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Monte-Carlo simulations of a flux tube model¹

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Abstract

We study, using Monte-Carlo simulations, a lattice version of the Patel flux tube model with the three-point baryonic vertices. Employing finite size scaling analysis for the heat-capacity we show that for large quark masses the model undergoes the second order phase transition.

The phase transition for the pure gauge $SU(N)$ theory is well understood in terms of spontaneous breaking of global Z_N symmetry. In the high temperature phase, $SU(N)$ theory can be described by an effective, three-dimensional, local, Z_N -symmetric action for Polyakov lines [1]. It is believed that locality and symmetry of the effective action specifies the transition and therefore one can simplify the considerations to three dimensional spin models. For $SU(2)$ theory, which undergoes a second order phase transition, one can make this statement more precise by saying that, on the universality grounds, the whole class of Z_2 symmetric local actions leads to the same critical indices. This was checked by high precision, comparative measurements of $SU(2)$ and 3d Ising model [4]. For $SU(3)$, the transition is first order, and the same is expected for a certain class of three dimensional, Z_3 symmetric, local models. In particular, it was shown by direct simulations that it is the case for the three states 3d Potts model [5].

The effective description of $SU(3)$ in terms of the three dimensional effective, Z_3 -symmetric action holds when the transition point is approached from high temperatures. It breaks down in the low temperature phase which is believed to be dominated by string like objects reflecting linear confinement. In the papers [2],[3] Patel analyzed a simple flux tube model as an effective mechanism driving the transition from the low temperature phase. The model assumes that strings have a constant, temperature independent, energy per unit length, σ . These strings are treated classically. Short distance string fluctuations are controlled by a cutoff a . The strings interact only via the baryonic vertices, which contribute certain energy to the system. Also heavy quarks can be introduced to the model by letting strings terminate. When a string terminates, a pair of quark and antiquark is created at the free ends of the string. Quarks contribute energy (mass) proportional to their number. There is no explicit two-quark interactions. It is a single flavour model and it does not take into account chiral phenomena and therefore only in the large mass quark limit it may be relevant to the deconfining phase transition in QCD.

The aim of studying the Patel's model is to get some understanding of the physical mechanisms responsible for the transition. In particular, it is very interesting if a flux tube model in a simple form of statistical ensemble of classical strings, reproduces correctly known facts about the transition. A positive answer to this question would mean that the transition can be explained in terms of the string entropy, and that the other features of the effective QCD string models, neglected on purpose in the Patel model, are only secondary contributions to the transition picture. Recently the question of a relation between string models and QCD, attracts a new interest [9].

In this paper we reexamine the order of the transition in the flux tube model with the three-point baryonic vertices. Our considerations are based on MC simulations of a lattice representation proposed by Patel. A configuration of flux tubes on the lattice is described by link occupation numbers, $n_{i,\mu}$'s, equal zero if a link emerging from a site i in a direction μ is empty, or ± 1 if it is occupied by a flux tube. The sign depends on the respective orientation of the link and the tube. Quarks/vertices are represented by site occupation numbers p_i 's/ q_i 's equal ± 1 if there is a quark/vertex (or an antiquark/antivertex) at a site i , or 0 otherwise. The energy of the system is :

$$E = \sigma a \sum_{i,\mu} n_{i,\mu}^2 + m \sum_i p_i^2 + v \sum_i q_i^2, \quad (1)$$

where the couplings σ, m, v are a string tension, a quark mass and a vertex energy, respectively. The occupation numbers are not independent variables. At each vertex, a constraint, being a discretized version of the Gauss law for $SU(3)$, is imposed :

$$G_i = \sum_{\mu} n_{i,\mu} - n_{i,-\mu,\mu} - p_i + 3q_i = 0, \quad (2)$$

so that, the partition function of the lattice model reads :

$$Z = \sum_{p_i, q_i, n_{i,\mu}} e^{-E/T} \prod_i \Delta(G_i), \quad (3)$$

where $\Delta()$ is the Kronecker delta. Summation over site variables p_i and q_i leads to a model with link occupation numbers, as only dynamical variables, weighted at each site i by a certain function $F(S_i)$ depending on a sum of links emerging from a site : $S_i = \sum_{\mu} n_{i,\mu} - n_{i,-\mu,\mu}$. The partition function now reads :

$$Z = \sum_{n_{i,\mu}} e^{-\sigma a/T \sum_i n_{i,\mu}^2} \prod_i e^{-F(S_i)/T}. \quad (4)$$

In particular, for the Patel model, at each vertex one can have one quark/antiquark and/or one baryonic vertex/antivertex, the function F takes the values : $F(0) = 0$, $F(\pm 1) = m$, $F(\pm 2) = m + v$, $F(\pm 3) = v$, $F(\pm 4) = m + v$. Sites from which emerge five or six strings are suppressed in the partition function : $e^{-F(\pm 5)} = e^{-F(\pm 6)} = 0$.

