BB

BCCNT 94/011/237

SW 94 17

Relation of QCD Sum Rules in Matter and
the Nuclear Many-Body Problem

C. M. Shakin

Department of Physics and Center for Nuclear Theory
Brooklyn College of the City University of New York
Brooklyn, N.Y. 11210

CERN LIBRARIES, GENEVA

P00022759

(January, 1994)

Submitted to Physical Review C

Abstract

The method of QCD sum-rules provides a powerful technique for the calculation of properties of hadrons in terms of a number of parameters that specify various condensate matrix elements. One may hope that it will be possible to calculate some of the condensate matrix elements using effective Lagrangians. If we concentrate on quark condensates, the Nambu–Jona-Lasinio model may provide a useful model. In this work we study the relation between relativistic nuclear many-body theory and the analysis of the nucleon self-energy in nuclear matter made using QCD sum rules. This is done by introducing the fields of a bosonized version of the Nambu–Jona-Lasinio (NJL) model. Using the simplest version of the QCD sum-rule analysis, we replace the QCD order parameters in matter with related order parameters describing a Lorentz scalar and a vector field. In our mean-field analysis we find that the many-body theory, based upon the bosonization of the NJL model, is consistent with the simplest version of the QCD sum-rule calculation of the nucleon self-energy in matter.

I. Introduction

A particularly extensive study of QCD sum rules in nuclear matter has been carried forward by the group associated with the University of Maryland [1-5]. They have calculated the nucleon self-energy in matter in terms of various quark and gluon condensates. The method of calculation involves the use of the operator product expansion to separate the short-distance (perturbative) behavior of operators from the long-distance (nonperturbative) behavior. It is the long-distance behavior that is expressed in terms of condensate matrix elements. These are of increasing complexity as the dimension of the operator studied is increased. In nuclear matter there are a number of condensates, such as $\langle q^\dagger q \rangle_\rho$, that vanish in the vacuum but which must be considered in nuclear matter. The large number of such condensates increases the complexity of the analysis relative to studies in vacuum. However, under certain assumptions it is found that the nucleon self-energy has a large (attractive) Lorentz scalar part and a large (repulsive) Lorentz vector part [1]. This body of work [1-5] may, therefore, provide support for the relativistic nuclear many-body theory that has been extensively studied in recent years [6,7].

The simplest version of the sum-rule calculations involves only the density-dependent condensates $\langle \bar{q}q \rangle_\rho$ and $\langle q^\dagger q \rangle_\rho$. (The latter condensate vanishes in vacuum.) The authors of refs. [1-5] have gone on to consider various gluon condensates, gluon-quark condensates and four-quark condensates. It is not known how the four-quark condensates should be calculated in matter and different results are obtained for the scalar part of the nucleon self-energy when different assumptions are made [5]. However, if we relate the QCD sum-rule calculations to nuclear many-body theory, it may be possible to gain some insight into how the density

dependence of the four-quark condensates is to be treated. One goal of this work is to make a first step in relating the sum-rule calculations to nuclear many-body theory. (Here we consider only the condensates of lowest dimension.)

Our analysis will be made by using a bosonized version of the Nambu–Jona-Lasinio (NJL) model [8]. Using this formalism, we can replace the order parameters, $\langle \bar{q}q \rangle_\rho$ and $\langle q^\dagger q \rangle_\rho$, by scalar and vector fields (in the simplest version of the QCD model). While this substitution does not represent a new calculation, it may allow for a more transparent description of some aspects of the sum-rule calculations.

A complete exposition of a momentum-space bosonization of the NJL model has been given in refs. [9] and [10]. In the latter reference the analysis is carried out for the Lagrangian

$$\begin{aligned} \mathcal{L}(q) = & \bar{q}(i\partial - m_q^0)q + \frac{G_S}{2}[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2] \\ & - \frac{G_V}{2}[(\bar{q}\gamma^\mu\tau_a q)^2 + (\bar{q}\gamma_5\gamma^\mu\tau_a q)^2] , \end{aligned} \quad (1.1)$$

where $\tau_a = (\tau_0, \tau_i)$ with $\tau_0 = \mathbb{I}$ and the $\tau_i (i = 1, 2, 3)$ are the Pauli matrices of the flavor group $SU(2)_f$. Further, $m_q^0 = \text{diag}(m_u^0, m_d^0)$ is a mass matrix of current quark masses. One may eliminate quartic quark interactions from the action by introducing boson fields in the standard manner.

We have [10]

$$\begin{aligned} \mathcal{L}(q, \bar{\sigma}, \bar{u}, \bar{v}, \bar{a}) = & \bar{q}(i\partial - m_q^0 - \bar{\sigma} + i\gamma_5\bar{\pi} + \gamma^\mu\bar{v}_\mu + \gamma_5\gamma^\mu\bar{a}_\mu)q \\ & - \left[\frac{\bar{\sigma}_a^2 + \bar{\pi}_a^2}{2G_S} \right] + \left[\frac{\bar{v}_{\mu a}^2 + \bar{a}_{\mu a}^2}{2G_V} \right] , \end{aligned} \quad (1.2)$$

where a summation over the isospin index a is implied ($a = 0, 1, 2, 3$). Here $\bar{\sigma} = \bar{\sigma}_a\tau_a$, $\bar{\pi} = \bar{\pi}_a\tau_a$, $\bar{v}_\mu = \bar{v}_{\mu a}\tau_a$, and $\bar{a}_\mu = \bar{a}_{\mu a}\tau_a$. The field $\bar{\sigma}_0$ has a nonzero vacuum value. (In Eq. (1.2) we have changed the sign of $\bar{\sigma}$ relative to that used in refs. [9,10].) It is useful to write

$$\bar{\sigma}_0 + m_q^0 = \tilde{\sigma}_0 + m_q , \quad (1.3)$$

where $\tilde{\sigma}_0$ has zero vacuum value, and where m_q is the constituent quark mass. (We neglect whatever small difference may exist between the constituent masses of the up and down quarks.)

