WIN 94-04
SW 9415

**Maxwell-Chern-Simons Scalar QED
with Magnetic Moment Interactions**

by

M.E. Carrington and G. Kunstatter

*Dept. of Physics and Winnipeg Institute of Theoretical Physics
University of Winnipeg, Winnipeg, Manitoba
Canada R3B 2E9*

ABSTRACT: The interaction of scalar particles in 2+1 dimensions with magnetic moment, λ and electric charge, e , is modeled by a generalized Chern-Simons term that couples the covariant conserved current of the scalar field to the dual of the electromagnetic field strength. In the non-relativistic limit the correct Aharonov-Bohm and Aharonov-Casher phases are recovered, leading to an “anomalous” statistical phase of $e\lambda$ even in the absence of the usual Chern-Simons term. Despite the power counting non-renormalizability of the model, it turns out to be fully one loop renormalizable provided that the usual Chern-Simons action is added as a counter-term.

WIN-94-04

February, 1994

It has been shown by Aharonov and Casher[1] that a particle with anomalous magnetic moment moving in the field of a line charge will develop a nonzero phase. This effect is the analogue of the Aharonov-Bohm phase[2] developed by a charged particle moving in the field of a solenoid. The Aharonov-Bohm phase is $\theta_{AB} = e\Phi$ where e is the charge of the particle and Φ is the flux contained in the solenoid. The Aharonov-Casher phase is $\theta_{AC} = \mu Q$ where μ is the magnetic moment of the particle and Q is the charge per unit length of the line charge.

Theories in two space and one time dimension are currently of great interest both theoretically[3] and because of their potential relevance to such physical effects as the fractional hall effect[4] and high T_c superconductivity[5]. In 2+1 dimensions, a solenoid is effectively a fluxon and a line charge is effectively a point charge. Thus the Lagrangian that describes the exchange of planar particles with both charge and magnetic moment should contain terms which lead to the generation of the Aharonov-Bohm phase and the Aharonov-Casher phase. Moreover, in contrast to the 3+1 dimensional case, it is possible to construct relativistic field theories in two spatial dimensions in which *spinless* particles possess magnetic moments[6]. The purpose of this Letter is to present the results of an analysis of a theory that describes the interactions of scalar particles with both magnetic and electric charge. The magnetic moment interaction term that must be added is closely connected with the Chern-Simons term usually considered in 2+1 dimensions, and gives rise to interesting behaviour at both the classical and quantum levels.

The Lagrangian we wish to consider is,

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) - \frac{m^2}{2}(\phi^*\phi) - \frac{\tau}{6!}(\phi^*\phi)^3 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{mag} \quad (1)$$

where,

$$\mathcal{L}_{mag} = -\frac{i}{4}\gamma\tilde{F}^\mu J_\mu \quad (2)$$

$$J_\mu[\phi, A] = \frac{1}{2}[\phi^*(D_\mu\phi) - (D_\mu\phi)^*\phi]. \quad (3)$$

$\phi = (\chi + i\eta)$ is a complex-valued scalar field, $\{\mu, \nu, \dots = 0, 1, 2\}$ are Lorentz indices and the covariant derivative is:

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi . \quad (4)$$

$\tilde{F}^\mu[A] = \epsilon^{\mu\nu\alpha} F_{\nu\alpha}$ is the dual of the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with $\epsilon^{\mu\nu\alpha}$ the totally antisymmetric tensor density, $\epsilon^{012} = 1$. The Lagrangian Eq.(1) was first considered in [7], where it was shown that the non-minimally coupled term in Eq.(2) was in fact a generalized Chern-Simons term¹. In particular, if the scalar field develops a non-zero vacuum expectation value v , then Eq.(2) takes the form:

$$\frac{1}{4}e\gamma v^2(A_\mu + \frac{1}{e}\partial_\mu\alpha)\tilde{F}^\mu \quad (5)$$

where α is the phase of the scalar field. The second term is a pure divergence, while the first is precisely the Chern-Simons term, with Chern-Simons mass $m_{CS} = e\gamma v^2$. It was also shown in [7] that in the theory with zero bare scalar mass, the non-minimal term induces spontaneous symmetry breaking by radiative corrections. As in the traditional Coleman-Weinberg mechanism[10], the resulting vector and scalar masses are functions of the (renormalized) couplings e , γ and τ .