The formula 4 is a starting point for MC simulations. The simulations are based on the procedure analogous to the BFACF algorithm used in the simulations of self avoiding random walks [6],[7]. In a single step of the algorithm we update simultaneously four links forming a square on the lattice. New occupation numbers on these four links are chosen in accordance with a heat-bath distribution induced by the partition function (4). Technically we do this in such a way, that we compute in advance the heat-bath weights for each type of neighborhood of a square and store

them in a matrix. On each square there are 3^4 new link configurations. We order them to store a heat-bath cumulative distribution function which we denote by D_j where j runs over these 3^4 configurations. To pick up a new link configuration for each square, we find its neighborhood, i , and read the distribution function, $\{D_j\}_i$, from the matrix. We choose a random number r from the uniform distribution on $[0, 1]$ and find the range $D_{j-1} \leq r < D_j$ which gives us an j -th configuration. We repeat this for all squares on the lattice. In principle, an elementary update of the algorithm can be generalized to objects consisting of more links. For example one could update in one step all edges of a cube, or even larger structures. This would make algorithm more mobile in the space of configurations. The choice of a square as the elementary object in the algorithm is dictated by technical reasons. Namely, a size of the matrix for heat bath weights grows, roughly speaking, exponentially with a number of links in the updated object, and for larger structures exceeds storage capacity. An update of a square takes $30\mu s$ on a HP-720 workstation.

We simulated the model in the range of heavy quarks, for a few sets of parameters $\sigma a, v, m$. One of this parameters, say $\sigma a = 1$ introduces a scale to the model. Here we present results for $v = 1$, and $m = 8$, typical for the case of heavy quarks (m a few times larger than σa). We have accumulated a better statistics for this case. We have performed series of MC simulation on different lattice sizes for different temperatures. A length of a run for a fixed temperature was of order of 10^5 sweeps. The main quantity we looked at was the heat capacity $c_v(T) = T^{-2}(\langle E^2 \rangle - \langle E \rangle^2)/V$, where V is the lattice volume. In the Figure 1 we depicted a c_v vs. T dependence for the lattice 15^3 . This curve has maximum $c_{v,MAX} = 8.93 \pm 0.12$ at $T_{MAX} = 0.657 \pm 0.002$. At T around T_{MAX} we observed also rapid increase of the integrated autocorrelation times (Fig. 2). To check whether this signal is related to a transition, and of which order, we studied a size dependence of $c_{v,MAX}$. To estimate T_{MAX} and $c_{v,MAX}$ we used multihistogram method of reweighting [10] combined with the jackknife procedure. The data presented in the Figure 3 shows $\log c_{v,MAX}(L)$ which fits well a straight line, indicating a second order phase transition. An estimate of its slope, equal to the ratio of Fisher exponents, gives :

$$\alpha/\nu = 0.73 \pm 0.05. \quad (5)$$

In principle, one could obtain ν and α separately by extracting ν from a three parameter fit to the formula $T_{MAX} = T_{cvt} + aL^{-1/\nu}$, but it would be very expensive in computer time. The fact that it is second order means that the model in this form does not describe effectively a mechanism which drives the QCD transition for heavy quarks. This conclusion is different from the one drawn by Patel, who suggested that this transition could be first order. His expectations were supported

by the mean-field consideration of a XY model and resulted from integration of the Gauss constraints in the modified model with occupation numbers running over all integers. Though the mean field theory is not conclusive about an order of a transition, his result prompts us to ask the question whether (and how) the order of the transition in the model of the type (4) depends on the function $F(s_i)$? In fact, the model can exhibit quite different behaviour depending on a choice of the function F . To see this, consider two extreme cases. The first one : $e^{-F(s)} = \text{const} \neq 0$: ie all F 's are the same for each $s = 0, \dots, 6$. In this case the model corresponds to a free bond percolation. The second one : $e^{-F(0)} = \text{const} \neq 0$ and $e^{-F(s)} = 0$ for $s = 1, \dots, 6$ which means that only sites with the same number of incoming and outgoing lines survive in the partition function. In some sense the second case is similar to random walks, and is, of course, quite different from a percolation problem. In our simulations we studied behaviour of the system for a few sets of parameters (yielding different functions F). We observed that, as long as m is a few times larger than σa , the behaviour of the system weakly depends on v , and the transition is second order. It seems that this region of parameters corresponds to another subclass of functions $F(s)$ which generate a different behaviour than in both the extreme cases. On the other hand, for small values of m (we looked at $m = 0.1$) we did not observe any signal in c_v , which means that the system does not undergo a transition, apart from a purely geometric one related to percolation.

At present it is not known how to classify types of critical behaviour of the model and how they depend on the function $F(s)$. To be in a position to address this kind of questions numerically one would need more sophisticated techniques, like the ones used for self avoiding walks, which offer a possibility of nonlocal updates and overcome problems with ergodicity, and which are much better suited to this type of problems [8]. In particular it would be very interesting to study another realization of the Patel model which additionally to the Gauss law (1) imposes another constraint at each site, namely that a baryonic vertex and a quark cannot occupy a site simultaneously. In this case, the only sites which contribute to the partition function are that for which $|s_i| = 0, 1, 3$, and the other ones are suppressed $e^{-F(s_i)} = 0$.

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Figure Captions

- Fig. 1. The heat capacity in the neighborhood of the pseudocritical point for the lattice 15^3 . The data were obtained from 7 runs for temperatures distributed between 0.645 and 0.68.
- Fig. 2. The integrated autocorrelation time τ_{int} (for energy) for the temperatures around the pseudocritical point for the lattice 15^3 .
- Fig. 3. The maximum value of the heat capacity, $c_{t,MAX}$ versus (linear) lattice size.

Fig.1

Fig.2

Fig.3