h = 0.5$, $C_3 = 5.64$, and different inner pressure boundary conditions. The central dimensionless temperatures (pressures) for the curves are $\tau_o = 5.9375, 6.0156, 6.0547, 6.0644, 6.0669, 6.0681, 6.0693, 6.0742$, and 6.0938 .

Fig. A2.— Inward integrations of equation (A1) for a model with $\beta = 0.7$, $C_2 = 0.99$, $b_g = 1.0$, $b_h = 0.5$, $C_3 = 5.64$, and different outer pressure boundary conditions. The dimensionless pressures at ∞ are $\psi_\infty = 0, 10^{-5}, 10^{-4}, 10^{-3}$, and 10^{-2} .

Fig. A3.— Same as Fig. A2, with contracted x-scale.

Fig. A4 Inward integrations of equation (A1) for a model with $\beta = 0.7$, $C_2 = 0.99$, $b_g = 1.0$, $b_h = 0.5$, $C_3 = 5.64$, and different truncation radii for the gravitational masses. The dimensionless values of the cutoffs are $x_t = 3, 5, 10$, and 100 .