Vibrations versus collisions and the iterative structure of two-body dynamics

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Abstract

We adopt a truncated version of two-body dynamics by neglecting three-body correlations, as is supported by microscopic numerical calculations. Introducing orthogonal channel correlations for the \(pp\)- and the \(ph\)-channel and integrating the latter in terms of vibrational RPA-states we derive a retarded two-body equation. Its solution is nonperturbative with respect to loops, ladders and mixed contributions. In the stationary limit we obtain an equation for a generalised effective interaction which iterates both the \(G\)-matrix and the polarisation matrix. An in-medium scattering approach transparently demonstrates the collisional damping of the vibrations.

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1 Introduction

A still outstanding problem in two-body dynamics concerns the simultaneous nonperturbative consideration of \( pp \) (\( hh \)) and \( ph \)-interactions, i.e. the consistent integration of both the \( pp \) and the \( ph \)-channel. As numerically tested in [1] within the concept of correlation dynamics ([2]-[6]) the short-range part of the two-body interaction produces correlations mainly in the \( pp \)-channel, while the long-range part favours the \( ph \)-channel. These results are in agreement with earlier discussions by Abrikosov et al. [7] for infinite nuclear matter and by Migdal [8] for finite nuclei which show that the sum of ladder diagrams should be essential at large momentum transfer whereas one expects loop diagrams to be dominant at small momentum transfer. In this sense, the simultaneous consideration of both channels corresponds to a consistent treatment of short- and long-range correlations (including their mixing), and therefore of collisional and collective aspects in nuclear dynamics.

Since a numerical solution of the equation of motion for the two-body correlations is available so far only for light nuclei and in a restricted single-particle basis it is of current interest to make the iterative structure of this equation more transparent, in particular with respect to the mutual influence of short- and long-range correlations which is not included in \( G \)-matrix [9]-[17] or RPA calculations [18]-[22] which specialize on short-range or long-range correlations, respectively. This should help for a better understanding of the interplay between collisions and vibrations during the evolution. Formally, the mutual influence shows up in mixed diagrams combining ladders and loops, a problem that has been also addressed within the framework of Green’s functions [23, 24]. The importance for the evaluation of occupation number distributions and for the damping width of single-particle and collective excitations (especially for finite temperatures as e.g. investigated experimentally in (c.f. [25, 26]) and theoretically in (c.f. [27]-[29])) has been demonstrated in [1] within a model appropriate for light nuclei as well as within the microscopic TDDM approach [30].

The paper is organised as follows: In section 2 we present microscopic studies on the relative importance of 3- and 4-body correlations and on the effect of channel mixing for the total energy in case of a light model system (\( ^{16}O \)). In section 3 coupled equations for channel correlations are derived and their iterative structure is made transparent by introducing vibrational RPA-states. Section 4 is devoted to the stationary limit while
section 5 investigates collisional damping within an in-medium scattering approach. The paper is summarised in section 6.

2 Microscopic numerical studies

The nuclear many-body problem on the two-body level can be formulated in terms of coupled equations of motion for the one-body density $\rho$ and the two-body correlation function $C$ which read in an arbitrary single-particle basis [1, 3]:

$$i \frac{\partial}{\partial t} \rho_{\alpha\alpha'} = \langle \alpha | \hat{h}(1), \rho | \alpha' \rangle + \sum_{\beta} \langle \alpha | \hat{h}(1) \rho | \beta \rangle \langle \beta | \hat{C} | \alpha' \rangle,$$

(2.1)

$$i \frac{\partial}{\partial t} C_{\alpha\beta\alpha'\beta'} = \langle \alpha | \hat{h}(1) + h(2), \hat{C} | \alpha' \beta' \rangle$$

$$+ \langle \alpha | v_\omega^\alpha \rho_\omega, \beta' \rangle \langle \alpha | \alpha' \beta' \rangle$$

$$+ \langle \alpha | v^\alpha \rho_\omega, \beta' \rangle \langle \alpha | \alpha' \beta' \rangle$$

$$+ A_{\alpha\beta} A_{\alpha'\beta'} \sum_{\gamma \gamma'} v_{\alpha\gamma\alpha'\gamma'} \gamma \gamma'$$

$$+ \sum_{\gamma \gamma'} \delta_{\gamma \gamma'} (P_{\alpha\gamma} P_{\alpha'\gamma} + P_{\beta\gamma} P_{\beta'\gamma})$$

$$\times \sum_{\lambda \lambda'} \{ v_{\alpha\beta\lambda\lambda'} C^{\beta_{\alpha\gamma\alpha'\gamma'}} - C^{\beta_{\alpha\gamma\alpha'\gamma'}} v_{\lambda\lambda'\alpha\alpha'} \},$$

(2.2a)

(2.2b)

(2.2c)

(2.2d)

or in short-hand notation (dropping the term (2.2d) in view of section 3)

$$i \dot{\rho} = [h(1), \rho] + tr[v, C]$$

(2.3)

$$i \dot{\hat{C}} = [h(1) + h(2), C] + [v^\omega, \rho_{20} + C] + A(1 + PP) v^\perp C.$$

