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RELIABILITY OF ONE-LOOP SPONTANEOUS SYMMETRY BREAKING

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ABSTRACT

The truncated one-loop effective potential indicates the occurrence of spontaneous symmetry breaking in massless ϕ^4 , scalar electrodynamics and standard electroweak theories. We show that renormalization-group analysis reveals the non-reliability of such a conclusion in all three cases.

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I. INTRODUCTION:

Work on spontaneous symmetry breaking (SSB) in field theories and, in particular, discussion of the nature and parameters of phase transitions, are usually formulated in terms of the effective potential (EP). Consideration of these matters would require knowledge of global properties of the EP since, for example, it is the absolute minimum of the EP which determines the true vacuum state of the theory and it is the total effective Euclidean action which determines the decay rate of the metastable phase. There are, however, no closed expressions for the EP of any of the physically interesting theories and one must therefore rely on semi-classical analysis - in particular on the systematic loop expansion [1] of the EP- to obtain any results. Thus great care should be exercised to avoid pitfalls and to draw reliable conclusions. In addition one needs to check that the approximate physical results obtained are gauge-independent in spite of the explicit gauge-dependence of the EP [2,3]. Although the existence of a minimum and the value of the decay rate are gauge-independent for the exact EP, a judicious choice of gauge may be required in an approximate calculation [3].

Use of the EP as a tool for the analysis of global properties of field theories has also recently been brought under suspicion by the claim [4] that an interacting EP may exist for a theory that is known to be trivial. This claim, within the gaussian approximation, has drawn some criticism and prompted current discussion of the role of the EP in field theory [5], where some authors have suggested that only effective action analysis may be trusted. In a recent contribution to this discussion [6], Tarrach arrives at the conclusion that even the exact EP of ϕ^4 in more than four dimensions - rigorously proven trivial - is in fact interacting. Such a conclusion needs to be closely re-examined since, if it holds, one would seriously question the utility of the concept of the EP in field theory.

In this paper we are modestly concerned with the validity of one-loop analysis of SSB in massless theories. Concern about the validity of the loop expansion of the EP for the investigation of SSB has previously been expressed by several authors. Appelquist and Heinz [7], for example, note that for an $O(N)$ model the loop-expansion, unlike the $\frac{1}{N}$ expansion, leads to the wrong conclusion on SSB. Callaway and Maloof [8] have proved, by non-perturbative arguments, that the loop expansion in ϕ^4 theory must fail for a range of ϕ if SSB occurs. This generalizes to $O(N)$ scalar theory [9] and the conclusion is reached that perturbative methods are not adequate for analysis of SSB.

In reference [10] it is shown that if one assumes SSB in a massless scalar theory in which either all couplings are asymptotically free (AF) or all couplings are not asymptotically free (NAF) then one can prove a general result on the behaviour of $\phi_o(g)$ the value of the classical field at the minimum of the EP, as the couplings g tend to vanish: $\phi_o(g) \rightarrow 0$ (∞) for AF (NAF) theories. This general result contradicts the one-loop result for both massless ϕ^4 and massless scalar electrodynamics (SED). That the one-loop result for ϕ^4 is unreliable is, of course, already known from the original work of Coleman and Weinberg [11].

Problems with the loop expansion have also been noted within the finite-temperature formalism. An example is the recent work of Brahm and Hsu [2] on the electroweak phase transition. They find that, due to the presence of non-abelian fields, the loop expansion must break down and needs to be modified before reliable conclusions are drawn.

Our tool of analysis in the present paper is the renormalization group equation (RGE) of the EP, which was also employed in reference [10] and in our recent work on the decay of the false vacuum [12]. Use of the RGE enables one to obtain, under certain conditions, essentially non-perturbative

results from perturbative expressions. One may, for example, obtain the asymptotic form of the exact EP $V(\phi)$ from its one-loop expression $V^{(1)}(\phi)$ and the one-loop renormalization group (RG) functions β, γ, γ_m . An example of this is the small- ϕ form of the exact EP of massless ϕ^4 theory, obtained in reference [13], and the corollary that, for $\lambda > 0$, it possesses a definite minimum at $\phi = 0$. Such results, for the EP, are similar to those on the large-momentum behaviour of green's functions in early applications [14] of the renormalization group.

It is our purpose to reconsider the question of the discrepancy between general RG considerations and results on SSB in a massless field theory based on the truncated one-loop EP $V^{(1)}(\phi)$. Since the asymptotic method of reference [10] is not applicable to the standard model (SM), we conduct a direct comparison between $V^{(1)}(\phi)$ and the RG-improved one-loop EP $V_{RG}^{(1)}(\phi)$, as regards analysis of SSB, for the three massless theories under consideration: ϕ^4 , SED and SM. The results confirm the conclusion that $V^{(1)}(\phi)$ is not, in general, a reliable probe of SSB and give insight as to why this is so.

After a brief review of the previous results on massless ϕ^4 and SED, we consider in section II analysis of SSB based on $V_{RG}^{(1)}(\phi)$, the exact solution of the RGE with one-loop RG functions, normalized to coincide with $V_{RG}^{(1)}(\phi)$ at $\phi = \mu$, the renormalization point. For massless ϕ^4 , one finds that $V_{RG}^{(1)}(\phi)$ does not admit SSB so that the result based on $V_{RG}^{(1)}(\phi)$ is false. For massless SED, the validity of $V_{RG}^{(1)}(\phi)$ as an approximation to $V(\phi)$ is limited to a finite range in ϕ . If the minimum in $V_{RG}^{(1)}(\phi)$ exists within this range, it is not - for general perturbative values of λ and e^2 - approximated by the minimum in $V^{(1)}(\phi)$, which exists for all such values. Further, a minimum in $V_{RG}^{(1)}(\phi)$ can exist only if e^2 exceeds a certain critical value e_c^2 .