We are now in a position to integrate out the quark fields. One obtains [9,10]

$$\begin{aligned} \mathcal{L}(\tilde{\sigma}, \tilde{\pi}, \tilde{v}, \tilde{a}) = & -i\text{Tr}\ln[1 + (i\partial - m_q)^{-1}(-\tilde{\sigma} + i\gamma_5\tilde{\pi} + \gamma^\mu\tilde{v}_\mu + \gamma_5\gamma^\mu\tilde{a}_\mu)] \\ & - \frac{\tilde{\sigma}_a^2 + \tilde{\pi}_a^2}{2G_S} + \frac{\tilde{v}_{\mu a}^2 + \tilde{a}_{\mu a}^2}{2G_V} . \end{aligned} \quad (1.4)$$

We will here concentrate on the scalar field $\tilde{\sigma}_0$ and the vector field $\tilde{\omega}_\mu \equiv \tilde{v}_\mu^0$, or the related fields σ_0 and ω_0 defined below. Following reference [10], we introduce momentum -

dependent coupling constants and wavefunction renormalization parameters. With use of the isospin index, a , we have

$$\tilde{\pi}^a(p) = Z_{\pi}^{-1/2} g_{\pi}(p^2) \pi^a(p) \quad , \quad (1.5)$$

$$\tilde{\sigma}^a(p) = Z_{\sigma}^{-1/2} g_{\sigma}(p^2) \sigma^a(p) \quad , \quad (1.6)$$

$$\tilde{v}^a(p) = Z_v^{-1/2} g_v(p^2) v^a(p) \quad , \quad (1.7)$$

$$\tilde{a}^a(p) = Z_a^{-1/2} g_a(p^2) a^a(p) \quad . \quad (1.8)$$

[Note that $v^0(p)$ is a $T = 0$ vector field.]

In this formalism, the wavefunction renormalization constants can be chosen to yield unit strength at the poles of the meson propagators [9,10]. (For our purposes, we can take the various Z factors to be equal to 1.) The fields σ_0 and $\omega_{\mu} \equiv v_{\mu}^0$ on the right-hand side of Eqs. (1.5)-(1.8) may then be identified with those fields usually used in relativistic many-body theory [6,7]. We will use the notation ω for the zeroth component of vector field ω_{μ} to avoid confusion with the isospin indices. Since we are here interested in $p^2 = 0$ for the meson fields, we will write $\sigma_0 = \sigma_0(0)$, with $\sigma_0 = f_{\pi} + \sigma$ and

$$f_{\pi} + \sigma = - \frac{G_S}{g_{\sigma}} \langle \bar{u}u + \bar{d}d \rangle_{\rho} . \quad (1.9)$$

We recall that $\sigma = 0$ in the vacuum. (Negative values of σ represent a partial restoration of chiral symmetry in matter.)

Further, we define $\omega \equiv \omega_0(0)$ and

$$\omega_{MF} = \frac{G_V}{g_{\omega}} \langle \bar{u}\gamma^0 u + \bar{d}\gamma^0 d \rangle_{\rho} , \quad (1.10)$$

where we have put $g_{\omega} = g_V$. As we will see, the ω field in nuclear matter is about four times larger than ω_{MF} of Eq. (1.10) due to vacuum polarization effects [11,12].

We adopt the notation used in refs. [1-5]:

$$\langle \bar{q}q \rangle_{\rho} = \frac{1}{2} \langle \bar{u}u + \bar{d}d \rangle_{\rho} , \quad (1.11)$$

and

$$\langle \bar{q}\gamma^0 q \rangle_{\rho} = \frac{1}{2} \langle u^{\dagger}u + d^{\dagger}d \rangle_{\rho} , \quad (1.12)$$

where the subscript ρ indicates that we are considering nuclear matter of baryon density ρ_B .

For the scalar field, we have

$$\sigma = - \frac{2G_S}{g_\sigma} [\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0] , \quad (1.13)$$

where $\langle \bar{q}q \rangle_0$ is the vacuum value. (Note that Eq. (1.13) is consistent with Eq. (1.9).) The last few equations suggest how we might go over to the use of $\sigma = \sigma_0 - f_\pi$ and ω as order parameters, rather than $\langle \bar{q}q \rangle_\rho$ and $\langle q^\dagger q \rangle_\rho$. However, to do this properly it is useful to understand how to calculate the values of $\langle q^\dagger q \rangle_\rho$ and $\langle \bar{q}q \rangle_\rho$ in terms of the valence (or constituent) quark sources of the nucleon. Proceeding in this manner, one then needs to take into account vacuum polarization effects [11,12]. In the NJL model these effects arise from a summation of a set of "bubbles" as shown in Fig. 1(c). That leads to replacing G_V in Eq. (1.10) by $G_V/(1 - G_V J_V(0))$. (Here we may replace $\langle q^\dagger q \rangle_\rho$ by $\langle q^\dagger q \rangle_{val}$.) Similar remarks pertain to the calculation of σ from Eq. (1.13). There one replaces G_S by $G_S/(1 - G_S J_S(0))$. In these expressions, $J_S(0)$ and $J_V(0)$ are the basic quark-loop integrals of the NJL model evaluated at $p^2 = 0$. (See Fig. 1 and reference [13].)

