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Abstract. We describe recent advances in understanding properties of matter in the crusts of neutron stars. Specific topics considered include the sequence of nuclides expected to exist below neutron drip density, and the existence of phases with very aspherical nuclei at densities just below nuclear matter density.

Key words: Neutron star, neutron-rich nuclei, dense matter.

1. Introduction

In this paper we shall review a number of developments in the theory of dense matter that have taken place during recent years, and more especially advances since the Agia Pelagia meeting in 1990^[1]. Our main focus will be on microscopic properties, and applications will be considered in other contributions to this volume. We refer to our earlier review^[2] for an introduction to the properties of matter at subnuclear densities.

Throughout our discussion we shall for the most part consider matter at zero temperature and in equilibrium. The assumption of zero temperature is a very good first approximation, since at temperatures characteristic of neutron stars the thermal energy, $k_B T$, is generally much less than characteristic microscopic energies. This is not the case in the outermost layers, where the density is relatively low, and the effects of finite temperature are important for understanding emission of radiation from the surfaces of neutron stars, and also, of course, for understanding transport of thermal energy within the star. The assumption of equilibrium may be less good, because processes by which matter can relax to the equilibrium state may take too long. Deviations from equilibrium can be especially important in the outer layers of neutron stars, where time scales for relaxation may be long because of the slowness of beta decay processes in finite nuclei at low temperatures, and the difficulty of changing the total number of heavy nuclei. This effect is particularly pronounced in neutron stars that are accreting matter, since the composition of the matter arriving at the surface of the neutron star will, as a rule, not correspond to that of matter in equilibrium under neutron star conditions.

At zero temperature the equilibrium state is the ground state. The task is therefore, for a given density of baryons, n , to determine the state of matter with the lowest energy per baryon. The number of baryons of each type is not specified *a priori*, but rather is determined by the condition that the energy be a minimum. The other important condition is that matter should be electrically neutral. If this were not the case enormous electrostatic potentials would build up, which is impossible because the many mobile charged particles present in the star would move to screen out the field. (If one were to remove the electrons from a neutron star, the electrostatic potential at the surface of the star would be of order $Qe/R \sim 10^{43}$ V. Here Q is the total number of electrons in the star, and R is the neutron star radius.)

2. Nuclei in the Outer Crust

In the outermost parts of a neutron star, matter is not so different from terrestrial matter, and the equilibrium nucleus is ^{56}Fe , but the temperature is rather high by terrestrial standards, with estimates lying in the range $10^5 - 10^7$ K, depending on the age of the star and on what processes occur in and around the star. With increasing depth in the star, the density increases and electrons soon become degenerate, and at a density of around 10^6 g cm^{-3} they become relativistic. As one proceeds to higher density, the electron Fermi energy is in the MeV range and the total energy per baryon can be lowered by replacing electrons, and the protons required for charge neutrality, by the same number of neutrons. In general it is also advantageous to change the atomic number of the nucleus.

The expected sequence of equilibrium nuclides as a function of density has been calculated by a number of authors. The procedure is to calculate the energy per unit volume, given by

$$E_{\text{tot}} = n_N \epsilon(A, Z) + E_e(n_e), \quad (1)$$

where Z and A are the atomic number and mass number of the nuclei, assumed to be identical, and n_N is their density. $\epsilon(A, Z)$ is the energy per nucleus, and $n_e = nZ/A$ is the electron density. The electrons are essentially free, because for relativistic electrons the effects of electron-ion interaction are of order $Z^{2/3}\alpha$ times the electron kinetic energy. Here α is the fine structure constant.

The most important piece of nuclear input to calculations of the sequence of nuclides is the energy per nucleus, which in the absence of effects of the nuclear environment, would be just the nuclear mass, times c^2 . The broad features of this sequence may be understood within the semi-empirical liquid drop model for nuclear masses. The equilibrium nuclear size is determined by a competition between Coulomb and surface energies, and one finds that the surface energy is twice the Coulomb energy. The bulk and symmetry energies in the nuclear mass formula do not enter here, because they are unaltered when the total number of nucleons and the proton fraction are fixed. If one uses standard parameters for the surface and Coulomb energies, one finds $A \simeq 12.5/x^2$, where $x = Z/A$ is the proton fraction of matter. Thus, with increasing density, and decreasing x , nuclei tend to become larger. Physically this is due to the fact that, because of the reduced proton fraction, the Coulomb energy, which tends to favor small nuclei, is reduced.