(2.4)

Here we have used the bare mean field $h = t + tr(v^\omega \rho)$, where $t$ is the kinetic energy operator, while $v^\omega$ is the antisymmetrised two-body interaction. Further, $\rho_{20} = A \rho \rho$ is the uncorrelated two-body density, and $A$ and $P$ denote the antisymmetrisation and permutation operator, respectively (c.f. [1]). The in-medium interactions $v^\omega = Q^\omega v$ and $v^\perp = Q^\perp v^\omega$ are density-dependent via the blocking operators which read in an arbitrary single-particle basis $Q^\omega_{\alpha\beta\alpha'\beta'} = \delta_{\alpha\alpha'} \delta_{\beta\beta'} - \delta_{\alpha\alpha'} \rho_{\beta\beta'} - \delta_{\beta\beta'} \rho_{\alpha\alpha'}$ and $Q^\perp_{\alpha\beta\alpha'\beta'} = \delta_{\alpha\beta} \rho_{\alpha'\beta'} - \delta_{\beta\beta'} \rho_{\alpha'\alpha'}$ (c.f. [32]). In a basis that diagonalises $\rho$ one verifies that $Q^\omega$ projects on $pp$- and $hh$-states in the Hartree-Fock-limit while $Q^\perp$ projects on $ph$- and $hp$-states. This leads to
the definition of two "orthogonal" channels: dropping the term with \( v^+ C \) in (2.2) constitutes the \( pp \) \((hh)\)-channel, while dropping of \( v^+ C \) constitutes the \( ph \)-channel. Hence an inclusion of (2.2a) and (2.2b) corresponds to a nonperturbative resummation of ladder diagrams (TDGMT\(^1\)) while (2.2a) and (2.2c) leads to a resummation of loop diagrams (RPA\(^2\)). The addressed channel mixing is then obtained by taking into account (2.2a-2.2c) (NQCD\(^3\)). The terms (2.2d) include the coupling to the three-body correlations. An explicit expression for \( C_3 \) as a function of \( \rho_2 \) and \( C_2 \) is given in \([1, 31]\) and corresponds to the lowest order three-body correlations which are needed to restore the fundamental trace relations dynamically for spin symmetric systems (c.f. \([1]\)). The respective limit (including (2.2a)-(2.2d) is denoted by SCD\(^4\).

The calculations in the following subsections are carried out to support the analytical considerations of this paper and are performed within a finite oscillator basis as explained in detail in \([1]\).

### 2.1 Importance of higher order correlations

According to the hierarchy of density matrices \([2]\) the two-body correlations \( C = C_2 \) are coupled dynamically with three-body correlations \( C_3 \), and so forth. Any truncation thus becomes only meaningful if the effect of higher order correlations decreases with increasing rank \( n \). In \([1]\) it has been shown in the framework of model calculations for light nuclei that the dynamical influence of the trace conserving terms (2.2d) in fact is quite small. To confirm this result and to give additional support for a neglect of \( C_3 \) for nuclear configurations close to the ground state in the following sections we compare the trace-relations between different levels of the hierarchy with each other. For this purpose we calculate the total traces of \( C = C_2, C_3 \) and \( C_4 \), respectively, starting from the trace relations for the density matrices \( \rho_n \) \([2]\)

\[
\rho_n = \frac{1}{(A-n)!} \mathrm{tr}_{(n+1),...,A} \rho A = \frac{1}{(A-n)} \mathrm{tr}_{(n+1)} \rho_{n+1}.
\]  

\(^1\)Time-Dependent-G-Matrix-Theory  
\(^2\)Random-Phase-Approximation  
\(^3\)Nuclear-Quantum-Correlation-Dynamics  
\(^4\)Selfconsistent-Correlation-Dynamics
Within the cluster expansion for $\rho_n$ [2] (c.f. Appendix A) this leads to the equivalent trace relations for the correlation functions

$$tr_{(2=2')} C_2(12, 1'2') = -tr_{(2=2)} (1/A - P_{12}) \rho(22') \rho(11'), \quad (2.6)$$

$$tr_{(3=3')} C_3(123, 1'2'3') = -tr_{(3=3)} (2/A - P_{13} - P_{23} - P_{13'}) \rho(33') C_2(12, 1'2'), \quad (2.7)$$

$$tr_{(4=4')} C_4(1234, 1'2'3'4') =$$

$$-tr_{(4=4)} (3/A - P_{14} - P_{24} - P_{34} - P_{14'} - P_{24'} - P_{34'}) \rho(44') C_3(123, 1'2'3')$$

$$+ tr_{(4=4)} (P_{14} + P_{24} + P_{14'} + P_{24'}) C_2(12, 1'2') \rho(22') C_2(12, 1'2')$$

$$+ tr_{(4=4)} (P_{34} + P_{34'}) C_2(24, 2'4') C_2(12, 1'2')$$

$$- tr_{(4=4)} (C_2(12, 1'2') C_2(34, 3'4') + C_2(23, 1'2') C_2(14, 2'3')$$

$$+ C_2(24, 1'3') C_2(13, 2'4')). \quad (2.8)$$

Using (2.6)-(2.8) the total traces over all particle coordinates are evaluated within a single-particle basis that diagonalises the one-body density, i.e.