We have used the fact that the baryon density is conserved, so that $\langle q^\dagger q \rangle_\rho = \langle q^\dagger q \rangle_{val}$. However, $\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0$ is not equal to $\langle \bar{q}q \rangle_{val}$. (See Sec. III.) As we will discuss in Sec. III, the σ and ω fields may be calculated in terms of their sources

$$\sigma = - \frac{2G_S}{g_\sigma} \left[\frac{1}{1 - G_S J_S(0)} \right] \langle \bar{q}q \rangle_{val} , \quad (1.14)$$

and

$$\omega = \frac{2G_V}{g_\omega} \left(\frac{1}{1 - G_V J_V(0)} \right) \langle q^\dagger q \rangle_{val} . \quad (1.15)$$

(Note that $\sigma \leq 0$, while $\omega \geq 0$.) Equations (1.14) and (1.15) express the values of the sigma and omega fields in matter in terms of the parameters of the NJL model and the coupling constants of these fields to quarks (g_σ and g_ω) that arise when one carries out the bosonization procedure [9,10]. As noted earlier, ω of Eq. (1.15) is about four times larger than ω_{MF} of Eq. (1.10). In these equations, $\langle q^\dagger q \rangle_{val} = 3\rho_B/2$, while $\langle \bar{q}q \rangle_{val}$ is somewhat less than $3\rho_B/2$ [14]. We remark that

$$\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0 = \left(\frac{1}{1 - G_S J_S(0)} \right) \langle \bar{q}q \rangle_{val} . \quad (1.16)$$

We may also write [2]

$$\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0 = \frac{\sigma_N}{2m_q} \rho_B , \quad (1.17)$$

if we keep terms linear in the density.

The last equation provides a model independent relation, expressed in terms of the pion-nucleon sigma term, σ_N . We recall that σ_N is defined in terms of the nucleon matrix element

of the quark scalar density and the average current quark mass, m_q^0 ,

$$\sigma_N = 2m_q^0 \langle N | \bar{q}q | N \rangle , \quad (1.18)$$

where

$$\langle N | \bar{q}q | N \rangle = \frac{1}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle . \quad (1.19)$$

Here in passing from Eq. (1.16) to (1.17) we have used the NJL model and identified $\langle N | \bar{q}q | N \rangle$ as

$$\langle N | \bar{q}q | N \rangle = \frac{1}{1 - G_S J_S(0)} \langle \bar{q}q \rangle_{val} . \quad (1.20)$$

From Eqs. (1.18)-(1.19) one might expect that $\langle N | \bar{q}q | N \rangle \approx 3/2$, so that $\sigma_N \sim 15$ MeV. However, the enhancement factor $(1 - G_S J_S(0))^{-1}$ yields a value of σ_N close to 45 MeV. (The calculation of σ_N using the NJL model is described in some detail in reference [11].) Uncertainty in the value of σ_N leads to a corresponding uncertainty for the value of the scalar field in matter, which may now be written as

$$\sigma = - \frac{2G_S}{g_\sigma} \left[\frac{\sigma_N}{2m_q^0} \rho_B \right] . \quad (1.21)$$

For example, if we use the values used in reference [12]: $G_S = 7.91 \text{ GeV}^{-2}$, $g_\sigma = 2.58$,

$\sigma_N = 45 \text{ MeV}$, $m_q^0 = 5 \text{ MeV}$ and $\rho_B = (0.109 \text{ GeV})^3$, we find $\sigma \approx -36 \text{ MeV}$.

For later reference, we remark that, if we identify σ and ω with the sigma and omega fields of relativistic many-body theory, we have [6,7]

$$\sigma = - \frac{G_{\sigma NN} \rho_S}{m_\sigma^2} , \quad (1.22)$$

and

$$\omega = \frac{G_{\omega NN} \rho_B}{m_\omega^2} , \quad (1.23)$$

in the simplest (Hartree) approximation. Here $G_{\sigma NN}$ and $G_{\omega NN}$ are the coupling constants of the sigma and omega fields to the nucleon, ρ_S is the nucleon scalar density and ρ_B is the baryon density. For consistency with Eq. (1.21), we should have $G_{\sigma NN} \approx 6.76$, if we put $\rho_S \approx \rho_B$.

Using a simple model developed earlier [14], we can calculate the coupling of the boson fields to the nucleon as in Fig. 2, where the small filled circle represents the meson-quark coupling. In such a model, we have $G_{\sigma NN} = g_\sigma F_S^{val}(0)$, where $F_S^{val}(0)$ is the scalar form factor for valence (or constituent) quarks of the nucleon [14]. We obtain $G_{\sigma NN} = 6.76$, if $g_\sigma = 2.58$ and $F_S^{val}(0) = 2.62$. We note that we expect $F_S^{val}(0) < 3$, if the valence quarks are described by four-component (Dirac) wavefunctions [14]. In the same model, we find $G_{\omega NN} = 3g_\omega = 14.2$, if we use $g_\omega = 4.73$, as determined via bosonization. (Note that

$F_V^{val}(0) = 3$, where $F_V^{val}(q^2)$ is the vector form factor for valence quarks as calculated in reference [14].)