One of the two main goals of the present work is to argue that γ has a natural physical interpretation as the magnetic moment of the scalar particle described by the field ϕ . The second objective is to prove that, despite power counting non-renormalizability, Maxwell Chern Simons scalar QED with the magnetic moment interactions is fully renormalizable at the one loop level. One of the more interesting features of the analysis is the appearance of the usual Chern-Simons Lagrangian as a counter term, even when it is absent from the bare Lagrangian.

It was first noted by Stern[6] that if J_μ is a one particle classical source, then a term in the matter Lagrangian proportional to $J_\mu \tilde{F}^\mu$ can be interpreted as a generalization of the familiar Pauli magnetic-moment coupling. He argued by analogy with

¹This work in turn was motivated by [8], [9] who considered a similar sort of non-minimal coupling.

the standard Pauli coupling for a Dirac field. By using the $SO(2,1)$ algebra of the $(2+1)$ -dimensional gamma matrices ($[\gamma^\mu, \gamma^\nu] = -2i\epsilon^{\mu\nu\alpha}\gamma_\alpha$), one can rewrite the Pauli coupling as follows:

$$\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi = \frac{1}{2}\tilde{F}^\mu J_\mu \quad (6)$$

where $J_\mu = \bar{\psi}\gamma_\mu\psi$. It is important to note that only in $(2+1)$ dimensions does the tensor structure allow magnetic moment couplings independent of the spin representation. Clearly, the term in Eq.(2) is the scalar field analogue of the right hand side of Eq.(6) above.

In the present case, we can further motivate our physical interpretation of this term as a magnetic moment interaction by taking the non-relativistic limit of Eq.(1). This leads to the Schrodinger equation describing the quantum mechanics of a particle in the presence of an electromagnetic field. In the nonrelativistic limit, the equation of motion for the scalar field has the form,

$$\left(i\mathcal{D}_0 + \frac{1}{2m}\mathcal{D}_i^2\right)\varphi = 0 \quad (7)$$

where $\mathcal{D}_\mu = \partial_\mu + ieA_\mu + i\frac{\gamma}{4}\tilde{F}_\mu$ and φ is the component of the scalar field ϕ that is large in the nonrelativistic limit. It is straightforward to extract the Hamiltonian from the Schrodinger equation above. We use the relations, $E^i = F^{i0}$, $B = -\frac{1}{2}\tilde{F}^0$ and $\epsilon^{ij}E^j = -\frac{1}{2}\tilde{F}^i$ to obtain,

$$H = eA_0 - \frac{\gamma}{2}B - \frac{1}{2m}\left(\nabla^i - ieA^i + i\frac{\gamma}{2}\epsilon^{ij}E^j\right)^2. \quad (8)$$

The first term in Eq.(8) is the potential energy and the second term is the anomalous magnetic interaction. The third term contains the non-minimally coupled covariant derivative that incorporates both the Aharonov-Bohm phase and the Aharonov-Casher phase. To verify that both of these effects are included, we calculate the phase of the wavefunction of the scalar field by looking for solutions of the equation of motion Eq.(7). If $\varphi^{(0)}$ solves the equation,

$$\left(i\partial_0 + \frac{1}{2m}\vec{\nabla}^2\right)\varphi^{(0)} = 0 \quad (9)$$

then the WKB approximation gives,

$$\varphi = \varphi^{(0)} \exp \left(-i \int dx_\mu (eA^\mu + \frac{\gamma}{4} \tilde{F}^\mu) \right). \quad (10)$$