$$tr \ C_2 = -\sum \alpha n_\alpha (1 - n_\alpha), \quad (2.9)$$

$$tr \ C_3 = 2 \sum \alpha n_\alpha (1 - n_\alpha)(1 - 2n_\alpha), \quad (2.10)$$

$$tr \ C_4 = - 6 \sum \alpha n_\alpha (1 - n_\alpha)(1 - 2n_\alpha) + 12 \sum \alpha n_\alpha^2 (1 - n_\alpha)^2 + 12 \sum \alpha n_\alpha n_\beta C_{\alpha \beta \alpha \beta}$$

$$+ \sum_{\alpha \beta \gamma \lambda} \{ C_{\alpha \beta \lambda \gamma} C_{\alpha \gamma \lambda \beta} + C_{\alpha \beta \lambda \gamma} C_{\beta \gamma \lambda \alpha} + C_{\lambda \beta \alpha \gamma} C_{\alpha \lambda \beta \gamma} + C_{\lambda \beta \alpha \gamma} C_{\gamma \lambda \alpha \beta} + C_{\gamma \beta \alpha \lambda} C_{\alpha \gamma \lambda \beta}$$

$$+ C_{\gamma \beta \lambda \alpha} C_{\alpha \gamma \lambda \beta} + C_{\lambda \beta \gamma \alpha} C_{\gamma \lambda \alpha \beta} - C_{\alpha \beta \lambda \gamma} C_{\gamma \lambda \alpha \beta} - C_{\alpha \beta \lambda \gamma} C_{\lambda \beta \alpha \gamma} C_{\alpha \gamma \lambda \beta} \}. \quad (2.11)$$

The time-averaged numerical results for $tr \ C_2$, $tr \ C_3$ and $tr \ C_4$ are displayed in Fig. 1 for the $^{16}O$ model system [1] as a function of the initialization temperature $T$ which via the Fermi distribution $\rho_{aa}(t = 0) = [1 + e^{x_p((\epsilon_\alpha - \epsilon_F)/T)]^{-1}$ is directly related to the initial thermal excitation energy. We observe that the effect of higher-order correlations decreases by about one order of magnitude in each case. This can be viewed as a further indication for the convergence of the cluster expansion within our model and supports the neglect of $C_3$ in the general considerations of the following sections. We note, however, that although $tr C_3$ and $tr C_4$ might remain small, individual matrix elements may become quite large and be responsible for new physical phenomena.
2.2 Effect of channel mixing on the correlated ground state

A central problem within correlation dynamics is the precise determination of the correlated ground state \cite{6}. We address this problem numerically and approach the correlated ground state energy by switching on the two-body interactions adiabatically in time. In order to define a proper adiabatic regime the following calculations are performed: we initialize a system of 16 nucleons in its Hartree-Fock-configuration at $T = 0$ MeV temperature. Then we switch on the two-body interaction in the term $vC$ in (2.1) and the terms $v^{=}C$ and $v^{=}C$ in (2.2) by multiplying the interaction with a common function $f(t) = t/t_s$ for $0 < t < t_s$ or 1 for $t > t_s$, respectively. Since for a fixed (time independent) two-body interaction the equations of motion (2.1) and (2.2) conserve the total energy for $t \geq t_s$ the ground state energy is extracted by evaluating $\langle E \rangle = tr\{t \rho + 1/2 tr\{t \rho \} \rho_2 \}$. This adiabatic approach is expected to work well for approximately spherical configurations as in case of $^{16}O$ but might become questionable for nuclei with large density fluctuations (shape coexistence of oblate and prolate configurations). Furthermore, due to the neglect of four-body correlations a mixing with alpha-cluster configurations is not expected to occur, too.

In Fig. 2 we show the resulting total asymptotic energy for $t > t_s$ as a function of the parameter $t_s$ for the limits BORN, TDGMT, RPA, NQCD and SCD which specifies the adiabatic regime by the condition $t_s \geq 10^{-21}s$ and thus guaranties the closest approach to the ground state energy of the correlated system in the respective limit. It becomes obvious that neither the $pp$-channel alone (TDGMT) nor the $ph$-channel alone (RPA) is sufficient to come close to the lowest possible energy which corresponds to the full theory (SCD). Thus channel-mixing is an important aspect of nuclear dynamics and demands for a deeper understanding. On the other hand the limits NQCD and SCD practically give the same result for the ground state energy such that the three-body terms in (2.2d) - which guarantee the dynamical conservation of the trace relations - can be neglected for the following investigations.
3 Vibrational retardation and the iterative structure of channel mixing