At the lowest densities the equilibrium nuclides are ones that can be studied in the laboratory, either stable ones or ones with sufficiently long lifetimes that measurements can be made on them. An interesting question is up to what density can one determine the equilibrium nuclide from empirical data on nuclear masses? Haensel and Pichon^[3] have recently looked into this question, and we show in Table 1 the equilibrium sequence they find, together with the maximum density at which a particular nuclide is the equilibrium one. For comparison we also show results from the work of Baym, Pethick, and Sutherland^[4] carried out two decades ago. What one sees is that the equilibrium nuclides at densities above that at which ^{56}Fe is stable are first a series of Ni isotopes, with the proton closed shell, $Z = 28$, followed by a sequence of nuclides with neutron closed shells, first with $N = 50$, and then with $N = 82$. It is interesting to note that the sequence of nuclides found by Haensel and Pichon is rather close to that reported by Haensel, Zdunik and Dobaczewski^[5] using the phenomenological droplet model of Myers^[6], which is basically a liquid drop model with shell corrections that is fitted to properties of known nuclei. Comparison of the two sets of results shown in Table 1 shows that the nuclear masses that have been determined over the past two decades enable one to extend by about an order of magnitude the range of densities over which the stable nuclides may be determined directly from measured nuclear masses.

The main objective of the work by Haensel, Zdunik and Dobaczewski^[5] just cited was a realistic microscopic calculation of nuclei near neutron drip. They carried out Hartree-Fock-Bogoliubov calculations of masses of nuclides using a Skyrme interaction, the SkP interaction of Dobaczewski, Flocard and Treiner^[7]. They found that the sequence of nuclides was dominated by Ni isotopes at all densities up to ones very close to neutron drip, but that for a limited density range the effect of the $N = 50$ closed shell could be seen. For a small range of densities, about 25% just below neutron drip, they found a sequence of Zr ($Z = 40$) isotopes.

What these results demonstrate is that the details of which nuclides are present in equilibrium matter is sensitive to shell effects, which in turn depend on the spin-orbit force in neutron-rich matter. We have already referred to one model for nuclear forces, the Skyrme interaction, that has been used in estimating nuclear masses over a wide range of conditions. Another is relativistic mean field theory (RMF), in which the basic couplings between nucleons are modelled in terms of couplings between nucleons and mesons. In both of these approaches the strength of the spin-orbit interaction is directly related to the empirical properties of nuclei close to the valley of stability. In the Skyrme models, the spin-orbit interaction is determined by the explicit spin-orbit term in the Hamiltonian, while in the relativistic mean field theory, the strength of the spin-orbit force is directly related to the specific assumptions made about the nuclear force being the consequence of the exchange of a restricted number of bosons, with parameters fitted to reproduce properties of known nuclei close to the valley of stability. It is still an open question what the spin-orbit interaction in neutron-rich nuclei really is, and it needs to be investigated from a more fundamental point of view.

Shell effects in nuclei are of interest in a number of different contexts. One is in connection with laboratory investigations employing radioactive beams, which make possible studies of nuclei close to the neutron drip line, at least for the lighter

Table 1
Nuclides in the ground state of cold matter as a function of density^a.

