ANTISHADOWING AND NUCLEAR OPTICS*

A. Białas and W. Czyż

Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Cracow, Poland
e-mail: bialas@cernvm, czyz86a.if.uj.edu.pl

Abstract. Using standard methods of nuclear optics, we investigate the recent suggestion by Kharzeev and Satz that antishadowing of photons in nuclear matter may be an effect of quantum interference. We show that the Kharzeev-Satz effect is controlled by the real part of the elastic scattering amplitude of the strongly interacting system coupled to the photon and traversing the nucleus. Phenomenological consequences of this observation are discussed.

In a recent paper [1], Kharzeev and Satz pointed out the role of quantum-mechanical interference in the process of nuclear shadowing. The most spectacular effect of their analysis is a prediction of nuclear antishadowing without the necessity of introducing any modification in the partonic structure of nucleons due to nuclear environment. Clearly, this prediction has important consequences for an analysis of data, as discussed in detail in [1].

The authors of Ref.[1] restricted their analysis to a semi-quantitative description of these phenomena. In the present paper we apply the well-known technique of nuclear optics [2] to estimate quantitatively the size of the effect and the kinematic region where it can be observed. To be definite, we consider the case of photoproduction where the corresponding formalism was developed some time ago [3],[4].

Detailed conclusions of our analysis are described at the end of the paper. Here, we only wish to point out one result which seems important: We have found that the effect of antishadowing depends critically on the value of the ratio α of the real to imaginary part of the amplitude for elastic scattering on a nucleon of the strongly interacting system coupled to photon and traversing the nucleus. In fact, antishadowing is not present if the real part of the elastic amplitude vanishes. The reason is unitarity, as can be seen from a careful analysis of the arguments given in [1]. Briefly, as argued by Kharzeev and Satz, antishadowing arises because in the coherent scattering on, e.g. two nucleons, the elastic amplitude may be multiplied by a factor up to 2 (for fully positive interference) and, therefore, the cross-section may get a factor 4. This argument is, however, valid only for the contribution from the real part. Indeed, since the imaginary part of the amplitude is (by unitarity) proportional to the total cross-section, multiplication of the imaginary part by factor 2 multiplies also the total cross-section by 2 and thus there is no reason for antishadowing.

Let us remark that this point does not invalidate the old analysis of high-energy electron scattering in purely electrodynamic processes [5] because in that case the dominant part of the elastic amplitude is described by one photon exchange and thus real. This is not the case, however, for the strongly interacting systems: here, one gluon exchange term cannot contribute to elastic scattering [6] and thus the imaginary part is, in general, dominant.

To summarize, we confirm the observation of Kharzeev and Satz that the quantum interference effects can play a role in photoproduction off nuclei. The detailed description is, however, modified by unitarity which implies that only the real part of the elastic amplitude is responsible for antishadowing.

Following [1], we consider photoproduction of quarkonium, denoted by V , on a nucleus of atomic number A :

$$\gamma + A \rightarrow V + \text{anything}. \quad (1)$$

Denoting the total cross-section for this process by $\sigma_V^\gamma(A)$, one sees that, by

unitarity, the total photoproduction cross-section can be expressed by the sum

$$\sigma^\gamma(A) = \sigma_{direct}^\gamma(A) + \sum_V \sigma_V^\gamma(A), \quad (2)$$

where the sum runs over all quarkonium states which couple directly to the photon. Thus $\sigma_V^\gamma(A)$ represents this part of the total photoproduction cross-section which contains V in the intermediate state.

Using this observation, it is not difficult — following the ideas of Ref. [3] — to construct the expression for $\sigma_V^\gamma(A)$ in terms of the amplitude for the “elementary” V -nucleon scattering and the parameters of the nucleus. The relevant general formula can be found, e.g. in the Appendix B of [4]*. If one neglects the internuclear correlations, the formula simplifies to

$$\begin{aligned} \sigma_V^\gamma(A) = & C_V^2 A \sigma_V - 2C_V^2 A(A-1) \\ & \times \text{Re} \left\{ \int d^2b \int_{-\infty}^{\infty} dz f_V \rho(b, z) \int_{-\infty}^z dz' e^{-i(z-z')/z_V} f_V \rho(b, z') \left[1 - f_V \int_{z'}^z \rho(z'') dz'' \right]^{A-2} \right\}, \end{aligned} \quad (3)$$

where C_V is the $\gamma - V$ coupling, f_V is the forward amplitude of V -nucleon elastic scattering, with

$$f_V = \frac{1}{2} \sigma_V (1 - i\alpha), \quad (4)$$

z_V is the “coherence length”

$$z_V = (k - k_V)^{-1} \approx \frac{2k}{m_V^2}, \quad (5)$$

k and k_V are momenta in the laboratory frame of the photon and of the quarkonium. $\rho(b, z)$ is the nuclear density normalized to unity and b is the impact parameter.

The second term in (3) represents nuclear corrections to the simple additive formula. One sees that these corrections vanish in the low-energy limit (small z_V) because of rapid oscillations of the integrand. On the other hand, in the limit of high energies (z_V very large) it can be shown by explicit integration that the first term is cancelled and the remainder gives the standard Glauber model formula for total quarkonium-nucleus cross-section (multiplied by C_V^2). All this was already explained in [3].