In order to associate with each channel its own correlation function we subdivide the correlation function $C$ into channel correlations according to

$$ C = C^= + C^\perp. \quad (3.1) $$

The channel correlations then follow the coupled set of equations

$$ i\dot{C}^= = [H_0^=, C^=] + [v^=, C^= + C^\perp + \rho_{20}] \quad (3.2) $$

$$ i\dot{C}^\perp = (1 + \mathcal{P}\mathcal{P})H_0^\perp C^\perp + \mathcal{A}(1 + \mathcal{P}\mathcal{P})v^\perp(C^\perp + C^=), \quad (3.3) $$

or, in an arbitrary single-particle basis,

$$ i\dot{C}_{\alpha\beta\alpha'\beta'}^\perp = \langle \alpha\beta | [H_0^\perp, C^=]|\alpha'\beta'\rangle + \langle \alpha\beta | [v^=, C^= + C^\perp + \rho_{20}]|\alpha'\beta'\rangle \quad (3.4) $$

$$ i\dot{C}_{\alpha\beta\alpha'\beta'}^\perp = (1 + \mathcal{P}_{\alpha\beta}\mathcal{P}_{\alpha'\beta'})\sum_{\gamma\gamma'} C_{\gamma\beta\gamma'\beta'}^\perp + \mathcal{A}_{\alpha\beta}(1 + \mathcal{P}_{\alpha\beta}\mathcal{P}_{\alpha'\beta'}) \sum_{\gamma\gamma'} v_{\alpha'\gamma'\alpha'\gamma}(C_{\gamma\beta\gamma'\beta'}^\perp + C_{\gamma\beta\gamma'\beta'}^=). \quad (3.5) $$

Observing that the mean-field contributions in (3.4) and (3.5) with

$$ \langle \alpha\beta | H_0^= |\gamma\gamma'\rangle = h_{\alpha\gamma}\delta_{\beta\gamma'} + \delta_{\alpha\gamma}h_{\beta\gamma'} \quad (3.6) $$

$$ H_{\alpha'\gamma'\alpha'\gamma}^0 = h_{\alpha\gamma}\delta_{\beta'\gamma'} - \delta_{\alpha\gamma}h_{\beta'\gamma'} \quad (3.7) $$

may be written either in “horizontal” or “vertical” form one easily proves that the sum of (3.4) and (3.5) reproduces the original equation (2.2) for $C_{\alpha\beta\alpha'\beta'}$ . Eqs. (3.4) and (3.5) describe the evolution of correlations in the pp- and ph-channel, respectively. Their mutual coupling is accounted for by the inhomogeneous terms which comprise $C^\perp$ in the pp- and $C^=$ in the ph-channel.

The explicit time-dependence of the channel correlations is controlled by the structure of the two-body equations (3.4) and (3.5). In addition we observe an implicit time-dependence due to the one-body density contained in the mean-field potentials and the in-medium interactions. To obtain a closer insight into the different time-scales we investigate
numerically, within the same model adopted in section 2, the monopole response in the occupation numbers and characteristic correlation matrix elements. From the comparison of Fig. 3 and Fig. 4 it is obvious that the relative change in time of the occupation numbers is small compared to that of the two-body matrix elements. To facilitate the discussion of the iterative structure of the correlations we thus can disregard the time-dependence of $\rho_{\alpha\alpha'}(t)$ by assuming a stationary $\rho$ in (3.4) and (3.5) which means a decoupling of $C$ from the one-body evolution given by (2.1). This allows to adopt a single-particle basis with $h_{a\gamma} \approx \delta_{a\gamma} c_a$ and $\rho_{a\gamma} = \delta_{a\gamma} n_a$. Neglecting the exchange term in the $ph$-channel by dropping $A_{a\beta}$, eq. (3.5) reads

$$i\dot{C}^\perp = (1 + \mathcal{P}\mathcal{P})H^\perp C^\perp + (1 + \mathcal{P}\mathcal{P})\nu^\perp C^\perp,$$  

(3.8)  

with the RPA-hamiltonian $H^\perp = H_0^\perp + \nu^\perp$ and its matrix elements (in the adopted single-particle basis)

$$H^\perp_{a\gamma' a'\gamma} = (\epsilon_a - \epsilon_{a'})\delta_{a\gamma\gamma'} + (n_{a'} - n_a) \nu_{a\gamma' a'\gamma}.$$  

(3.9)  

For an integration of (3.8) it is suggestive to transform this equation into the RPA-basis. The RPA-eigenstates $\chi^\mu$ and $\chi^{\mu*}$ (collective phonons) are given by the secular equations

$$\sum_{\gamma\gamma'} H^\perp_{a\gamma' a'\gamma} \chi^\mu_{\gamma\gamma'} = \Omega^\mu_{\chi^\mu_{aa'}},$$  

(3.10)  

$$\sum_{\gamma\gamma'} \chi^{\mu*}_{\gamma\gamma'} H^\perp_{a\gamma' a'\gamma} = \Omega^\mu_{\chi^{\mu*}_{\beta\beta'}},$$  

(3.11)  

where the matrix elements of the hermitean conjugate operator $H^\dagger_{\gamma'\beta'\gamma}$ are related to those of $H^\perp$ by

$$H^\dagger_{\gamma'\beta'\gamma} = -H^\perp_{\beta'\gamma'\gamma}.$$  

(3.12)  