One may argue that the sigma meson does not appear in the data tables and is not a physical meson. That is true; however, as we have discussed extensively [15,16], the sigma "meson" appears as a useful degree of freedom, if one restricts oneself to spacelike, or zero, values of p^2 ($p^2 \leq 0$). This may be understood, if one extends the bosonization theory developed here (at one-loop order) to include coupling to the two-pion continuum [15,16]. For timelike p^2 , the coupling to the two-pion continuum moves the sigma pole to high energy. (The $f_0(1400)$ may be a good candidate for the first $S = 0$, $T = 0$, $\bar{q}q$ resonance.) On the other hand, for $p^2 \leq 0$, the sigma is sufficiently far from the two-pion continuum so as to appear as a reasonably pure $\bar{q}q$ excitation. Indeed, for spacelike p^2 , a sigma-dominance model provides an excellent approximation for the scalar-isoscalar hadronic correlation function and the $q\bar{q}$ T matrix [15,16]. Our studies indicate that the sigma meson may be assigned a mass of about 500 MeV, if we consider only momenta with $p^2 \leq 0$.

The bosonization scheme of references [9,10] leads to the result $m_\omega^2(p^2) = g_\omega^2(p^2)/4G_V$, or $m_\omega^2(0) = g_\omega^2(0)/4G_V$. In our studies of the bosonization of the NJL model we found $g_\omega(0) = 4.73$ and $G_V = 9.14 \text{ GeV}^{-2}$. These values yield $m_\omega(0) = 782 \text{ MeV}$, which is in agreement with the experimental value of the omega mass.

Our work is based upon the observation that the NJL model provides a good description of the long-range quark dynamics. The bosonized version of this model exhibits the various meson fields (σ , π , ρ , ω) that play an important role in relativistic nuclear many-body theory [7]. Of the various effective Lagrangians available, the relation of the NJL effective

Lagrangian and nuclear many-body theory is most easily described. (The NJL model has some advantages over the nonlinear sigma model in that, upon bosonization of the NJL model, one obtains a sigma meson. As noted earlier, in nuclear dynamics the sigma meson is usually spacelike ($p^2 \leq 0$) and is a useful degree of freedom [15,16], even though there is no physical low-mass sigma meson.)

The organization of our work is as follows. In Sec. II, using some results of Sec. III, we rewrite the nucleon correlator of Ioffe currents in terms of the scalar (σ) and vector (ω) fields of the bosonized NJL model. In Sec. III we relate the σ and ω fields to the condensates and provide further insight into the bosonization procedure. We also point out that generally consistent results are obtained if the nucleon self-energy is calculated by different methods. Section IV contains some discussion of four quark condensates, while Section V contains some further discussion and conclusions.

II. Self-Energy of the Nucleon in Nuclear Matter

We consider the correlator of two Ioffe currents [1-5],

$$\Pi(x) = \langle T[\eta_N(x)\bar{\eta}_N(0)] \rangle_\rho, \quad (2.1)$$

which is related to the propagator of a nucleon in nuclear matter. Here $\Pi(x)$ has two Dirac indices corresponding to the indices of the Ioffe currents. After transformation to momentum space, one has

$$\Pi(q) = \Pi_1(q^2, q \cdot u) + \not{q}\Pi_q(q^2, q \cdot u) + \not{u}\Pi_u(q^2, q \cdot u), \quad (2.2)$$

where u_μ describes the flow of the nuclear matter. [In the rest frame, we have $u_\mu = (1, 0, 0, 0)$.]

It is found that, for the nucleon

$$\Pi_1(q^2, q \cdot u) = \frac{1}{4\pi^2} q^2 \ln(-q^2) \langle \bar{q}q \rangle_\rho + \dots, \quad (2.3)$$

$$\Pi_q(q^2, q \cdot u) = -\frac{1}{64\pi^2} (q^2)^2 \ln(-q^2) + \frac{1}{3\pi^2} q \cdot u \ln(-q^2) \langle q^\dagger q \rangle_\rho + \dots, \quad (2.4)$$

$$\Pi_u(q^2, q \cdot u) = \frac{2}{3\pi^2} q^2 \ln(-q^2) \langle q^\dagger q \rangle_\rho + \dots. \quad (2.5)$$

Here the nucleon correlator is taken as the average of the proton and neutron correlators, with

$$\langle \bar{q}q \rangle_\rho = \frac{1}{2} [\langle \bar{u}u \rangle_\rho + \langle \bar{d}d \rangle_\rho] , \quad (2.6)$$

$$\langle q^\dagger q \rangle_\rho = \frac{1}{2} [\langle u^\dagger u \rangle_\rho + \langle d^\dagger d \rangle_\rho] , \quad (2.7)$$

defined as above.

We note that polynomials in q^2 are not written in Eqs. (2.2)-(2.4), since these do not contribute if a Borel transformation is made. Also, higher-dimensional condensates will affect the result. These have been extensively studied in references [3-5].

We now rewrite Eqs. (2.3)-(2.5) in terms of the fields σ and ω found in nuclear matter.

We use some results derived in the next section, to obtain

$$\Pi_1(q^2, q \cdot u) = \frac{1}{4\pi^2} q^2 \ln(-q^2) \langle \bar{q}q \rangle_0 + \frac{1}{4\pi^2} q^2 \ln(-q^2) \left[-\frac{g_\sigma}{2G_S} \right] \sigma + \dots , \quad (2.8)$$

$$\Pi_q(q^2, q \cdot u) = -\frac{1}{64\pi^2} (q^2)^2 \ln(-q^2) + \frac{1}{3\pi^2} q \cdot u \ln(-q^2) \left[\frac{g_\omega}{2G_V} \frac{1}{\kappa_V} \right] \omega + \dots , \quad (2.9)$$

and

$$\Pi_u(q^2, q \cdot u) = \frac{2}{3\pi^2} q^2 \ln(-q^2) \left[\frac{g_\omega}{2G_V} \frac{1}{\kappa_V} \right] \omega + \dots \quad (2.10)$$

(If we put $\sigma = \omega = 0$, we obtain the results for the vacuum.) In these expressions $\kappa_V \approx 4.0$. In Sec. III we will explain why this factor appears in the case of the ω field [see Eqs. (2.9) and (2.10)] and a corresponding factor does not appear in the case of the σ field. The corrections to the vacuum result are linear in the density, if σ and ω are given by the mean-field expressions, Eqs. (1.22) and (1.23), with $\rho_S \approx \rho_B$. (We note that ρ_S and ρ_B only differ by a few percent [6,7].)