For massless SM, one may take a value for the top quark mass m_t such that $V^{(1)}(\phi)$ exhibits SSB for all perturbative values of the scalar and gauge couplings. As for SED, $V_{RG}^{(1)}(\phi)$ is a valid approximation within a finite range in ϕ . One finds that, for very small values of the scalar coupling λ , the minimum of $V^{(1)}(\phi)$ lies well outside the range of validity of $V_{RG}^{(1)}(\phi)$ and therefore also of $V^{(1)}(\phi)$. Thus the conclusion, based on $V^{(1)}(\phi)$, of the occurrence of SSB for general perturbative values of the couplings, is unreliable.

In section III we give a summary of the results and a discussion of their significance. In particular we point out that a minimum in $V^{(1)}(\phi)$ approximates a true minimum of the exact effective potential $V(\phi)$ only when a certain condition on the couplings is satisfied. This condition requires that the one-loop quantum correction be of the same order as the tree-level classical contribution. The postulate that such a condition holds would lead to a determination of the scalar coupling in terms of the gauge and fermion couplings. In the case of the massless SM this would yield an estimate of the Higgs boson mass. There is, however, no physical justification for such a postulate.

We also remark in section IV on the implication of the SSB condition $e^2 > e_c^2$ for the phase structure of massless SED. It is pointed out that numerical computations show that in massless QED the broken phase occurs in $e^2 > e_c^2$ where $e_c^2 \approx 2.8$. It is thus probable that, for perturbative λ , massless SED may have a phase diagram similar to that of massless QED.

II FORMALISM AND APPLICATION:

It has been shown in reference [10] that general results that follow from the RGE for massless theories with SSB could be at variance with deductions based on the perturbative one-loop EP. Of direct relevance to the reliability of the one-loop EP in analysis of SSB is the result on the small-coupling behaviour of the minimum ϕ_0 of the EP in these theories.

Consider a renormalizable massless scalar field theory with a single dimensionless coupling λ . The RGE of the EP in the mass-independent formulation [15],

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma(\lambda) \phi \frac{\partial}{\partial \phi} \right\} V(\phi) = 0 \quad , \quad (2.1)$$

admits the solution

$$V(\phi, \lambda, \mu) = \mu^4 F\left(\frac{\phi^2}{\mu^2} \exp. \left[2 \int_{\lambda}^1 \frac{\gamma(x) - 1}{\beta(x)} dx \right] \right) \exp. \left[4 \int_{\lambda}^1 \frac{dx}{\beta(x)} \right] \quad , \quad (2.2)$$

where $F(x)$ is arbitrary and x is dimensionless.

Assuming SSB, $V(\phi)$ possess an absolute minimum at $\phi = \phi_0 \neq 0$. Thus $F(x)$ has an absolute minimum at $x = x_0 \neq 0$, where x_0 is a pure number, and equation (2.2) implies that

$$\phi_0 = x_0 \mu^2 \exp. \left[2 \int_{\lambda}^1 \frac{1 - \gamma(x)}{\beta(x)} dx \right] \quad . \quad (2.3)$$

This equation determines the dependence of ϕ_0 on the coupling λ if the RG functions $\beta(\lambda)$ and $\gamma(\lambda)$ are known. In perturbation theory both $\beta(\lambda)$ and $\gamma(\lambda)$ vanish at $\lambda=0$. Thus $(1 - \gamma(x)) \beta^{-1}(x) \approx \beta^{-1}(x)$ as $x \rightarrow 0$. This behaviour dominates the integral in equation (2.3) as $\lambda \rightarrow 0$ and one finds

$$\phi_0^2 \approx x_0 \mu^2 \exp. \left[2 \int_{\lambda}^1 \frac{dx}{\beta(x)} \right] \quad (2.4)$$

as $\lambda \rightarrow 0$. In particular the sign of $\beta(\lambda)$ near $\lambda=0$ determines whether in the small-coupling limit $\phi_0 \rightarrow 0$ or ∞ : For AF theories $\phi_0 \rightarrow 0$ while for NAF

theories $\phi_0 \rightarrow \infty$ as $\lambda \rightarrow 0$. Although it is based on the perturbative expression for $\beta(\lambda)$, this general result constitutes a significant check on any approximation that indicates SSB in the EP.