Making use of the orthogonality and completeness relation (c.f. [33])

$$\sum_{a a'} \tilde{\chi}_a^{\mu*} \chi_{a a'}^{\mu} = \delta_{\mu\nu} N_{\mu},$$  

(3.13)  

$$\bar{N}_{\mu} \chi_\mu^{\mu*} \chi_{\beta\beta'}^{\mu*} = \delta_{\alpha\beta} \delta_{\alpha'\beta'} (n_{a'} - n_a),$$  

(3.14)  

the relations between matrix elements in the single-particle- and the RPA-basis read

$$C^\perp_{a\beta a'\beta'} = \sum_{\mu\nu} N_{\mu} N_{\nu} \chi_{\mu a a'} \chi_{\beta\beta'}^{\nu*} C_{\mu\nu},$$  

(3.15)  

$$C_{\mu\nu} = \sum_{a a' \beta \beta'} \tilde{\chi}_a^{\mu*} \tilde{\chi}_{a a'}^{\nu} \tilde{C}_{a a' \beta \beta'}^{\perp},$$  

(3.16)
with \( N_\mu = 1 \ (-1) \) for \( \mu > 0 \ (< 0) \) and \( \chi_{a\alpha'}^\mu = (n_{a'} - n_a)^{-1} \chi_{a\alpha'}^\mu \). The equation for \( C_{\mu\nu} \)
follows from (3.16) using (3.8)
\[
\begin{align*}
 i \dot{C}_{\mu\nu} &= (\Omega_\mu - \Omega_\nu) C_{\mu\nu} + \sum_{\gamma'\lambda'} K_{\gamma'\lambda'}^{\mu\nu} C_{\gamma'\lambda'}^{\nu}, \tag{3.17}
\end{align*}
\]

where we have introduced the phonon-particle vertices
\[
\begin{align*}
 \theta^\mu_{\gamma'\lambda'} &:= \sum_{a\alpha'} v_{\alpha'\gamma'}^a \chi_{a\alpha'}^\mu = (\Omega_\mu - \epsilon_\gamma + \epsilon_{\gamma'}) \chi_{\gamma'\lambda'}, \tag{3.19}
\end{align*}
\]

\[
\begin{align*}
 \theta_{\lambda'\gamma'}^{\mu} &:= \sum_{\beta\beta'} v_{\beta'\beta}^\nu \chi_{\beta'\beta}^{\nu} = (\Omega_{-\nu} - \epsilon_\lambda + \epsilon_{\lambda'}) \chi_{\lambda'\gamma'}, \tag{3.20}
\end{align*}
\]

with \( \Omega_{-\nu} = -\Omega_\nu \). These relations may be proved using the RPA-equations (3.10) and (3.11). We note that a conceptionally similar approach has been given by Belyaev almost three decades ago [35, 36] starting from an expansion of the two-body interaction in terms of spherical tensor operators. This leads directly to phonon-particle vertices which induce a nonlocality in time for the particle propagation due to the phonon exchange diagrams. Especially the quadrupole-quadrupole term in the two-body interaction is found to be important for the nuclear structure at low excitation energy thus also pointing out the importance of mixed diagrams for nuclear configurations close to the ground state. In this respect our present analysis can be regarded as an alternative formulation of the influence of channel mixing, however, emerges quite naturally within the NQCD equations without any severe restrictions.

After integration of (3.17) the resulting solution \( C_{\mu\nu}(t) \) is transformed back into the single-particle basis yielding (\( \Omega_{\mu\nu} = \Omega_\mu - \Omega_\nu \))
\[
\begin{align*}
 C_{\alpha\beta\alpha'\beta'}^{\pm}(t) &= \sum_{\mu\nu} N_\mu N_\nu \chi_{a\alpha}^{\mu} \chi_{\beta\beta'}^{\nu} e^{-i\Omega_{\mu\nu}t} C_{\mu\nu}(0) +
 +\frac{1}{i} \int_0^t dt' \sum_{\mu\nu} N_\mu N_\nu \chi_{a\alpha}^{\mu} \chi_{\beta\beta'}^{\nu} e^{-i\Omega_{\mu\nu}(t-t')} \sum_{\gamma'\lambda'} K_{\gamma'\lambda'}^{\mu\nu} C_{\gamma'\lambda'}^{\nu}(t'). \tag{3.21}
\end{align*}
\]

The first term is a superposition of oscillations around the initial values \( C_{\mu\nu}(0) \) and represents a solution of the homogeneous equation that follows when dropping the term with \( C^{\pm} \) in (3.8). The second term comprises the coupling with the \( pp \)-channel.