element	Z	N	Z/A	$\rho_{\max}$ (g cm ⁻³)	μ_e (MeV)	$\Delta\rho/\rho$ (%)
(i) From Haensel and Pichon, Ref.[3].						
Using experimental nuclear masses.						
⁵⁶ Fe	26	30	0.4643	7.96 10 ⁶	0.95	2.9
⁶² Ni	28	34	0.4516	2.71 10 ⁸	2.61	3.1
⁶⁴ Ni	28	36	0.4375	1.30 10 ⁹	4.31	3.1
⁶⁶ Ni	28	38	0.4242	1.48 10 ⁹	4.45	2.0
⁸⁶ Kr	36	50	0.4186	3.12 10 ⁹	5.66	3.3
⁸⁴ Se	34	50	0.4048	1.10 10 ¹⁰	8.49	3.6
⁸² Ge	32	50	0.3902	2.80 10 ¹⁰	11.44	3.9
⁸⁰ Zn	30	50	0.3750	5.44 10 ¹⁰	14.08	4.3
⁷⁸ Ni	28	50	0.3590	9.64 10 ¹⁰	16.78	4.0
From the mass formula of Möller (1992), unpublished.						
¹²⁶ Ru	44	82	0.3492	1.29 10 ¹¹	18.34	3.0
¹²⁴ Mo	42	82	0.3387	1.88 10 ¹¹	20.56	3.2
¹²² Zr	40	82	0.3279	2.67 10 ¹¹	22.86	3.4
¹²⁰ Sr	38	82	0.3167	3.79 10 ¹¹	25.38	3.6
¹¹⁸ Kr	36	82	0.3051	(4.33 10 ¹¹)	(26.19)	
(ii) From Baym, Pethick and Sutherland, Ref.[4].						
Using experimental nuclear masses.						
⁵⁶ Fe	26	30	0.4643	8.1 × 10 ⁶	0.95	2.9
⁶² Ni	28	34	0.4516	2.7 × 10 ⁸	2.6	3.1
⁶⁴ Ni	28	36	0.4375	1.2 × 10 ⁹	4.2	7.9
⁸⁴ Se	34	50	0.4048	8.2 × 10 ⁹	7.7	3.5
From the mass formula of Myers and Swiatecki, Ref.[8].						
⁸² Ge	32	50	0.3902	2.2 × 10 ¹⁰	10.6	3.8
⁸⁰ Zn	30	50	0.3750	4.8 × 10 ¹⁰	13.6	4.1
⁷⁸ Ni	28	50	0.3590	1.6 × 10 ¹¹	20.0	4.6
⁷⁶ Fe	26	50	0.3421	1.8 × 10 ¹¹	20.2	2.2
¹²⁴ Mo	42	82	0.3387	1.9 × 10 ¹¹	20.5	3.1
¹²² Zr	40	82	0.3279	2.7 × 10 ¹¹	22.9	3.3
¹²⁰ Sr	38	82	0.3166	3.7 × 10 ¹¹	25.2	3.5
¹¹⁸ Kr	36	82	0.3051	(4.3 × 10 ¹¹)	(26.2)	-

^a $\rho_{\max}$ is the maximum density at which the nuclide is present, μ_e is the electron chemical potential (including electron rest mass) at that density, and $\Delta\rho/\rho$ is the fractional increase in the mass density in the transition to the next nuclide. The value of $\rho_{\max} = 4.3 \times 10^{11}$ g cm⁻³ is the density at which neutron drip begins. The lines with $\rho_{\max}$ in parentheses corresponds to the neutron drip point.

nuclei. A second is the rapid neutron capture process (r-process) in astrophysics, which is thought to be an important source of many of the heavier elements. Peaks in abundances are associated with closed nuclear shells, and therefore the position and strength of shell closings is of crucial importance.

The need to understand shell effects is further underlined by the results of recent calculations for nuclei near the particle drip lines. Calculations employing Skyrme interactions (see, e.g., Smolańczuk and Dobaczewski^[9]) generally give less marked shell effects near the neutron drip line compared with those obtained from relativistic mean field theory (see, e.g., Sharma, Lalazissis, Hillebrandt and Ring^[10]). A comparison of calculations of shell effects in Hartree-Fock theory (that is without pairing) and relativistic mean field theory have been made by Dobaczewski, Hamamoto, Nazarewicz, and Sheikh^[11], who arrive at similar conclusions regarding the reduction of shell effects in Hartree-Fock theory (HF) as compared with relativistic mean field theory. For example, for $A = 150$, they find that the splitting between the $h_{11/2}$ and $h_{9/2}$ orbitals for $N = 100$ in RMF calculations with the NL-SH interaction is roughly twice what it is in HF theory with the SkP Skyrme interaction. More progress on this subject must await the development of a more fundamental understanding of the spin-orbit interaction.

One recent piece of work directly relevant to this question is the calculation of the spin-orbit splitting in ^{15}N by Pieper and Pandharipande^[12]. They calculated the energies of the $p_{1/2}$ and $p_{3/2}$ configurations, allowing for many-body forces and many-body correlations, as well as the usual two-body ones. They found that the calculated splitting agrees with the observed one, but that only about one half of the total comes from two-body force and two-body correlations, with the remainder coming roughly equally from three-body forces, and three-body correlations. This indicates the dangers inherent in using simple models to estimate the spin-orbit interaction in neutron-rich nuclei by extrapolation from the properties of nuclei near the valley of stability. It would be extremely valuable to have results of microscopic calculations for the spin-orbit splitting for neutron-rich nuclei similar to those of Pieper and Pandharipande^[12]. This would enable one to construct effective interactions that are better able to reflect the complexities of the underlying physics.