III. Relation of Condensates to Sigma and Omega Fields

In this Section we wish to determine the relation that exists between the condensates $\langle \bar{q}q \rangle_\rho$ and $\langle q^\dagger q \rangle_\rho$ and the sigma and the omega fields of the bosonized NJL model. We may relate $\langle \bar{q}q \rangle_\rho$ to the sigma field using the relation

$$f_\pi + \sigma = - \frac{2G_S}{g_\sigma} \langle \bar{q}q \rangle_\rho, \quad (3.1)$$

given previously. Equation (3.1) may be used as it stands; however, it is useful to introduce valence (or constituent) quark sources of the nucleon and then calculate the vacuum polarization effects to obtain $\langle \bar{q}q \rangle_\rho$ [11]. In that procedure we have, with $\sigma = \sigma_0 - f_\pi$,

$$\sigma = - \frac{2G_S}{g_\sigma} \left[\frac{1}{1 - G_S J_S(0)} \right] \langle \bar{q}q \rangle_{val} \quad (3.2)$$

$$= - \frac{2G_S}{g_\sigma} \left[\frac{1}{1 - G_S J_S(0)} \right] \langle N | \bar{q}q | N \rangle_{val} \rho_B \quad (3.3)$$

and, with $\langle q^\dagger q \rangle_\rho = \langle q^\dagger q \rangle_{val}$,

$$\omega = \frac{2G_V}{g_\omega} \left[\frac{1}{1 - G_V J_V(0)} \right] \langle q^\dagger q \rangle_{val}, \quad (3.4)$$

$$= \frac{2G_V}{g_\omega} \left[\frac{1}{1 - G_V J_V(0)} \right] \frac{3}{2} \rho_B . \quad (3.5)$$

We have put $\langle q^\dagger q \rangle_{val} = 3\rho_B/2$, since we have defined

$$\langle q^\dagger q \rangle_\rho = \frac{1}{2} \langle u^\dagger u + d^\dagger d \rangle_\rho \quad (3.6)$$

to be an average of the up and down quark densities. We note that the (simplest) momentum-space bosonization scheme requires [17],

$$\frac{G_S}{1 - G_S J_S(p^2)} = - \frac{g_\sigma^2(p^2)}{p^2 - m_\sigma^2(p^2)} , \quad (3.7)$$

and

$$\frac{G_V}{1 - G_V J_V(p^2)} = - \frac{g_\omega^2(p^2)}{p^2 - m_\omega^2(p^2)} . \quad (3.8)$$

For $p^2 = 0$, we have, therefore,

$$\frac{G_S}{1 - G_S J_S(0)} = \frac{g_\sigma^2(0)}{m_\sigma^2(0)} \quad (3.9)$$

and

$$\frac{G_V}{1 - G_V J_V(0)} = \frac{g_\omega^2(0)}{m_\omega^2(0)} . \quad (3.10)$$

For example, with $g_\omega = 4.73$ and $G_V = 9.14 \text{ GeV}^{-2}$, $J_V(0) = 0.082 \text{ GeV}^2$ [17], we have $\kappa_V \equiv [1 - G_V J_V(0)]^{-1} = 4.00$. Then, with $\rho_B = (0.109 \text{ GeV})^3$, we find $\omega = 30 \text{ MeV}$ in nuclear matter, upon use of Eq. (3.5). (We note that, when carrying out the bosonization procedure for the ρ meson [17], we had $g_\rho = 4.31$ and $G_V = 7.58 \text{ GeV}^{-2}$. These values differ somewhat from those obtained for the ω meson. This difference has its origin in the fact that coupling to the two-pion continuum shifts the resonance position downward by about 100 MeV in the case of the ρ meson [17]. This effect is not present for the ω meson, which has a quite small width. Therefore, a larger value of G_V is needed to obtain the experimental value of the ω mass upon bosonization. Note that the constituent quark mass, m_q , is determined by G_S , which remains the same for all mesons studied.)

We now write the nucleon self-energy in the nuclear matter rest frame as $\Sigma = \Sigma_S + \gamma^0 \Sigma_V$. The vector part is found to be [1]

$$\Sigma_V = \frac{64\pi^2}{3M^2} \langle q^\dagger q \rangle_\rho , \quad (3.11)$$

where M is the Borel mass. We may rewrite this equation as

$$\Sigma_V = \frac{64\pi^2}{3M^2} \left[\frac{g_\omega}{2G_V} \frac{1}{\kappa_V} \right] \omega \quad . \quad (3.12)$$

With $\rho_B = (0.109 \text{ GeV})^3$, we find $\Sigma_V = 407 \text{ MeV}$, if we put the Borel mass equal to 1 GeV in Eq. (3.11). For consistency, we should have $\Sigma_V = \hat{\Sigma}_V$, with the definition

$$\hat{\Sigma}_V \equiv G_{\omega NN} \omega \quad , \quad (3.13a)$$

$$= (g_\omega F_V(0)) \omega \quad , \quad (3.13b)$$

$$= (3g_\omega) \omega \quad . \quad (3.13c)$$

[See Fig. 2.] We have seen that $G_{\omega NN} = 14.2$, so that $\hat{\Sigma} = 426 \text{ MeV}$, which is reasonably close to the value $\Sigma_V = 407 \text{ MeV}$ obtained from Eq. (3.11) when $M = 1 \text{ GeV}$. (A small decrease in the value of M will bring the values obtained for Σ_V and $\hat{\Sigma}_V$ into agreement.)