Insertion of (3.21) into (3.4) leads to a retarded two-body equation in the \( pp \)-channel,
\[
\begin{align*}
 i \dot{C}_{a\beta a'\beta'}^{=} &= <\alpha|H^=|\alpha'\beta'> + <\alpha\beta|\{v^=, \rho_{20} + C_{10}\}|\alpha'\beta'> +
\end{align*}
\]
\[ + \int_0^t dt' \sum_{\gamma \lambda ' \gamma ' \lambda} \alpha \beta |M^{\gamma \lambda' \gamma'}(t - t')| \alpha' \beta' > C^=_{\gamma \lambda' \gamma'}(t'), \tag{3.22} \]

with \( H^= = H^0_0 + v^= \), the memory-kernel

\[ M^{\gamma \lambda' \gamma'}(t - t') = 1/i \sum_{\mu \nu} N_\mu N_\nu K^{\mu \nu}_{\gamma \lambda' \gamma'}[v^=, \chi^\mu \circ \chi^\nu * e^{-i\Omega_{\mu \nu}(t - t')}], \tag{3.23} \]

and the initial correlations

\[ C^\perp 0 = \sum_{\mu \nu} N_\mu N_\nu C_{\mu \nu}(0) \chi^\mu \circ \chi^\nu * e^{-i\Omega_{\mu \nu}t}. \tag{3.24} \]

Here, we have used the notations \([A, B] = AB - BA\) and \((A \circ B)_{1234} = A_{13}B_{24}\). The coupling with the \(ph\)-channel produces two contributions in the \(pp\)-channel: (1) A source term originating from the initial correlations \(C_{\mu \nu}(0)\) in the \(pp\)-channel, and (2) a retardation term comprising memory effects due to phonon-particle couplings. The non-locality in time of the memory-kernel – characterised by a memory-time \(\tau^*\) – is controlled by the number of collective states contributing to the sum \(\sum_{\mu \nu}\) in (3.23). On the other hand, the change in time of \(C^= (t')\) – characterised by a relaxation time \(\tau_{rd}\) – is determined by the strength of the interaction. Only for \(\tau^* \ll \tau_{rd}\) one can replace \(t'\) by \(t\) in \(C^= (t')\) and extend the integration to infinity (Markov-limit).

The channel-mixing is described by the retardation term in (3.22). In order to allow for a separate treatment of this mixing we introduce a corresponding mixing correlator \(\Delta C\) by \(C^= = c^= + \Delta C\), with

\[ ic^= = [H^=, c^=] + [v^=, \rho_{20} + C^\perp 0]. \tag{3.25} \]

Without \(C^\perp 0(t)\) eq. (3.25) accounts for a resummation of ladders as known from time-dependent G-matrix theory [34]. The additional driving term \([v^=, C^\perp 0]\) modifies this by coupling with oscillations due to the initial correlations in the \(ph\)-channel. Now, from (3.22) we obtain for the correlations due to a mixing of ladders and loops

\[ i\Delta C = [H^=, \Delta C] + \int_0^t dt' \sum_{\gamma \lambda' \gamma'} M^{\gamma \lambda' \gamma'}(t - t')(\Delta C(t') + c^= (t'))_{\gamma \lambda' \gamma'}. \tag{3.26} \]

In order to study the iterative structure it is more convenient to use the equivalent integral equation

\[ \Delta C(t) = \Delta c^= (t) + \int_0^t dt' \sum_{121'2'} W^{121'2'}(t - t')\Delta C_{121'2'}(t'), \tag{3.27} \]
with

$$\Delta c^\sigma(t) = \int_0^t dt' \sum_{121'2'} W^{121'2'}(t-t') c_{121'2'}^\sigma(t').$$  \hspace{1cm} (3.28)$$

The integral kernel $W(t-t')$ may be cast into the form

$$W^{121'2'}(t-t') = 1/i \int_0^{t-t'} d\tau e^{-iH\tau} M^{121'2'}(t-t' - \tau) e^{iH\tau},$$

which is of first order in the phonon-particle vertices (c.f. (3.23)), but of infinite order in the blocked interaction $v^\sigma$. In deriving (3.28) and (3.29) we have used the identity

$$\int_0^t dt' \int_0^{t-t'} dt'' = \int_0^t dt' \int_0^{t-t''} dt''.$$

Applying this relation repeatedly we may arrange the solution of (3.27) in powers of $W$,

$$\Delta C_{\alpha\beta\alpha'\beta'}(t) = \Delta c_{\alpha\beta\alpha'\beta'}(t) + \int_0^t dt' \sum_{121'2'} [W(t-t')]$$

$$+ \int_0^t dt_1 W(t-t_1) W(t_1-t_2) + \int_0^t dt_1 \int_0^{t_1} d\tau_1 \int_0^{t_1} d\tau_2 \Delta c_{121'2'}(t')$$

$$W(t-t_1) W(t_1-t_2) W(t_2-t') \cdots c_{121'2'}(t')$$

(3.30)

where products of $W$ are formed according to

$$(WW)_{\alpha\beta\alpha'\beta'}^{121'2'} = \sum_{343'4'} W_{\alpha\beta\alpha'\beta'}^{343'4'} W_{343'4'}^{121'2'}. \hspace{1cm} (3.31)$$

The solution (3.30) makes the iterative structure transparent: the resummation of ladders together with a coupling to vibrations contained in each $W$ is followed by a resummation of $W$ and thus of phonon-particle vertices.

Combining all relations with the solution of (3.25) we are able within our approximations to trace back the total two-body correlations $C$ to the uncorrelated two-body density $\rho_{20}$ and the initial (vibrational) correlations $C^{i0}$ in the $ph$-channel.