3. Nuclei in the inner crust

At a density of about $4 \times 10^{11} \text{ g cm}^{-3}$, the most energetic occupied neutron orbitals in the equilibrium nucleus become unbound, and at higher densities nuclei are immersed in a sea of neutrons, as well as an essentially uniform background of electrons. The properties of nuclei in this density range have been explored by many authors, including Langer *et al.*^[13], Bethe, Börner and Sato^[14], Baym, Bethe, and Pethick^[15], Arponen^[16], and Buchler and Barkat^[17]. In this regime the nuclear matter is very different from that which can be directly studied in the laboratory. At neutron drip, the proton fraction of matter is $x \simeq 0.3$, corresponding to a neutron excess $\delta = 1 - 2x \simeq 0.4$. With increasing density the matter inside nuclei becomes more neutron rich, and the pure neutrons outside become denser. Eventually nuclei become so closely packed that they merge to form a uniform liquid of neutrons and protons, together with a neutralizing background of electrons. When nuclei merge, the proton

fraction of matter in nuclei is about 0.1, and the proton fraction averaged over the matter inside nuclei and that outside is about 0.04, and therefore conditions are very different from those encountered in most nuclei that can be studied in the laboratory. Since matter inside nuclei and that outside become increasingly similar, it is important to describe matter in the two phases in a consistent way and this is done by evaluating properties of the two phases from one and the same microscopic interaction.

In addition to the modifications in the properties of bulk matter at high densities, there are a number of changes in finite-nucleus properties. One of these is that the nuclear surface is modified by the presence of the neutron liquid outside. This reduces the surface tension, an effect that becomes more marked as the properties of the two phases become increasingly similar. The Coulomb energy is also modified because the spacing between nuclei is not so very different from the nuclear radius, and therefore in evaluating the Coulomb energy it is important to include contributions from ion-ion and electron-ion interactions, which are of order r_N/r_c times the Coulomb energy of an isolated nucleus. Here r_N is the nuclear radius, and r_c is the radius of the Wigner-Seitz cell, a sphere whose volume is equal to the average volume per nucleus, $1/n_N$. Within a generalized liquid-drop model, the equilibrium nuclear size is still determined by the virial-like condition relating the Coulomb and surface energies, just as it was below neutron drip, but the surface tension to be used must be that appropriate for the neutron excess of the nuclear matter in the nucleus, and the Coulomb energy must include the "lattice" corrections from the ion-ion and electron-ion terms. In a model in which one replaces the unit cells having the symmetry of the lattice by spherical cells having the same volume, the Coulomb energy is given by

$$E_C = \frac{3}{5} \frac{Z^2 e^2}{r_N} \left(1 - \frac{3}{2} \frac{r_N}{r_c} + \frac{1}{2} \left[\frac{r_N}{r_c} \right]^3 \right). \quad (2)$$

The reduction of the Coulomb energy tends to increase the size of the equilibrium nucleus, while the decrease of the surface tension for increasing neutron richness tends to decrease the size of the equilibrium nucleus. Detailed calculations by Ravenhall, Bennett, and Pethick^[18] based on a generalized liquid drop model showed that the atomic number, Z , remains rather close to 37 for most of the density range between neutron drip and roughly one third of nuclear matter density. In the more recent work of Lorenz, Pethick and Ravenhall^[19], which uses a nuclear interaction that fits observed nuclear masses, as well as the properties of neutron matter as calculated by Friedman and Pandharipande^[20], Z is slightly smaller, around 35.

The calculations of Ravenhall, Bennett, and Pethick^[18], and subsequent liquid-drop explorations^[21, 19], do not allow for shell effects. As will be clear from our discussion of shell effects in neutron-rich nuclei in lower density matter, it is not possible at this time to say with any degree of certainty how large shell effects are in nuclei in neutron stars when the density is so high that the relevant nuclei cannot be investigated experimentally. The most thorough investigation of nuclear properties over most of the density range is still that of Negele and Vautherin^[22], who performed Hartree-Fock calculations for a Skyrme-like interaction. They found that Z was 40 for densities from drip, at about $4 \times 10^{11} \text{ g cm}^{-3}$, until it jumped to 50 at a density of $\sim 3 \times 10^{12} \text{ g cm}^{-3}$, and then fell to 40 and 28 for the last two points

at densities of about $8 \times 10^{13} \text{ g cm}^{-3}$ and $1.3 \times 10^{14} \text{ g cm}^{-3}$. This work suggested that the dominant shell effects are those for *protons*. This should be contrasted with the somewhat confusing situation at lower densities, where some calculations point to proton shell effects being dominant, while others point to neutron shell effects. This controversy will be resolved only when a better fundamental understanding of the spin-orbit interaction has been achieved.