We should note that Σ_V of Eq. (3.11) depends only on M^2 and $\langle q^\dagger q \rangle_\rho$. It is of interest that the bosonization scheme that yields Eqs. (3.5), (3.12) and (3.13), gives a value of $\hat{\Sigma}_V$ quite close to the value of Σ_V obtained from the QCD sum rule calculation.

Now we consider the scalar part of the self-energy, Σ_S , and note that the value of the nucleon mass in matter is $m_N^* = m_N + \Sigma_S$, with

$$\Sigma_S = \frac{8\pi^2}{M^2} \left(\frac{g_\sigma}{2G_S} \right) \sigma . \quad (3.14)$$

(Recall that $\sigma_0 = f_\pi + \sigma$.) For ease of reference, we again use the values of g_σ and G_S used in reference [12]: $G_S = 7.91 \text{ GeV}^{-2}$ and $g_\sigma = 2.58$. We have

$$\sigma = - \frac{2G_S}{g_\sigma} \left(\frac{1}{1 - G_S J_S(0)} \right) \langle \bar{q}q \rangle_{val} , \quad (3.15)$$

$$\simeq - \frac{2G_S}{g_\sigma} \kappa_S \left(\frac{3}{2} \rho_B \right) , \quad (3.16)$$

$$\simeq - 40 \text{ MeV} , \quad (3.17)$$

where the enhancement factor $\kappa_S \equiv [1 - G_S J_S(0)]^{-1} = 3.37$. We have put $\langle \bar{q}q \rangle_{val} \simeq \langle q^\dagger q \rangle_{val}$ in Eq. (3.16) to obtain a simple estimate for σ in Eq. (3.17). That is probably an overestimate since relativistic effects (small components of the quark wavefunctions) reduce the value of $\langle \bar{q}q \rangle_{val}$ relative to $\langle q^\dagger q \rangle_{val}$ [14]. If we consider a 10 percent reduction, so that $\sigma = - 36 \text{ MeV}$, we would have $\Sigma_S = - 418 \text{ MeV}$, which is of the order of magnitude of the empirical scalar part of the self-energy ($\Sigma_S \sim - 400 \text{ MeV}$). (Modifications of the self-energy due to short-range nucleon-nucleon correlations and exchange

(Fock) terms are discussed in reference [7], where an analysis of the nucleon self-energy based upon relativistic Brueckner-Hartree-Fock theory is given.)

We note the different structure of Eqs. (3.12) and (3.14). In Eq. (3.14) there is an enhancement factor present in the expression for σ . (See Eq. (3.15).) On the other hand, the factor κ_V in ω is canceled by the factor of κ_V^{-1} in Eq. (3.12). That this feature is correct may be seen from the standard analysis that makes Σ_S proportional to σ_N , the pion-nucleon sigma term. We note that the enhancement factor, κ_S , also appears when one calculates the value of σ_N in a sigma dominance model. [See Fig. 1(c).] We have [11],

$$\sigma_N \approx 2m_q^0 \kappa_S \langle N | \bar{q}q | N \rangle_{val} \quad (3.18)$$

$$\approx 2m_q^0 \kappa_S \langle N | q^\dagger q | N \rangle_{val} \quad (3.19)$$

$$\approx 3m_q^0 \kappa_S \quad (3.20)$$

$$\approx 50 \text{ MeV} \quad , \quad (3.21)$$

if $m_q^0 = 5 \text{ MeV}$. Actually $\langle N | \bar{q}q | N \rangle$ is expected to be somewhat smaller than $\langle N | q^\dagger q | N \rangle$, so that values of $\sigma_N \sim 45 \text{ MeV}$ are easily found. (Theoretical analysis of experimental data suggests $\sigma_N = 45 \pm 8 \text{ MeV}$ [18].)

We may make contact with some familiar formulas, if we note that

$$\frac{G_S}{g_\sigma} = \frac{m_q^0}{m_\pi^2 f_\pi} . \quad (3.22)$$

Equation (3.22) follows from the use of the Goldberger-Treiman relation and a relation obtained in the bosonization procedure [9]:

$$m_\pi^2(p^2) = \frac{m_q^0}{m_q G_S} g_\sigma^2(p^2) . \quad (3.23)$$

(When using the last relation, we take $p^2 = 0$.) We may use Eq. (3.22) and Eq. (1.9) and consider the vacuum, where $\sigma_0 = f_\pi$ and $\sigma = 0$, to obtain

$$f_\pi^2 m_\pi^2 = - 2m_q^0 \langle \bar{q}q \rangle_0 , \quad (3.24)$$

which is the Gell-Mann-Oakes-Renner [19] relation. The generalization to nuclear matter is made by replacing f_π^2 by $f_\pi \sigma_0$:

$$\sigma_0 f_\pi m_\pi^2 = - 2m_q^0 \langle \bar{q}q \rangle_\rho . \quad (3.25)$$

In writing Eq. (3.25), we have assumed that m_π^2 has only a very weak density dependence [13].