Consider the phase generated by traversing a spatial loop:

$$\theta = \int dx^i (eA^i + \frac{\gamma}{4} \tilde{F}^i). \quad (11)$$

A self-consistent solution for A^i and $\tilde{F}^i$ would have to be obtained from the field equations for the gauge fields. From Eq.(1) we have,

$$\partial^\tau F_{\tau\mu} = ie\mathcal{J}_\mu + \frac{i}{2} \gamma \epsilon_{\mu\tau\lambda} \partial^\tau J^\lambda \quad (12)$$

where

$$\mathcal{J}_\mu = J_\mu + \frac{i}{4} \gamma \phi^* \phi \tilde{F}_\mu \quad (13)$$

is the conserved current associated with the gauge invariance. As a first approximation we will consider $F_{\mu\nu}$ to be the background field generated by a stationary delta-function source. Ignoring higher order interactions, we have $\mathcal{J}_0 = J_0 = -i\delta(R)$; $\mathcal{J}_i = J_i = 0$, where $\delta(R)$ is the two dimensional delta function, (see also [6]). We define $\lambda \equiv \gamma/2$ and rewrite Eq.(12) as,

$$\nabla \cdot E = e_B \delta(R); \quad \nabla B = \lambda_B \nabla \delta(R) \quad (14)$$

Solving for the background field configurations we obtain,

$$e_B A^i = \frac{\lambda_B}{2} \tilde{F}^i = -e_B \lambda_B \epsilon^{ij} \frac{R^j}{R^2}. \quad (15)$$

These solutions correspond to the vector potential, and localized magnetic field strength of a point-like, electrically charged fluxon, with charge e_B and magnetic moment λ_B .

Substituting this background field solution into Eq.(11) gives,

$$\theta = -(e_B \lambda + \lambda_B e) \oint dx^i \epsilon^{ij} \frac{R^j}{R}. \quad (16)$$

Using Stokes' Law and the fact that $\nabla^i(R^i/R) = 2\pi\delta(R)$ in two dimensions we find $\theta = \pm(\lambda_B e + e_B \lambda)$. If we interpret λ as the magnetic moment of the particle (and e as its electric charge), then the first term in this expression gives the contribution from the Aharonov-Bohm effect and the second term gives the contribution from the Aharonov-Casher effect. Thus, our Lagrangian includes both of these effects as claimed, and we conclude that the term we have introduced is necessary to describe the dynamics of charged magnetic particles in 2+1 dimensions. It is interesting to note that if we were to consider the exchange of two scalar particles with charge e and magnetic moment λ then, in the background field approximation $e_B = e$ and $\lambda = \lambda_B$, so that our result shows that there is a statistical phase²

$$\theta = \lambda e. \quad (17)$$

Thus it is in principle possible for such particles to exhibit anomalous statistics without the standard Chern-Simons term in the Lagrangian³.

As a further motivation for the interpretation of our coupling γ as a 2+1 dimensional magnetic moment, we consider the Lagrangian for a fermion theory in 3+1 dimensions with charge $\bar{e}$ and magnetic moment μ . The Lagrangian is,

$$\mathcal{L} = \bar{\psi} \left(i\gamma^\mu \bar{D}_\mu - m - \frac{\mu}{2} \sigma_{\mu\nu} \bar{F}^{\mu\nu} \right) \psi \quad (18)$$

where $\bar{D}_\mu = (\partial_\mu + i\bar{e}\bar{A}_\mu)$. In these expressions, the bars indicate 3+1 dimensional quantities. We rewrite this expression using the relations $\bar{F}^{i0} = \bar{E}^i$ and $\bar{F}^{ij} = -\epsilon^{kij}\bar{B}^k$, and take the nonrelativistic limit of the equation of motion for the spinor field. We take the planar limit by imposing axial symmetry so that $\bar{B}^k = \delta^{k3}\bar{B}$ and $\bar{E}^i = (1 - \delta^{i3})\bar{E}^i$ (*no summation convention*). This limit corresponds to integrating over a

²The factor of two disappears because exchange of identical particles is represented by moving one particle halfway around the other.