4. Unusual nuclear shapes

In essentially all the above discussion it has been assumed that nuclei are basically round. Recently it has become clear that in the crusts of neutron stars, nuclei may have very different shapes. A possibility pointed out in one of the early investigations^[15] is that, still within the realm of spherical geometry, nuclei could turn “inside out”, thereby giving roughly spherical drops of neutron matter surrounded by nuclear matter, in this case a rather dilute solution of protons in neutrons. The configuration is thus similar to Swiss cheese, with the holes ordered on a bcc lattice, the cheese corresponding to regions that contain both protons and neutrons, and the holes corresponding to regions with neutrons alone. Within a liquid drop model, the Swiss cheese or “bubble” phase is energetically favorable, compared with the conventional nuclear phase, if nuclei fill more than one half of space, since the sum of surface and Coulomb energies is lower for the bubble phase. The calculations made at the time led to the conclusion that the bubble phase was not the thermodynamically most favorable state at any density, and that, with increasing density, the transition to the uniform phase occurred directly from the phase with roughly round nuclei, without an intermediate region with the bubble phase.

The conclusion that the bubble phase was unimportant in neutron stars received support from general arguments for estimating the density range over which the phase might be important. Consider a situation when nuclei occupy one half of space. The average density is then $(n_{nm} + n_n)/2$, where n_{nm} is the density of the nuclear matter phase, and n_n is the density of the neutron matter phase. If the neutron matter density were $\simeq 0.6$ times the nuclear matter density, and the nuclear matter density were close to the saturation density of symmetric nuclear matter^[15], this indicates that the bubble phase might play a role at densities in excess of $0.8 n_{nm}$. However this density is likely to be quite close to the density at which nuclei dissolve completely, and form the uniform phase. Consequently it was argued^[15] that the bubble phase was likely to be unimportant, since it could affect properties over only a limited density range.

The study of non-spherical nuclear shapes was initiated by work on the equation of state of matter in stellar collapse. This differs from matter in neutron stars in that the proton fraction is typically ~ 0.35 . The proton fraction is thus above that for neutron drip for cold matter at zero temperature, and the temperatures encountered in the initial collapse are low enough that thermally evaporated nucleons outside the nuclear matter can be neglected. The proton fraction is maintained at this high value because neutrinos are trapped in the star on the timescale of the collapse, and their density builds up to the point where electron captures on protons are hindered for lack of final states for the neutrinos that would be emitted. Ravenhall, Pethick, and

Wilson^[23] pointed out the possibility of a number of aspherical shapes for nuclei and bubbles. The ones they considered are rod-like nuclei (dubbed “spaghetti”) together with the bubble analog, and plate-like nuclei (“lasagna”), which are identical to the bubble analog. They found that with increasing density the equilibrium configurations for nuclear matter are round nuclei, rod-like nuclei, plate-like nuclei, the bubble analog of the rod-like phase, the round bubble phase, and, finally, uniform matter.

The non-spherical nuclear shapes may be understood as being due to instability of nuclei to fission. The Bohr-Wheeler condition for fission of isolated nuclei is^[24]

$$E_C^0 \geq 2E_{surf} \quad (3)$$

where the superscript ‘0’ on the Coulomb energy denotes that it must be evaluated for an isolated nucleus. Corrections to the fission condition in the presence of other nuclei have been calculated by Brandt^[25], who finds that the leading corrections are of order $(r_N/r_c)^3$, in contrast to the equilibrium condition (2), which shows that the leading corrections to the nuclear size are of order r_N/r_c . To understand the basic physics, let us consider just the leading corrections to the equilibrium and fission conditions. If the Coulomb energy of a nucleus becomes sufficiently large compared with the surface energy, a spherical nucleus will become unstable to quadrupolar deformation. The fission condition (3) may be combined with the equilibrium condition