Taking the ratio of the last two equations yields

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = \frac{\sigma_0}{f_\pi} , \quad (3.26)$$

$$= \left[1 + \frac{\sigma}{f_\pi} \right] , \quad (3.27)$$

$$= 1 - \frac{\sigma_N}{f_\pi^2 m_\pi^2} \rho_B , \quad (3.28)$$

a result given previously by several authors [2,20]. It is readily seen that

$$\frac{m_N^*}{m_N} = 1 - \frac{\sigma_N}{f_\pi^2 m_\pi^2} \rho_B , \quad (3.29)$$

since the nucleon mass is proportional to the scalar condensate order parameter in models of the type considered here. With $\sigma_N = 45$ MeV, Eq. (3.29) describes a 35 percent reduction of m_N in matter.

IV. Four-Quark Condensates

A major difficulty in the use of QCD sum rules in matter is the lack of knowledge concerning the proper calculation of four-quark condensates [5]. In general, a factorization scheme of the form

$$\begin{aligned} \langle \bar{q}q\bar{q}q \rangle_\rho &= \langle \bar{q}q \rangle_\rho \langle \bar{q}q \rangle_\rho \\ &= \langle \bar{q}q \rangle_0^2 + 2\langle N|\bar{q}q|N \rangle \langle \bar{q}q \rangle_0 \rho_B + \dots \end{aligned} \quad (4.1)$$

is used. Such a procedure yields a strong density dependence for the 4-quark condensates. That strong density dependence is an undesirable feature, if one is to reproduce the empirical values of the nucleon self-energy in matter [5].

For the calculations in vacuum, the factorization hypothesis is represented by the approximation

$$\langle \bar{q}_{i\alpha} q_{j\beta} \bar{q}_{k\gamma} q_{l\delta} \rangle_0 \simeq \langle \bar{q}_{i\alpha} q_{j\beta} \rangle_0 \langle \bar{q}_{k\gamma} q_{l\delta} \rangle_0 - \langle \bar{q}_{i\alpha} q_{l\delta} \rangle_0 \langle \bar{q}_{k\gamma} q_{j\beta} \rangle_0, \quad (4.2)$$

where $i, j = 1, \dots, 3$ are color indices and $\alpha, \beta = 1, \dots, 4$ are Dirac indices. We consider the quantity

$$C^{vac} = \langle \bar{q} \Gamma \lambda^a q \bar{q} \tilde{\Gamma} \lambda^a q \rangle_0, \quad (4.3)$$

where Γ and $\tilde{\Gamma}$ are Dirac matrices and the $\lambda^a (a = 1, \dots, 8)$ are Gell-Mann color matrices.

If all the quarks have the same flavor one finds [21]

$$C^{vac} \simeq -\frac{1}{9} Tr(\Gamma\tilde{\Gamma}) <\bar{q}q>_{vac}^2 . \quad (4.4)$$

The factorization hypothesis used for four-quark condensates in matter would give rise to

$$\begin{aligned} C_\rho &\equiv <\bar{q}\Gamma\lambda^a q\bar{q}\tilde{\Gamma}\lambda^a q>_\rho \\ &\simeq -\frac{1}{9} Tr(\Gamma\tilde{\Gamma}) <\bar{q}q>_\rho^2 . \end{aligned} \quad (4.5)$$

If we use $<\bar{q}q>_\rho = <\bar{q}q>_0 + <N|\bar{q}q|N>_{\rho_B}$, we find that this expression yields a strong density dependence of the four-quark condensates. As noted above, a strong density dependence of the four-quark condensates is an undesirable feature, if one is to reproduce the empirical values of the self-energy in matter [5].

V. Discussion

The NJL model provides an excellent representation of some of the low-energy properties of QCD. That is particularly true of an extended model, such as that given in Eq. (1.1), that can describe the rho and omega mesons. For example, a study has been made of the effective chiral Lagrangian that may be derived from the extended NJL model [22]. The chiral Lagrangian is given in terms of increasing factors of momentum. The model-independent part is the usual nonlinear sigma model. At the next level one has ten new terms (of order p^4) that require the specification of ten constants, $L_i (i = 1, \dots, 10)$. The extended NJL model is seen to reproduce the empirical values of the L_i quite well [22]. That is strong evidence that the low-energy (or large distance) features of QCD may be well represented by the extended NJL model.

The NJL model may also be generalized to include a description of confinement [17]. For a complete analysis, however, one may also wish to include order parameters of the gluon condensate in a generalized NJL model. Since it is found that the gluon condensate has a weaker density dependence than the quark condensate $\langle \bar{q}q \rangle_\rho$ [2], in a first approximation it might be possible to avoid reference to the gluon condensate.

As mentioned earlier in this work, the greatest theoretical uncertainty arising when one considers QCD sum rules in matter is the treatment of the four-quark condensates. For example, in reference [5] one writes the scalar-scalar four-quark condensate so that it interpolates between its factorized form in vacuum and its factorized form in nuclear matter,

$$| \quad \langle \bar{q}q \rangle^2 \equiv (1 - f) \langle \bar{q}q \rangle_0^2 + f \langle \bar{q}q \rangle_\rho^2, \quad (5.1)$$

where f is a free parameter. The calculations made for nuclear matter seem to prefer $f \approx 0$, if one is to reproduce the scalar and vector fields of relativistic many-body theory [6,7]. This feature of the calculations is not well understood and further analysis is needed to clarify the treatment of four-quark condensates.