In the case of two dimensionless couplings λ_1, λ_2 with RG functions $\beta_1(\lambda_1, \lambda_2)$, $\beta_2(\lambda_1, \lambda_2)$, and $\gamma(\lambda_1, \lambda_2)$, the solution to the RGE is

$$V(\phi, \lambda_1, \lambda_2, \mu) = \mu^4 e^{4\zeta} F\left(\frac{\phi^2}{\mu^2} \exp. [2 \int_0^\zeta (\gamma(t) - 1) dt], u\right) \quad , \quad (2.5)$$

where $\zeta \equiv \zeta(\lambda_1, \lambda_2)$ is defined by

$$\int_0^\zeta \beta_2(t) dt = 1 - \lambda_2 \quad , \quad (2.6)$$

and $u \equiv u(\lambda_1, \lambda_2)$ is given by

$$u = \lambda_1 + \int_0^\zeta \beta_1(t) dt \quad . \quad (2.7)$$

The notation is such that $\beta(t)$ denotes $\beta(\lambda_1(t), \lambda_2(t))$ where $\lambda_1(t), \lambda_2(t)$ are the couplings. For the occurrence of SSB we require the existence of an absolute minimum of $F(x, y)$ at $x = x_0(y)$. Then $V(\phi)$ has an absolute minimum at $\phi = \phi_0$ where

$$\phi_0^2 = x_0(u) \mu^2 \exp. [2 \int_0^\zeta (1 - \gamma(t)) dt] \quad . \quad (2.8)$$

This implies that on the trajectory $u(\lambda_1, \lambda_2) = \text{constant}$, the dependence of ϕ_0 on the couplings λ_1, λ_2 is specified by the RG functions. For small λ_1 and λ_2 the one-loop RG functions can be used to determine $\zeta(\lambda_1, \lambda_2)$ and $u(\lambda_1, \lambda_2)$. It would then be possible to approach the limit $(\lambda_1, \lambda_2) \rightarrow (0, 0)$ with $u(\lambda_1, \lambda_2)$ fixed at a constant value u_0 if λ_1 and λ_2 are both AF or both NAF. In the first case $\phi_0 \rightarrow 0$ while in the later $\phi_0 \rightarrow \infty$.

These RG results are directly applicable to the cases of massless ϕ^4 and massless SED. One finds [10] that analysis based on the one-loop EP is in conflict with these general results for both theories. In each case the one-loop EP $V^{(1)}(\phi)$ possesses a minimum at $\phi = \phi_1$, say, which does not exhibit the expected RG behaviour. These minima are therefore false and one must reject the conclusion of SSB based on them.

It is known, however, that a perturbative expression $V^{(1)}(\phi)$ for the one-loop EP may be extended into an exact solution $V_{RG}^{(1)}(\phi)$ of the RGE. Since $V_{RG}^{(1)}(\phi)$ is an exact solution of the RGE, any minima ϕ_0 it may possess would automatically satisfy the RG results on small-coupling behaviour discussed in section II. Possible conflict, on SSB, with $V^{(1)}(\phi)$ may therefore be seen by direct comparison of $V_{RG}^{(1)}(\phi)$ and $V^{(1)}(\phi)$. It may even be possible to determine whether SSB occurs in $V_{RG}^{(1)}(\phi)$ within its region of validity. Since these considerations do not require that all couplings be AF or NAF, they are also applicable to the case of the massless SM.

For a renormalizable massless field theory with a scalar field of dimensionless self-coupling λ and other fields with dimensionless couplings g_k , the RGE for the EP $V(\phi, \lambda, g, \mu)$ is

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta(\lambda, g) \frac{\partial}{\partial \lambda} + \beta_k(\lambda, g) \frac{\partial}{\partial g_k} + \gamma(\lambda, g) \phi \frac{\partial}{\partial \phi} \right\} V(\phi) = 0, \quad (2.9)$$

where, for gauge theories, we assume the Landau gauge. The form of the solution to equation (2.9) which we use in this section is [16]

$$V(\phi) = F(\bar{\lambda}(t), \bar{g}(t)) \phi^4 \exp. \left[4 \int_0^t \frac{\gamma(t')}{1 - \gamma(t')} dt' \right], \quad (2.10)$$

where $t = \log \frac{\phi}{\mu}$ and

$$\frac{d\bar{\lambda}(t)}{dt} = \frac{\beta(t)}{1 - \gamma(t)}, \quad \frac{d\bar{g}_k(t)}{dt} = \frac{\beta_k(t)}{1 - \gamma(t)}, \quad (2.11)$$

with $\bar{\lambda}(0) = \lambda$, $\bar{g}_k(0) = g_k$. The notation is such that $\gamma(t) = \gamma(\bar{\lambda}(t), \bar{g}(t))$. The arbitrary function $F(\lambda, g)$ is fixed if the value of $V(\phi)$ at $\phi = \mu$ is given:

$$F(\lambda, g) = \mu^{-4} V(\mu, \lambda, g, \mu). \quad (2.12)$$

Since we are going to use the solution (2.10) when the RG functions are given by their one-loop expressions, we shall normalize $V(\phi)$ of (2.10) so that it agrees with the perturbative one-loop expression $V^{(1)}(\phi)$ at $\phi = \mu$. With this

normalization $F(\lambda, g) = \mu^{-4} V^{(1)}(\mu)$. For the theories under consideration - massless ϕ^4 , SED and minimal SM - the one-loop EP is of the form

$$V^{(1)}(\phi) = c\lambda\phi^4 + f(\lambda, g)\phi^4 \log \frac{\phi^2}{\mu^2} \quad (2.13)$$

where c is a constant. The normalization $V(\mu) = V^{(1)}(\mu)$ then yields $F=c\lambda$ so that (2.10) reduces to

$$V(\phi) = c\bar{\lambda}(t)\phi^4 \exp. [4 \int_0^t \frac{\gamma(t')}{1-\gamma(t')} dt'] . \quad (2.14)$$