4 Stationary limit and effective interactions

In the stationary limit $\dot{C} = 0$ the two-body dynamics can be traced back to integral equations for frequency-dependent effective interactions. We meet the stationary limit imposing $\dot{C} = 0$ and $\dot{C} = 0$ in (3.4) and (3.5), respectively. Strictly speaking, due to the exchange term equation (3.5) splits up into two equations for the $ph$-channels ($\alpha, \alpha'$) and ($\alpha, \beta'$). Since, however, the equation for channel ($\alpha, \beta'$) does not provide new aspects with respect to channel mixing it is neglected in the following by dropping $A_{\alpha\beta}$ in (3.5).
The price for this simplification is the loss of antisymmetry with respect to the labels \((\alpha, \beta)\) and \((\alpha', \beta')\) in the ph-channel. Considerations with three coupled equations are made e.g. in [5] using Green’s-function techniques.

Equations (3.4) and (3.5) are solved in the stationary limit by the correlations \(C^{=0}\) and \(C^{\perp 0}\) which can be determined from the two coupled equations

\[
(\omega - \epsilon_\alpha - \epsilon_\beta)C^{=0}_{\alpha \beta \alpha' \beta'} = \langle \alpha \beta | v (C^{=0} + C^{\perp 0} + \rho_{20}^0) | \alpha' \beta' \rangle
\]
\[
(\omega - \epsilon_\alpha + \epsilon_\alpha')C^{\perp 0}_{\alpha \beta \alpha' \beta'} = \sum_{\gamma \gamma'} v^\perp_{\alpha \gamma' \alpha' \gamma} (C^{\perp 0} + C^{=0})_{\gamma \beta \gamma' \beta'}
\]

(4.1)

(4.2)

where \(\rho_{20}^0\) is the stationary uncorrelated two-body density. This can be seen by subtracting from (4.1) and (4.2) the respective hermitean conjugate equation. By introducing effective interactions \(G(\omega)\) and \(\Pi(\omega)\) via the definitions

\[
\langle \alpha \beta | v (C^{=0} + C^{\perp 0} + \rho_{20}^0) | \alpha' \beta' \rangle = \langle \alpha \beta | G (C^{=0} + \rho_{20}^0) | \alpha' \beta' \rangle
\]
\[
\sum_{\gamma \gamma'} v^\alpha_{\alpha \gamma' \alpha' \gamma} (C^{\perp 0} + C^{=0})_{\gamma \beta \gamma' \beta'} = \sum_{\gamma \gamma'} \Pi_{\alpha \gamma' \alpha' \gamma} C^{=0}_{\gamma \beta \gamma' \beta'}
\]

(4.3)

(4.4)

and insertion of (4.3) and (4.4) into (4.1) and (4.2), respectively, we obtain integral equations for the \(G\)-matrix and the polarisation matrix \(\Pi\) (c.f. also [1])

\[
G_{\alpha \beta \alpha' \beta'} = v_{\alpha \beta \alpha' \beta'} + \langle \alpha \beta | v (\omega^{(+)} - H_0^\perp)^{-1} Q^= G | \alpha' \beta' \rangle
\]
\[
\Pi_{\alpha \beta \alpha' \beta'} = v^\perp_{\alpha \beta \alpha' \beta'} + \sum_{\gamma \gamma'} [v^\alpha_{\alpha \gamma' \alpha' \gamma} (\omega^{(+)} - H_0^\perp)^{-1} Q^\perp]_{\alpha \gamma' \alpha' \gamma} \Pi_{\gamma \beta \gamma' \beta'}
\]

(4.5)

(4.6)

with \(\omega^{(+)} = \omega + i \eta, \eta \to 0^{(+)}\). Assuming again \(\rho^0\) and \(h(\rho^0)\) to be diagonal, the matrix elements of the blocking operators simplify to \(Q^=_{\lambda \lambda' \lambda' \gamma} = \delta_{\lambda \lambda'} \delta_{\gamma' \gamma} (1 - n_\lambda - n_{\gamma'}\) and \(Q^\perp_{\lambda \lambda' \lambda' \gamma} = \delta_{\lambda \lambda'} \delta_{\gamma' \gamma} (n_{\lambda'} - n_\lambda)\) and the mean-field two-body propagators \(g^{pp}(\omega) = [\omega^{(+)} - H_0^\perp]^{-1}\) and \(g^{ph}(\omega) = [\omega^{(+)} - H_0^\perp]^{-1}\) reduce to

\[
gen^{pp}_{\alpha \beta \alpha' \beta'} = \delta_{\alpha \alpha'} \delta_{\beta \beta'} [\omega^{(+)} - \epsilon_\alpha - \epsilon_\beta]^{-1}
\]
\[
gen^{ph}_{\alpha \beta \alpha' \beta'} = \delta_{\alpha \beta} \delta_{\alpha' \beta'} [\omega^{(+)} - \epsilon_\alpha + \epsilon_\alpha]^{-1}
\]

(4.7)

(4.8)

A representation of (4.5) and (4.6) in terms of diagrams is given in Fig. 5 and Fig. 6, respectively. The iterative structure makes explicit that \(G\) accounts for a resummation of ladders while \(\Pi\) accounts for a resummation of loops.