We believe our work represents a first step in relating QCD sum-rule calculations in matter to the nuclear many-body problem. We have shown how a mean-field calculation can be put into correspondence with the simplest version of the QCD sum-rule analysis for a nucleon in nuclear matter. It is unclear whether our procedure can be generalized beyond the mean-field approximation.

Acknowledgement

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Research Award Program.

References

- [1] T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. Lett. 67, 961 (1991).
- [2] T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. C45, 1881 (1992).
- [3] R.J. Furnstahl, D.K. Griegel and T.D. Cohen, Phys. Rev. C46, 1057 (1992).
- [4] X. Jin, T.D. Cohen, R.J. Furnstahl, and D.K. Griegel, Phys. Rev. C47, 2882 (1993).
- [5] X. Jin, M. Nielsen, T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. C49, 464 (1994).
- Xuemin Jin, Ph.D. Thesis, Univ. of Maryland (1993).
- For recent work dealing with the role of instantons in QCD sum rules for the nucleon see, H. Forkel and M.K. Banerjee, Univ. of Maryland Report No. 93-172 (1993).
- [6] B.D. Serot and J.D. Walecka, in Advances in Nuclear Physics, edited by J.W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
- [7] L.S. Celenza and C.M. Shakin, Relativistic Nuclear Physics: Theories of Structure and Scattering. (World Scientific, Singapore, 1986).
- [8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).
- [9] V. Bernard, A.A. Osipov and Ulf-G. Meissner, Phys. Lett. B285, 119 (1992).
- [10] V. Bernard, Ulf-G. Meissner and A.A. Osipov, preprint of the Center for Nuclear Research and the Univ. Louis Pasteur de Strasbourg, CRN 93-57 (1993).
- [11] Nam-Wei Cao, C.M. Shakin and Wei-Dong Sun, Phys. Rev. C46, 2535 (1992).
- [12] L.S. Celenza, C.M. Shakin, Wei-Dong Sun, and Xiquan Zhu, Phys. Rev. C48, 159 (1993).

- [13] For a recent review, see V. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).
- [14] L.S. Celenza, A. Rosenthal and C.M. Shakin, Phys. Rev. C31, 212 (1985).
- [15] L.S. Celenza, C.M. Shakin and J. Szweda, Intl. J. Mod. Phys. E2, 437 (1993).
- [16] L.S. Celenza, C.M. Shakin, Wei-Dong Sun, J. Szweda and Xiquan Zhu, Intl. J. Mod. Phys. E2, 603 (1994).
- [17] L.S. Celenza, C.M. Shakin, Wei-Dong Sun, J. Szweda and Xiquan Zhu, Brooklyn College Report: BCCNT 93/071/232R (1993) - submitted to Physical Review D.
- [18] J. Gasser, H. Leutwyler, and M.E. Sainio, Phys. Lett. B253, 252 (1991).
- [19] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).
- [20] E.G. Drukarev and E.M. Levin, Prog. Part. Nucl. Phys. 27, 77 (1991); Nucl. Physics A511, 679 (1990); Nucl. Phys. A516, 715(E) (1990).
- [21] See, for example, D.K. Griegel, Ph.D. Thesis, University of Maryland (1991), Appendix D.
- [22] J. Bijnens, Ch. Bruno and E. deRafael, Nucl. Phys. B390, 501 (1993).

Figure Captions

- Fig. 1 (a) Basic quark loop integral of the NJL model in the scalar $q\bar{q}$ channel. The solid lines are quark propagators for (constituent) quarks of mass m_q [13].
- (b) Basic quark-loop integral for the vector $q\bar{q}$ channel. Here

$$J_V^{\mu\nu}(p) = - (g^{\mu\nu} - p^\mu p^\nu / p^2) J_V(p^2)$$

defines the function $J_V(p^2)$ in terms of the tensor defined in the figure.

- (c) Calculation of a matrix element such as $\langle N | \bar{q}q | N \rangle$, or $\langle N | q^\dagger q | N \rangle$, in terms of $\langle N | \bar{q}q | N \rangle_{val}$ or $\langle N | q^\dagger q | N \rangle_{val}$. The summation of the $q\bar{q}$ "bubbles" introduces factors such as $\kappa_S = [1 - G_S J_S(0)]^{-1}$ or $\kappa_V = [1 - G_V J_V(0)]^{-1}$. (See the text and references [11, 12].)

- Fig. 2 (a) Calculation of the omega or sigma fields with valence (constituent) quarks of a nucleon as a source [14]. The wavy line is the meson propagator and the filled dot is the quark-meson coupling (g_σ or g_ω) as obtained from the bosonization scheme. In this model, with $\rho_S \simeq \rho_B$,

$$\sigma = - \frac{g_\sigma F_S(0)}{m_\sigma^2} \rho_B ,$$

and

$$\omega = \frac{g_\omega F_V(0)}{m_\omega^2} \rho_B \quad ,$$

where F_S and F_V are quark scalar and vector form factors of the nucleon defined in reference [14]. (We also have defined $G_{\sigma NN} = g_\sigma F_S(0)$ and $G_{\omega NN} = g_\omega F_V(0) = 3g_\omega$.)

- (b) Modification of the self-energy of a nucleon of momentum p due to the sigma and omega fields present in matter. Here the heavy, solid line denotes a nucleon in an occupied state, $\bar{k}$. An integral over $\bar{k}$ is implied, where $\bar{k}$ is the momentum of a nucleon in the Fermi sea.

$$-iJ_S(p^2)$$

(a)

$$-iJ_V^{\mu\nu}(p)$$

(b)

(c)