We now note that when the RG functions are restricted to their one-loop expressions, one cannot distinguish $\beta(1-\gamma)^{-1}$ and $\gamma(1-\gamma)^{-1}$ from β and γ for ϕ^4 theory, and also for SED and the SM when $\lambda = 0(g^2)$. The one-loop RG-improved EP then takes the form:

$$V_{RG}^{(1)}(\phi) = c\lambda(t)\phi^4 \exp. [4 \int_0^t \gamma(t') dt'] , \quad (2.15)$$

where, in this expression, one must use the one-loop RG functions, while $\gamma(t) = \gamma(\lambda(t), g(t))$ with $\lambda(t)$, $g(t)$ the usual running couplings and $t = \log \frac{\phi}{\mu}$. The range of validity of $V_{RG}^{(1)}(\phi)$ as an approximation to $V(\phi)$ includes $t = 0$, where it is normalized to $V^{(1)}(\mu)$, and extends over values of t such that $\lambda(t)$ and $g_k(t)$ remain perturbative with first-order dominance.

We discuss the possibility of SSB in $V_{RG}^{(1)}(\phi)$ for each of the theories under consideration.

2.1 Massless ϕ^4

In this case $\lambda(t) = \lambda(1 - \frac{3\lambda}{(4\pi)^2} t)^{-1}$. Thus $V_{RG}^{(1)}(\phi)$ is positive throughout its range of validity and there is no possibility of SSB in $V_{RG}^{(1)}(\phi)$. The one-loop EP

$$V^{(1)}(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\lambda\phi^4}{256\pi^2} \log \frac{\phi^2}{\mu^2}, \quad (2.16)$$

possesses a minimum at $\phi \equiv \phi_1 = \mu \exp. \left[\frac{-16\pi^2}{3\lambda} - \frac{1}{4} \right]$ where $V^{(1)}(\phi_1) < 0$.

This minimum cannot approximate a true minimum of the exact EP $V(\phi)$.

The reason for the failure of the prediction of $V^{(1)}(\phi)$ is obvious : Since $V^{(1)}(\phi_1) < 0$, while $V_{RG}^{(1)}(\phi) \geq 0$ for all ϕ , $\phi = \phi_1$ lies outside the range of validity of $V^{(1)}(\phi)$.

One further notes that the validity of $V_{RG}^{(1)}(\phi)$, which is given by

$$V_{RG}^{(1)}(\phi) = \frac{1}{4!} \frac{\lambda \phi^4}{1 - \frac{3\lambda}{16\pi^2} \log \frac{\phi}{\mu}} \exp. \left\{ \frac{\lambda^2 \log \frac{\phi}{\mu}}{3(4\pi)^4 [1 - \frac{3\lambda}{16\pi^2} \log \frac{\phi}{\mu}]} \right\} , \quad (2,17)$$

extends down to $\phi = 0$. For fixed λ the asymptotic behaviour near $\phi = 0$ is

$$V_{RG}^{(1)}(\phi) \approx \frac{2\pi^2}{9} \exp. \left(\frac{-\lambda}{144\pi^2} \right) \frac{\phi^4}{|\log \frac{\phi}{\mu}|} , \quad (2,18)$$

which coincides with that of the exact EP as given in reference [13]. It is obvious from (2.18) that $V^{(1)}(\phi)$ is not valid near $\phi = 0$.

We should perhaps stress that no claim is made here to the effect that SSB does not occur in massless ϕ^4 theory. Our result is that $V_{RG}^{(1)}(\phi)$ does not display SSB and therefore its indication by $V^{(1)}(\phi)$ is false and should not be trusted. There are indeed recent claims [17] that the Gaussian approximation shows massless ϕ^4 theory to be both asymptotically free and spontaneously broken. While it is not easy to understand asymptotic freedom of massless ϕ^4 in contradiction to the perturbative result (except perhaps in the context of its triviality), the conclusion on SSB is not in disagreement with our analysis which is limited to the utility of the one-loop EP in investigating SSB.

2.2 Massless Scalar Electrodynamics:

In this case one finds

$$V_{RG}^{(1)}(\phi) = \frac{1}{4!} \lambda(t) \phi^4 \left[1 - \frac{e^2}{24\pi^2} \log \frac{\phi}{\mu} \right]^{-\frac{9}{2}} , \quad (2,19)$$

where $t = \log \frac{\phi}{\mu}$ and

$$\lambda(t) = \lambda\eta(t) \frac{1 - A \tan[\alpha \log \eta(t)]}{1 - B \tan[\alpha \log \eta(t)]},$$

$$e(t) = e[\eta(t)]^{\frac{1}{2}}, \eta(t) = [1 - \frac{e^2}{24\pi^2} t]^{-1},$$

$$\alpha = \frac{1}{2}\sqrt{719}, A = \frac{1}{2\alpha}(19 - 108\frac{e^2}{\lambda}), B = \frac{1}{2\alpha}(10\frac{\lambda}{e^2} - 19). \quad (2.20)$$

The function $\lambda(t)$ blows up at $t=t_L$ and $t=t_R$ where $t_L < 0 < t_R$. Thus the validity of $V_{RG}^{(1)}(\phi)$ is limited to a region about $\phi = \mu$ that cannot extend to the points $\phi = \phi_L$ and $\phi = \phi_R$ to the left and right of $\phi = \mu$. Further, $\beta(\lambda) > 0$ so that $\lambda(t)$ is an increasing function of t which vanishes at $t=t_0$ and has the general shape shown in Fig. 1.