Following Kharzeev and Satz, we have taken $z_V = (mx_t)^{-1}$, where m is the nucleon mass and x_t is the Bjorken x of the target gluon.

* Note a misprint in their formula (B16), where $\sum_{j \neq i, k}$ should be replaced by $\prod_{j \neq i, k}$.

The numerical studies of the formula (3) were performed for what seems to us a reasonable range of the parameters. The results are illustrated in Fig. 1, where the ratio

$$r_A = \frac{\sigma_V^\gamma(A)}{C_V^2 A \sigma_V} \quad (6)$$

is plotted for $A = 208$, $\sigma_V = 10 \text{ mb}$ and different values of α . They can be summarized as follows:

- (i) For $\alpha = 0$, *i.e.* vanishing real part of the elastic amplitude, there is no antishadowing. As explained in the introduction, this is a direct consequence of unitarity.
- (ii) For positive α , the antishadowing shows up in the region which starts at relatively small x_t and extends to rather high values of x_t (for $\alpha \neq 0$ the integral in (3) falls only as z_c for $z_c \rightarrow 0$, and even for $x_t = 1$, z_c is still $\approx 0.2f$).
- (iii) The point of transition from shadowing to antishadowing depends mostly on value of α and decreases with increasing α .
- (iv) The magnitude of antishadowing depends of course on the value of α : the deviation of the maximal value of r_A from unity is approximately proportional to α^2 . For “reasonable” values of α in the region $\alpha \leq 0.25$, the effect of antishadowing is rather weak, however, the ratio r_A being rather close to 1.
- (v) For negative α there is no antishadowing, at least for $|\alpha|$ smaller than 1. On the contrary, the shadowing is stronger and extends to larger values of x_t .

These results indicate that the mechanism of antishadowing suggested in Ref. [1] is not very effective for dominantly absorptive quarkonium cross-section. It is worth to emphasize, however, that little is known about the real part of the elastic amplitude in heavy quarkonium-nucleon scattering. In fact, since in this case the corresponding cross-sections are much smaller than those of standard meson-nucleon scattering (due to colour cancellations [6]), it is not excluded that real parts may actually be quite large. At the moment it is only a matter of speculation, but perhaps it can turn into an interesting problem if a reliable method of estimating α becomes available. A particularly interesting phenomena may arise is α varies with energy. For example, if α is positive at high energies and negative at low energies, the region of antishadowing should be followed by a second region of shadowing (apart from the one at $x \approx 0$). It is perhaps too early to speculate that this phenomenon may be, at least partly, responsible for experimentally observed structures (compare *e.g.* [7]).

To summarize, we have found that the real part of the elastic amplitude is responsible for quantum interference effects suggested by Kharzeev and Satz [1].

For amplitudes with a substantial real part the interference phenomena may produce rich patterns, qualitatively resembling experimental findings. It remains an open and very exciting question if indeed this mechanism is important in the physics of nuclear photoproduction.

One comment is necessary. To simplify the presentation, we have neglected in our discussion two technical complications. First, the possible transitions between different quarkonia is neglected in Eq. (3). Secondly, we also neglected fluctuations of the elastic cross-section, *i.e.* we ignored the possibility of diffractive excitations. Therefore our discussion applies — strictly speaking — only to scattering of the absorption eigenstates of the $q\bar{q}$ system [8]. We have checked, however, that the qualitative results do not depend on the value of the cross-section σ_V and therefore we believe that inclusion of these two effects cannot change qualitatively our conclusions.

ACKNOWLEDGEMENTS

We would like to thank Helmut Satz for showing his results prior to publication. We also thank Kacper Zalewski for helpful comments.

REFERENCES

1. D. Karzeev and H. Satz, The quantum mechanical origin of nuclear shadowing, preprint CERN-TH 7115/93.
2. R.J. Glauber, High energy collision theory, Proc. of the Theoretical Physics Institute, Boulder (1958), p. 315.
3. K. Gottfried and D.R. Yennie, *Phys. Rev.* **182** (1985) 1969.
4. T.H. Bauer, R.D. Spital, D.R. Yennie and F.M. Pipkin, *Rev. Mod. Phys.* **50** (1978) 261.
5. L.D. Landau and I.Ya. Pomeranchuk, *Dokl. Akad. Nauk SSSR* **92** (1953) 535 and 735; E.L. Feinberg and I.Ya. Pomeranchuk, *Suppl. Nuovo Cim.* **III**, ser. X, No 4 (1956) 652.
6. F. Low, *Phys. Rev.* **D12** (1975) 163; S. Nussinov, *Phys. Rev. Lett.* **34** (1975) 1286.
7. P. Amandruz *et al.*, *Nucl. Phys.* **B371** (1992) 553; J. Gomez *et al.* Measurements of the A -dependence of deep inelastic electron scattering, Preprint SLAC-PUB-5813, August 1993.
8. M.L. Good and W.D. Walker, *Phys. Rev.* **120** (1960) 1857; H.I. Miettinen and J. Pumplin, *Phys. Rev.* **D18** (1978) 1696.

Figure caption

Figure 1. The ratio r_A defined by Eq. (6), calculated from the Eq. (3) for $A = 208$, using Saxon-Woods nuclear density with the radius $R = 7.1f$ and the surface thickness parameter $a = 0.54f$. The cross-section $\sigma_V = 10 \text{ mb}$ and $z_V = (mx_t)^{-1}$, as suggested in [1].