$$E_{surf} \simeq 2E_C^0(1 - \frac{3}{2} \frac{r_N}{r_c}). \quad (4)$$

Thus one can see that if $r_N/r_c \gtrsim 1/2$, the equilibrium nucleus is unstable to fission. This corresponds to nuclei filling 1/8 of space. The more detailed calculations show that the transition from round nuclei to rod-like ones occurs at a filling factor rather close to this value. Physically, the reduction of the total Coulomb energy due to the lattice contribution increases the size of the equilibrium nucleus, and eventually this becomes so large that it is unstable to fission. It is not difficult to imagine that if nuclei on a lattice all elongate spontaneously, the cigar-shaped nuclei would eventually join up to form string-like structures such as those envisioned in the spaghetti phase.

The conclusions of Ref. [23] regarding the existence of very aspherical nuclear shapes in hot, dense matter were confirmed by other groups who used different techniques and interactions. Hashimoto and colleagues^[26] adopted a variational approach to a simplified version of the liquid drop model, with specified simple geometries, and came to similar conclusions. Williams and Koonin^[27] used a three-dimensional Thomas-Fermi theory, in which the nuclear density distribution is not constrained to a specific shape, but can have arbitrary geometry. This was done at the expense of assuming charge symmetric nuclear matter, a constrained situation different from what one encounters in stellar collapse or neutron stars. They confirmed the general picture outlined above, with the feature that the rods and plates in general do not have a constant cross section, but rather retain a vestige of the modulation in three dimensions that exists for a lattice of separated nuclei. Las-saut *et al.* ^[28], using the three-dimensional Thomas-Fermi approach and the SkM interaction^[29], considered thermodynamically stable matter of arbitrary charge character at finite temperature, under conditions relevant to stellar collapse, and they

too obtained the aspherical shapes. It may be conjectured that the conditions in stellar collapse, so far as the nuclei are concerned, are close enough to laboratory nuclei that all approaches that reproduce known nuclear properties will encounter these 'funny phases'. Lassaut *et al.* [28] considered also the question of other or intermediate shapes that the matter could adopt. They found as the lowest energy states the spherical, cylindrical or planar shapes described above, and an interesting 'mixed' phase with somewhat higher energy that went continuously from the nucleus to the bubble configurations.

It is interesting to observe that in matter in stellar collapse the non-spherical phases can be the ground state of matter over a range in density of almost one decade. This may be seen from our simple estimate, $1/8$, of the filling factor at which round nuclei become unstable to fission, because the density of matter corresponding to this is roughly $1/8$ of the nuclear saturation density, since in stellar collapse the density of matter outside nuclei is much less than nuclear density. The reasons for this density range being so much larger than that for neutron star matter given above is that, first, in neutron stars the density of matter outside nuclei is much closer to that of nuclear matter for the conditions of interest here, and, second, the nonspherical nuclei become energetically favorable at a filling factor of $\sim 1/8$, rather than $1/2$, as one would expect if the only shapes of importance were spherical nuclei and spherical bubbles.

Following this work for matter in stellar collapse, Oyamatsu^[30] and Lorenz, Ravenhall and Pethick^[31] reexamined whether or not one would expect states with non-spherical nuclei to be the equilibrium ones for neutron star matter. Lorenz *et al.* calculated the energies of various inhomogeneous phases in the spirit of the liquid drop model. The basic interaction, denoted by FPS, was of the Skyrme type, as described above, and the interface energy was evaluated using this interaction. Figure 1 shows the energy of the various phases as a function of baryon density, relative to the energy of a two-phase state with no Coulomb and surface contributions. One sees that, with increasing density, all the non-spherical nuclear shapes we have considered are the energetically favorable ones in some density range.