³If one were to interpret the fluxons as projections of Nielson-Olesen vortex type solutions, then the magnetic flux would be quantized so that $e\lambda = 2\pi n$, and no anomalous statistics would be observed. It is not clear that this constraint on the magnetic moment is necessary in the planar theory, however.

direction of axial symmetry, which we choose to be the 3 direction. Finally, we rewrite the result in terms of the 2+1 dimensional quantities. The relationship between the 3+1 dimensional quantities and the 2+1 dimensional quantities can be obtained as follows. From the form of the covariant derivative, we have $\bar{e}\bar{A}_\mu = eA_\mu$. Substituting this expression into the Maxwell term we find $\bar{e} = e\sqrt{d}$. Similarly, λ is related to the 3+1 dimensional magnetic moment μ by: $\lambda = \mu/\sqrt{d}$. All of these expressions are consistent dimensionally. The result is,

$$H = eA_0 - \lambda\sigma_3 B - \frac{1}{2m} \left[\left(\nabla^i - ieA^i + i\lambda\sigma_3\epsilon^{ij}E^j \right)^2 - e\sigma_3 B + \lambda\vec{\nabla} \cdot \vec{E} \right] \quad (19)$$

A comparison of Eq.(8) and Eq.(19) leads us to identify $\gamma/2 = \lambda$ as before. Notice that Eq.(19) contains a term $e\sigma_3 B/2m$ that corresponds to the usual magnetic moment interaction for particles with intrinsic spin, in addition to the term $-\lambda\sigma_3 B$ which gives the anomalous interaction, while the Hamiltonian for the scalar theory Eq.(8) contains only the anomalous interaction.

We now summarize our results concerning the quantum field theoretic behaviour of the Lagrangian in Eq.(1) (details can be found in [11]). First we note that the coupling λ has mass dimension of $-\frac{1}{2}$, so that $\mathcal{L}_{mag}$ is power counting non-renormalizable: one expects different divergent diagrams at all orders in perturbation theory. Remarkably, the theory is nonetheless fully renormalizable at one loop. We have checked this explicitly in renormalizable gauge, which allows the straightforward use of power counting arguments. There are several key factors that lead to this result. In 2+1 dimensions all diagrams with superficial degree of divergence $\Delta = 0$ vanish by symmetric integration. In addition, many diagrams with $\Delta > 0$ turn out to be finite because they contain explicit dependence on the external momenta, which reduces the actual degree of divergence. This situation is analogous to that of the box diagram for photon scattering in 3+1 dimensional QED, which is finite despite the fact that it is superficially divergent. Finally, we note that the contribution to the superficial degree of divergence from vertices from $\mathcal{L}_{mag}$ is always less than indicated by power

counting arguments: There are two derivatives in $\mathcal{L}_{mag}$ but, in any given one loop diagram, the vertex from $\mathcal{L}_{mag}$ can only contribute one power of momentum to the loop integral because of the presence of the epsilon tensor. Note that this argument does not hold for higher loop diagrams and thus our proof of renormalizability is not valid beyond one loop.

There are 31 divergent diagrams. We use Pauli-Villars regularization to avoid the problems associated with using dimensional regularization in theories involving epsilon tensors. Using the BPHZ formulation of the renormalization procedure we obtain the following one-loop counterterms[11]:

$$\begin{aligned}\mathcal{L}_{CT} = & (Z_\phi^{-1} - 1)\frac{1}{2}|D_\mu\phi|^2 - (Z_A^{-1} - 1)\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - \frac{5M\delta}{24\pi}\epsilon_{\mu\alpha\nu}A^\mu\partial^\alpha A^\nu - \frac{e^2M}{4\pi}\phi^*\phi - \frac{M}{2\pi 4!}\left(\frac{3\lambda}{10} + 6\delta^2\right)(\phi^*\phi)^2\end{aligned}\quad (20)$$

where,

$$Z_\phi^{-1} = 1 + \frac{\gamma_0^2 M}{4\pi}; \quad Z_A^{-1} = 1 + \frac{\gamma_0^2 M}{12\pi}; \quad Z_e = Z_A^{1/2} \quad (21)$$

and M is the Pauli-Villars mass.

It is interesting to note that the magnetic moment interaction Eq.(2) induces a counterterm of the form of the Chern Simons term. Since a Chern Simons term represents a non-zero photon mass [12] and since the theory does not contain any non-zero diagrams with degree of divergence greater than one, a Chern Simons term in the original Lagrangian will not effect the divergence structure of the theory, and will not effect renormalizability. It is equivalent to say that if we had included a Chern Simons term in our original Lagrangian, we would simply obtain a shift in the counterterms.