The constants A, B of equation (2.20) are such that $B > A$ for all values of e^2/λ . To be definite let us take $\frac{e^2}{\lambda} < \frac{19}{108}$. Then $B > A > 0$ and for $k = 0, R, L$:

$$t_k = \frac{24\pi^2}{e^2} \left\{ 1 - \exp. \left(\frac{-\theta_k}{\alpha} \right) \right\}, \quad (2.21)$$

where

$$\tan \theta_0 = 1/A, -\pi < \theta_0 < \frac{-\pi}{2}; \tan \theta_R = \tan \theta_L = \frac{1}{B}, \quad (2.22)$$

$$0 < \theta_R < \pi/2, \theta_L = \theta_R - \pi$$

Note that the Landau pole at $t = \frac{24\pi^2}{e^2}$ is outside the interval (t_L, t_R) .

To compare $V_{RG}^{(1)}(\phi)$ and $V^{(1)}(\phi)$ where

$$V^{(1)}(\phi) = \frac{\lambda}{4!} \phi^4 + \left(\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2} \right) \phi^4 \log \frac{\phi^2}{\mu^2} \quad (2.23)$$

let us assume that the region of validity of $V_{RG}^{(1)}(\phi)$ extends to include $\phi = \phi_0$ given by

$$\phi_0 = \mu \exp. \left[\frac{24\pi^2}{e^2} \left\{ 1 - \exp. \left(\frac{-\theta_0}{\alpha} \right) \right\} \right] \quad (2.24)$$

where $V_{RG}^{(1)}(\phi_o) = 0$. Both $V_{RG}^{(1)}(\phi)$ and $V^{(1)}(\phi)$, are assumed to be valid at $\phi = \mu$ where they have been set equal. Further, $V^{(1)}(\phi)$ also possess a zero at $\phi = \phi_o^1$ where

$$\phi_o^1 = \mu \exp. \left[\frac{-24\pi^2 \lambda}{5\lambda^2 + 54e^4} \right] \quad (2.25)$$

and $\phi_o, \phi_o^1 < \mu$. It is, however, clear that ϕ_o^1 cannot - for general small values of λ and e^2 - be an approximation to ϕ_o . For example, as $e^2 \rightarrow 0$ at fixed λ , $\phi_o \rightarrow 0$ whereas ϕ_o^1 tends to the non-vanishing λ -dependent value $\mu \exp. \left(\frac{-24\pi^2}{5\lambda} \right)$.

Thus the validity of $V^{(1)}(\phi)$ does not extend from $\phi = \mu$ down to its zero at $\phi_o = \phi_o^1$. Since its minimum, at $\phi = \phi_1$ where

$$\phi_1^2 = \mu^2 \exp. \left[\frac{-\lambda}{4!} \left(\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2} \right)^{-1} - \frac{1}{2} \right], \quad (2.26)$$

is even below $\phi = \phi_o^1$, it is definitely outside the range of validity of $V^{(1)}(\phi)$ and is not a reliable approximation to a true minimum of the exact EP.

One may now ask whether $V_{RG}^{(1)}(\phi)$ possesses a minimum. It is easily seen [16] that the condition for a turning point in the EP with the integral representation (2.14) is the existence of a solution $t=t_m$ to

$$\beta_\lambda(\bar{\lambda}(t_m), \bar{e}(t_m)) + 4\bar{\lambda}(t_m) = 0. \quad (2.27)$$

Using the explicit expression for β_λ we find that equation (2.27) requires at the one-loop level

$$\lambda(t_m) < 0, \left(\frac{63}{32\pi^2} e^2(t_m) + 9 \right)^2 e^2(t_m) > \frac{75}{8\pi^2}. \quad (2.28)$$

These conditions imply that $t_m < t_o$ and $e^2(t_m) > 0.0117$. But $e^2(t)$ is an increasing function. Thus $e^2(t_o) > 0.0117$. In particular

$$e^2(t_o(\max)) > 0.0117 \quad (2.29)$$

where, from (2.21) ,

$$t_o(\max) = \frac{-24\pi^2}{e^2} \left\{ \exp\left(\frac{\pi}{2\alpha}\right) - 1 \right\} . \quad (2.30)$$

Equations (2.29) and (2.30) give $e^2 > 0.013$.

Thus a necessary condition for the existence of any turning points in $V_{RG}^{(1)}(\phi)$ is that e^2 should exceed a certain critical value $e_c^2 \geq 0.013$. The fact that a minimum does not exist when $e^2 < e_c^2$ contradicts the analysis based on $V^{(1)}(\phi)$ which indicates the existence of a minimum for all perturbative λ and e^2 . It is, however, a result which is consistent with the absence of a minimum in $V_{RG}^{(1)}(\phi)$ for ϕ^4 theory since SED merges into ϕ^4 as $e^2 \rightarrow 0$.