The question of whether or not non-spherical nuclear shapes can ever be the equilibrium ones is a delicate one, as one can understand from the following considerations. The problem is to determine whether uniform matter with a particular proton fraction has a higher or lower energy than a state with an inhomogeneous distribution of nuclear matter. If one neglects any difference in proton fractions between the two possible phases, the energy difference may be regarded as being made up of two types of contribution. The first is the bulk energy gain resulting from the matter undergoing a transition to a two-phase system, a transition analogous to a liquid-gas transition in a two component system consisting of protons and neutrons. (Since the characteristic scales of inhomogeneities in the nuclear matter are small compared with electron screening lengths, the electrons form a uniform background and play little role in the energetics.) The second type of contribution to the energy difference comes from the Coulomb and surface energies, which are a consequence of the inhomogeneities in the non-uniform phase. Only if the energy gain from the bulk contributions exceeds the Coulomb and surface contributions will the inhomogeneous phase be favorable. In the case of stellar collapse, the bulk energy gained

Fig. 1. Energies per unit volume of matter in beta equilibrium for the uniform phase and for various nuclear phases. All energies are measured with respect to the energy of a phase with two fluids in equilibrium when Coulomb and surface effects are neglected. Results are shown for two different Skyrme-type interactions, as described in the text.

from the phase transition is considerable, since the two phases (nuclear matter and a very diffuse gas of nucleons) that coexist in the inhomogeneous state are very different, while for the neutron star case the bulk term is much less, since the two coexisting phases in the inhomogeneous state (pure neutrons and a dilute solution of protons in neutrons) are very similar. Even though surface energies in neutron star matter are less than those in matter in stellar collapse, the reduction of the total Coulomb and surface energies in neutron star matter compared with matter in stellar collapse is less than the corresponding reduction in the bulk energy gain from the phase separation. In fact, the bulk terms in neutron star matter are comparable with the surface and Coulomb terms.

For comparison, we also show in Fig. 1 results obtained using the SkM interaction^[29], a simpler Skyrme-type interaction. For the SkM interaction the phases with non-spherical nuclei are never the equilibrium state at any density. It is also important to note that the characteristic energy differences between the nuclear phases and the uniform phase in the case of the FPS interaction are about twice those for the SkM interaction.

These calculations indicate just how sensitive results for the non-spherical nu-

clear phases are to what is assumed about pure neutron matter, and dilute solutions of protons in neutrons. The properties of nuclei in the laboratory provide rather little direct information about this, and thus one must rely heavily on theoretical investigations of neutron matter. Fortunately this is theoretically a rather well-defined problem, since the nucleon-nucleon two-body interaction at low energies is well characterized, and at sub-nuclear densities, three- and higher- body interactions play little role, even though at higher densities the poor knowledge of these components of the interaction contributes in a major way to uncertainties in the equation of state and composition of matter. It is important to put the properties of neutron matter at subnuclear densities on as firm a footing as possible, not only for astrophysical applications, but also for interpreting terrestrial experiments with coming radioactive beam facilities.

The advances in understanding of matter at subnuclear densities have a number of consequences for observational properties of neutron stars. First, the inner edge of the crust, where the transition between the nuclear phase and the uniform liquid takes place, is now estimated to be at a density of $\sim 0.6n_s$, rather than at about n_s as earlier work suggested^[15]. As a consequence of the lower transition density, the mass of the crust and its moment of inertia, important quantities in models of glitches and of thermal evolution, are reduced compared with earlier estimates. Second, the elastic properties of the part of the crust where nuclei are aspherical are very different from those of an ordinary solid. In fact they will behave like liquid crystals, and there will be no linear restoring forces for certain types of shear. Rod-like phases resemble discotic liquid crystals, while the plate-like phase is similar to a smectic liquid crystal. Thus the question of how large a strain the crust can sustain needs to be reconsidered. Third, the non-spherical nuclear shapes will affect properties of matter, such as pinning of vortices and neutrino emission.

5. Conclusion

In recent years there have been a number of developments in theoretical understanding of matter in neutron star crusts, and in this paper we have dwelt on two of them. First, measurements that have been made on increasingly neutron-rich nuclei over the past two decades allow a determination of the equilibrium nuclides up to densities close to that at which neutron drip occurs. Second, close to nuclear matter density it has been shown that nuclei may well be very aspherical, a fact that will have important consequences for observational properties of neutron stars.

In order to further improve our understanding of neutron star crusts, better physical information about neutron-rich matter is necessary. Of particular relevance for the non-spherical phases are properties of pure neutron matter and of dilute solutions of protons in neutrons, where one has to rely heavily on theoretical investigations. On the other hand, at lower densities, the equilibrium nuclides are influenced by shell effects, which depend on the spin-orbit interaction in neutron-rich nuclei, which is at present not understood at a fundamental level. In the future one can foresee a continuing fruitful interplay between the study of matter in neutron star crusts, and experimental and theoretical studies of neutron-rich nuclei.

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