At finite temperature, our model exhibits a phase transition at one loop that appears to be first order. The physics of the transition (specifically, the mass ratios) is affected by the presence of Eq.(2). In contrast, the inclusion of a Chern Simons term would only result in a shift of the zero point of the effective potential [13].

Finally, we note that although gauge invariance allows γ in Eq.(2) to be an arbitrary function of the magnitude of the scalar field, $\gamma = \text{constant}$ yields the only term of this form that can be added to the Lagrangian without destroying renormalizability. As shown in [11], if $\gamma = \gamma(|\phi|)$, then renormalizability of the one loop effective potential requires,

$$\gamma = \gamma + \frac{m_{CS}}{e\phi\phi^*} \quad (22)$$

or $\gamma = c/|\phi|$. In Eq.(22), the first term corresponds to $\mathcal{L}_{mag}$ and the second term gives the ordinary Chern Simons term. The second possibility depends on $|\phi|$ and thus does not respect the structure of the original Lagrangian.

To summarize, we have considered scalar QED with magnetic moment interactions. We have shown that it gives rise to both the AB and AC effect in the non-relativistic limit and therefore makes possible anomalous statistics without the standard CS term. The quantum theory, remarkably, is fully one-loop renormalizable, provided a CS counterterm is included. Additional motivation for considering scalar particles with anomalous magnetic moment comes from the work of Kogan[14] who showed that a magnetic moment interaction of the type considered here is induced at one loop in Maxwell-Chern-Simons scalar QED. These results suggest a remarkable duality between CS interactions and magnetic moment interactions (also first noticed by Stern), and indicates that such magnetic moment interactions may be important to our understanding of planar physics.

Acknowledgements We are grateful to M. Burgess for useful discussions and for bringing references[6], [14] to our attention. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

References

- [1] Y. Aharonov and A. Casher, Phys. Rev. Letts. **53**, 319 (1984); For recent experimental verication of this effect see K. Sangster *eta l*, Phys. Rev. Letts. **71**, 3641 (1993).
- [2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
- [3] R. Jackiw and S. Templeton, Phys. Rev. **D23**, 2291 (1981); W. Siegel, Nucl. Phys. **B156**, 135 (1979); J. Schonfeld, Nucl. Phys. **B185**, 157 (1981); S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. **140**, 372 (1982).
- [4] R.B. Laughlin, Science **242**, 525 (1988); R.B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988); A. Fetter, C. Hanna, and R.B. Laughlin, Phys. Rev. **B39**, 9679 (1989); Y.-H. Chen, F. Wilczek, E. Witten and B. Halperin, Int. J. Mod. Phys. **B 3**, 1001 (1989).
- [5] S. Girvin and R. Prange, *The Quantum Hall Effect* (Springer-Verlag, Berlin, 1987); Z. Zhang, T. Hansson and S. Kivelson, Phys. Rev. Lett. **62**, 980 (1989).
- [6] J. Stern, Phys. Letts. **B265**, 119 (1991).
- [7] M.E. Carrington and G. Kunstatter, Phys. Lett.**B321**, 223 (1994).
- [8] S.K. Paul and A. Khare, Phys. Letts. **B193**, 253 (1987).
- [9] J. Kapusta, M.E. Carrington, B. Bayman, D. Seibert, C.S. Song, Phys. Rev. **B44**, 7519 (1991).
- [10] S. Coleman and E. Weinberg, Phys. Rev. **D7**, 1888 (1973).

- [11] M.E. Carrington and G. Kunstatter, University of Winnipeg Preprint WIN-93-10.
- [12] R.D. Pisarski and S. Rao, Phys. Rev. **D32**, 2081 (1985).
- [13] M. Burgess, D.J. Toms and N. Tveten, “Is there a phase transition in Maxwell-Chern-Simons Theory”, University of Oslo preprint Oslo-TP 2-93 (to appear in Phys. Rev. D).
- [14] I.I. Kogan, Phys. Letts. **B262**, 83 (1991).