We now consider the significance, for SSB, of a minimum in $V_{RG}^{(1)}(\phi)$ when such a minimum exists and lies within the region of validity of the approximation. In this respect one observes that condition (3.15) implies that $V_{RG}^{(1)}(\phi)$ can have at most one minimum, at $\phi = \phi_m$ say, and one minimum at $\phi = \phi_M$ where $0 < \phi_M < \phi_m < \phi_o$ and $V_{RG}^{(1)}(\phi) < 0$ at both $\phi = \phi_m$ and $\phi = \phi_M$: Fig. 2. It is then clear that one cannot ascertain that $\phi = \phi_m$ is an absolute minimum of $V_{RG}^{(1)}(\phi)$. Furthermore, it is not possible to link this minimum to the value of the EP at $\phi = 0$ since $V_{RG}^{(1)}(\phi)$ is not valid at $\phi = 0$. It thus appears that no reliable conclusion on SSB of massless SED can be drawn on the basis of $V_{RG}^{(1)}(\phi)$.

There is, however, in this situation a global observation which is worth noting. In the neighbourhood of $\phi = \phi_o$, where $\lambda(t_o) \approx 0$ and $e(t_o) < e$, one expects $V_{RG}^{(1)}(\phi)$ to be a reliable approximation to the exact EP $V(\phi)$ (which is assumed to be normalized so that $V(\mu) = \frac{1}{4!} \lambda \mu^4$). One may then state that $V(\phi)$ assumes negative values for a range of ϕ . If $V(0)=0$ and $V(\phi) \rightarrow \infty$ - or some non-negative value - as $\phi \rightarrow \infty$, then $V(\phi)$ would possess an absolute minimum at which it is negative. Thus SSB probably occurs in massless SED. One notes, however, that the validity of $V_{RG}^{(1)}(\phi)$ is limited to a finite range that excludes the origin and there is no guarantee that $V_{RG}^{(1)}(\phi)$ is

reliable at this absolute minimum. The minimum of $V_{RG}^{(1)}(\phi)$ may thus be irrelevant to the analysis of SSB in massless SED.

2.3 Massless Standard Model:

The one-loop perturbative EP of the massless minimal standard model is

$$V^{(1)}(\phi) = \frac{1}{4} \lambda \phi^4 + f \phi^4 \log \frac{\phi^2}{\mu^2} \quad , \quad (2.31)$$

where

$$f = \frac{1}{64\pi^2} \left[\frac{3}{16} (g_1^4 + 3g_2^4 + 2g_1^2 g_2^2) - 3h^4 + 12\lambda^2 \right] \quad , \quad (2.32)$$

assuming, as usual, that the top-quark coupling h is dominant over the other fermion couplings. This potential possesses a turning point at $\phi = \phi_1$ where

$$\phi_1 = \mu \exp. \left[-\frac{1}{4} - \frac{1}{8} \frac{\lambda}{f} \right] \quad , \quad V^{(1)}(\phi_1) = -\frac{1}{2} f \phi_1^4 \quad . \quad (2.33)$$

It is a maximum with $V^{(1)}(\phi_1) > 0$ if $f < 0$ and a minimum with $V^{(1)}(\phi_1) < 0$ if $f > 0$. Thus the one-loop EP exhibits SSB provided that

$$4\lambda^2 - h^4 + \frac{1}{16} (g_1^4 + 3g_2^4 + 2g_1^2 g_2^2) > 0. \quad (2.34)$$

This condition translates into an inequality that involves the top and Higgs masses:

$$m_t^4 < \frac{1}{9} m_H^4 + (130 \text{Gev})^4 \quad , \quad (2.35)$$

using the physical values of the couplings and gauge boson masses. Thus (3.23) is always satisfied if $m_t \leq 130 \text{ Gev}$ and gives a lower bound on m_H if $m_t > 130 \text{ Gev}$. One may therefore assume the condition (3.23) to be satisfied, so that a minimum in $V^{(1)}(\phi)$ exists.

It is our purpose now to investigate SSB in the massless standard model using the RG-improved one-loop EP $V_{RG}^{(1)}(\phi)$ and, in particular, to

determine whether the conclusion based on $V^{(1)}(\phi)$ and the condition (2.34) stands the scrutiny of RG analysis.

The one-loop RG functions are, in the Landau gauge, given by

$$\begin{aligned} 16\pi^2\beta_\lambda &= 24\lambda^2 + 3\lambda(4h^2 - g_1^2 - 3g_2^2) + \frac{3}{8}(g_1^4 + 3g_2^4 + 2g_1^2g_2^2) - 6h^4, \\ 16\pi^2\beta_h &= h\left(\frac{9}{2}h^2 - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \\ 16\pi^2\gamma &= -3\left(h^2 - \frac{1}{4}g_1^2 - \frac{3}{4}g_2^2\right). \end{aligned} \quad (2.36)$$

For the running gauge couplings the one-loop expressions are

$$\begin{aligned} g_1^2(t) &= g_1^2 \left(1 - \frac{41g_1^2 t}{48\pi^2}\right)^{-1}, \quad g_2^2(t) = g_2^2 \left(1 + \frac{19g_2^2 t}{48\pi^2}\right)^{-1}, \\ g_3^2(t) &= g_3^2 \left(1 + \frac{7g_3^2 t}{8\pi^2}\right)^{-1}. \end{aligned} \quad (2.37)$$

One then finds

$$h^2(t) = h^2 \frac{X(t)}{1 - \frac{9}{16\pi^2} h^2 \int_0^t X(t') dt'}, \quad (2.38)$$

where

$$X(t) = \left\langle \frac{g_1^2(t)}{g_1^2} \right\rangle^{-\frac{17}{82}} \left\langle \frac{g_2^2(t)}{g_2^2} \right\rangle^{\frac{27}{38}} \left\langle \frac{g_3^2(t)}{g_3^2} \right\rangle^{\frac{8}{7}}, \quad (2.39)$$

and

$$V_{RG}^{(1)}(\phi) = \frac{1}{4} \lambda(t) \frac{\left(1 + \frac{19g_2^2 t}{48\pi^2}\right)^{\frac{27}{19}}}{\left(1 - \frac{41g_1^2 t}{48\pi^2}\right)^{\frac{9}{41}}} \phi^4 \exp\left[-\frac{3}{4\pi^2} \int_0^t h^2(t') dt'\right] \quad (2.40)$$

where $t = \log \phi/\mu$. Inspection of $V_{RG}^{(1)}(\phi)$, using equations (2.36) - (2.39) reveals that the region of its validity is limited to an interval about $\phi = \mu$ within (ϕ_L, ϕ_R) where $\phi = \phi_L$ and $\phi = \phi_R$ are the points at which the expression (2.40) blows up:

$$\phi_L = \mu \exp\left(\frac{-8\pi^2}{7g_3^2}\right), \quad \phi_R = \mu \exp\left(\frac{48\pi^2}{41g_1^2}\right). \quad (2.41)$$

Thus, as for massless SED, the validity of $V_{RG}^{(1)}(\phi)$ does not extend to $\phi = 0$.

To simplify the comparison of $V^{(1)}(\phi)$ and $V_{RG}^{(1)}(\phi)$ take, at $t=0$, $h^4 = \frac{1}{16} (g_1^4 + 3g_2^4 + 2g_1^2 g_2^2)$. Then condition (2.34) is always satisfied and $V^{(1)}(\phi)$ exhibits SSB and

$$\phi_1 = \mu \exp. \left(-\frac{1}{4} - \frac{2\pi^2}{3\lambda} \right), \quad (2.42)$$

for all values of the gauge couplings. It is clear that, for small λ , ϕ_1 is well outside the region of validity of $V_{RG}^{(1)}(\phi)$ and therefore also of $V^{(1)}(\phi)$. In fact for all values of λ such that

$$\lambda < \frac{7g_3^2}{12(1 - \frac{7g_3^2}{32\pi^2})}, \quad (2.43)$$

one has $\phi_1 < \phi_L$, so that ϕ_1 lies in a region where $V_{RG}^{(1)}(\phi)$ is not even defined. Thus the existence of this minimum is not relevant to SSB and the conclusion based on $V^{(1)}(\phi)$ is not reliable.

Let us now consider $V_{RG}^{(1)}(\phi)$ for the same value of h^2 . In this case one can show that $\frac{dV_{RG}^{(1)}}{d\phi} \big|_{\phi=\mu} > 0$ and the expression in equation (2.40) must have at least one minimum in (ϕ_L, μ) . For the existence of a minimum of $V_{RG}^{(1)}$ in (ϕ_L, μ) within the region of validity of $V_{RG}^{(1)}(\phi)$, with $V_{RG}^{(1)} < 0$, $\lambda(t)$ must vanish at $t = t_0 < 0$ where $|t_0| \ll |t_L| = \frac{8\pi^2}{7g_3^2}$. A linear approximation to $\lambda(t)$ for $t \in (t_0, 0)$, $\lambda(t) = \lambda + \beta_\lambda t$, yields $t_0 = \frac{-\lambda}{\beta_\lambda}$. The

condition $t_L \ll t_0 < 0$ gives

$$\lambda \gg \frac{7g_3^2}{12} + \frac{1}{8} \left[(g_1^2 + 3g_2^2) - (g_1^4 + 3g_2^4 + 2g_1^2 g_2^2)^{\frac{1}{2}} \right]. \quad (2.44)$$

Thus, as anticipated from (2.43), negative values of $V_{RG}^{(1)}(\phi)$ are realized within the range of validity of the approximation only for values of λ above a critical limit. Since the validity of $V_{RG}^{(1)}(\phi)$ is, in any case, restricted to a finite region about $\phi = \mu$ that does not include $\phi = 0$, analysis of its minima appears to be irrelevant to SSB.

III RESULTS AND DISCUSSION;

(1) The main result of this paper is that the indication of SSB in massless ϕ^4 , scalar electrodynamics and minimal electroweak theories by the truncated one-loop effective potential $V^{(1)}(\phi)$ is revealed to be generally false when examined under the constraints of the renormalization group equation.

(2) For massless ϕ^4 the inadequacy of $V^{(1)}(\phi)$ for SSB was pointed out in the original paper of Coleman and Weinberg [11] who observed that the predicted minimum was, for small λ , outside the range of validity of $V^{(1)}(\phi)$. Then reference [10] found the minima predicted by $V^{(1)}(\phi)$ for both massless ϕ^4 and SED to be in conflict with the RGE regarding their behaviour as the couplings tend to vanish along prescribed trajectories.

The conclusion of reference [10] that one cannot generally accept the minimum of $V^{(1)}(\phi)$ as an approximate minimum of $V(\phi)$ for small couplings is not in disagreement with the earlier result of reference [11] that the minimum of $V^{(1)}(\phi)$ for massless SED is reliable when $\lambda = ke^4$, $k=0(1)$. In fact when $\lambda = ke^4$ it is not possible to reduce (λ, e^2) to $(0,0)$ along the trajectory specified in reference [10] for massless SED. It remains true that the minimum of $V^{(1)}(\phi)$ for massless SED is not generally reliable for arbitrary small values of the independent parameters λ and e^2 .

(3) The reliability of $V^{(1)}(\phi)$ as a probe of SSB may be directly examined by comparing it to the RG-improved, one-loop effective potential $V_{RG}^{(1)}(\phi)$. From the analysis, in section II, of the three theories under consideration, it emerges that $V_{RG}^{(1)}(\phi)$ approximates the exact effective potential $V(\phi)$ in a finite region that includes $\phi = \mu$ while $V^{(1)}(\phi)$ approximates $V_{RG}^{(1)}(\phi)$, and therefore $V(\phi)$, in a small neighbourhood of $\phi = \mu$. Thus a minimum of $V^{(1)}(\phi)$ approximates a minimum of $V(\phi)$ only when it lies close to $\phi = \mu$.

This requirement imposes a condition on the couplings that is not generally satisfied in the small-coupling regime. From the general expression (2.13) of $V^{(1)}(\phi)$, this condition is

$$c\lambda/2f(\lambda, g) \approx 0(1) \quad (3.1)$$

The loop expansion suggests that such a condition will not be generally satisfied for arbitrary small couplings since one expects $f(\lambda, g) \approx 0(\lambda^2)$. In fact one finds, for the theories under consideration, that $\frac{\lambda}{f(\lambda, g)} \rightarrow +\infty$ as $(\lambda, g) \rightarrow (0, 0)$ on any path that satisfies $\lambda = 0(g^2)$.

(4) Condition (3.1) requires the one-loop contribution to the effective potential to be of the same order as the zero-loop term, i.e. of order λ . This means that in this semi-classical treatment the first quantum correction should be of the same order as the classical term if the minimum of $V^{(1)}(\phi)$ is to approximate a true minimum of the exact effective potential $V(\phi)$.

The Coleman-Weinberg relation, $\lambda = \frac{33}{8\pi^2} e^4$, may indeed be considered a solution to condition (3.1) for massless SED. It gives the value $\frac{11}{6}$ for the left hand side of (3.1) and renders the one-loop contribution $0(\lambda)$.

A similar relation for the massless standard model, giving $\lambda \approx \lambda(h, g_1, g_2)$ may also be obtained on imposing condition (3.1). There is however, no reason why such a relation should be physically significant since there is no principle that forces $V^{(1)}(\phi)$ to possess a minimum in its region of validity.

(5) One may further elaborate on this point by noting that the constraint on the couplings, implied by the assumption that $V^{(1)}(\phi)$ approximates the exact EP $V(\phi)$ near its minimum, determines the scalar coupling λ in terms of the gauge couplings for both massless SED and the massless standard model. This leads to a definite value for the Higgs boson mass. There is

obviously no justification for the postulate that the Higgs mass must be of such a value as to enable $V^{(1)}(\phi)$ to be a valid approximation to $V(\phi)$ near its minimum. A similar postulate on, for example, the $1/N$ - expansion would yield a different value for the Higgs mass. This point is worthy of further consideration.

(6) We observe that in all three theories $V^{(1)}(\phi)$ does not approximate $V(\phi)$ near $\phi = 0$. In the case of massless SED and standard model this is seen from the fact that the region of validity of $V_{RG}^{(1)}(\phi)$ does not extend to $\phi = 0$. In the case of massless ϕ^4 , $V_{RG}^{(1)}(\phi)$ approximates $V(\phi)$ near $\phi = 0$ but its asymptotic behaviour as $\phi \rightarrow 0$ is not approximated by that of $V^{(1)}(\phi)$.

A consequence of this observation is that $V^{(1)}(\phi)$ does not convey information on what happens to the minimum of $V_0(\phi)$, the tree potential, at $\phi = 0$ when one-loop corrections are taken into consideration. Thus the statement that " $V^{(1)}(\phi)$ shows that the minimum at $\phi = 0$ turns into a maximum due to radiative corrections" is not valid, since $V^{(1)}(\phi)$ does not approximate $V(\phi)$ at $\phi = 0$.

(7) Our analysis shows that a necessary condition for SSB in massless SED is $e^2 > e_c^2$ where $e_c^2 \geq 0.013$. This indicate that massless SED may possess a phase diagram with a symmetric phase for $e^2 < e_c^2$ and a broken phase for $e^2 > e_c^2$. This would then be similar to the phase diagram recently obtained [18] for massless quantum electrodynamics (QED) by numerical computation. It is found that the chiral condensate $\langle \bar{\psi} \psi \rangle$, for massless QED, vanishes when $e^2 < e_c^2$ and is non-vanishing when $e^2 > e_c^2$ where $e_c^2 \approx 2.8$. Our analysis suggests this is probably the general picture for massless SED as well. There is in fact an earlier numerical computation [19] which indicates that, for perturbative λ , the phase diagram of massless SED is similar to that of pure lattice gauge theory: it undergoes a second

order phase transition at $e_c^2 \approx 1$. Clarification of the status of symmetry-breaking in massless SED awaits further work.

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FIGURE CAPTIONS:

Figure 1: The running coupling $\lambda(t)$ for massless SED.

Figure 2: The RG-improved one-loop EP for massless SED.

Fig. 1.

Fig. 2.