Equations (4.5) and (4.6) describe the stationary limit in the absence of channel mixing, i.e. when dropping \(C^{\perp 0}\) in (4.1) and \(C^{=0}\) in (4.2). In order to account for the mutual
influence of both channels it is tempting to construct a total $G$-matrix $G^{\text{tot}}(\omega)$ in terms of $G(\omega)$ and $\Pi(\omega)$. This can be achieved by the definition

$$G^{\text{tot}}_{\rho_{20}} = G(C^{\perp} + \rho_{20}), \quad (4.9)$$

which yields, together with (4.3) and (4.1), a relation between $C^{=0}$ and $G^{\text{tot}}$. Furthermore, insertion of (4.4) into (4.2) allows to express $C^{\perp}$ in terms of $\Pi$ and $C^{=0}$. Finally, combining all relations with (4.9) one arrives at an equation for $G^{\text{tot}}$ which may be cast into the form

$$G^{\text{tot}}_{a\beta\alpha'\beta'} = G_{a\beta\alpha'\beta'} + \sum_{\lambda\kappa\kappa'} \Gamma(\omega)_{a\beta\kappa'\alpha'\kappa} \frac{1 - n_{\kappa} - n_{\lambda}}{\omega + i} G^{\text{tot}}_{\kappa\lambda\kappa'\beta'}. \quad (4.10)$$

The frequency-dependent 6-label interaction $\Gamma(\omega)$ is a combination of $G$, $\Pi$ and a $ph$-propagator,

$$\Gamma(\omega)_{a\beta\kappa'\alpha'\kappa} = \sum_{\gamma} G_{a\beta\gamma} \frac{n_{a'} - n_{\gamma}}{\omega + i} \Pi_{\gamma\kappa'\alpha'\kappa}. \quad (4.11)$$

Eq. (4.10) is not an usual Bethe-Goldstone equation since not only two but three labels of $G^{\text{tot}}$ are involved in the summation on the r.h.s.. Its iterative structure is displayed in Fig. 7. The successive iteration proceeds in powers of $\Gamma$ connected by a respective $pp$-propagator. A diagram of second order in $\Gamma$ is shown in Fig. 8.

The effective interaction (4.11) emerging from the limit NQCD in the stationary limit accounts for the lowest order parquet diagrams as pointed out in [5]. In general, the parquet formalism [38, 39, 40] addresses the problem of summing up irreducible interaction diagrams (including the mixed channels) with proper symmetry (for bosons) or antisymmetry (for fermions) without double counting of interactions. It thus includes potentially much higher classes of diagrams than those considered in (4.11) due to the neglect of $C_3$ and higher order correlations. Within the notation of [40] our effective interaction $\Gamma(\omega)$ corresponds to the parquet diagrams including the $s, t$ and $u$ channels of the bare interaction, however, without "left-" and "right-hand" vertex corrections. It is presently still a matter of debate if the higher-order diagrams – that potentially are included in the parquet formalism – are actually needed for a "proper" description of two-body dynamics in the nuclear physics context. We note in passing that the parquet theory also has been formulated on the three-body level in [40] while a corresponding correlation dynamics of Green’s functions up to the three-body level has been presented in [41].
Equation (4.10) describes nonperturbatively the channel mixing in the stationary limit of two-body dynamics. For transparency ρ₀ was assumed to be diagonal. This assumption can easily be dropped replacing 1 − n_α − n_β in (4.10) and n_α' − n_β' in (4.11) by the non-diagonal expressions for Q^= and Q^⊥, respectively. For a complete description of a stationary system the equations for the effective interactions must be completed by the stationary one-body equation

$$0 = [\hat{t}, \rho_0] + tr_2[v, \rho_{20}^0 + C^0].$$

(4.12)

Here, $\hat{t}$ is the kinetic energy operator, and $tr_2$ means the trace over the second particle.

With $C^0 = C^{0=0} + C^{0=}$ and the definitions (4.3) and (4.9) we obtain

$$0 = [\hat{t}, \rho_0] + tr_2[G^{tot}, \rho_{20}^0].$$

(4.13)

This equation for $\rho^0$ together with the equations for $G^{tot}$, $G$ and II represent a closed set of equations for the stationary limit of the system which includes channel mixing and therefore accounts nonperturbatively for the interplay of long- and short-range correlations.

5 In-medium scattering approach

Short-range correlations are associated with multiple $pp$- (or $hh$-) collisions mediated by the in-medium interaction $v^=$. The in-medium scattering approach is based on the assumption that these collisions happen on a time-scale which is short as compared with the time in between the collisions. Neglecting, for a moment, long-range correlations and approximating the “collision-free” evolution on the long time-scale by the uncorrelated two-body density,

$$i\dot{\rho}_{20} = [H_0^=, \rho_{20}],$$

(5.1)

we obtain for the short-range correlations [32, 34]

$$C^= = -\rho_{20} + \Omega(E)\rho_{20}\Omega(E)^\dagger,$$

(5.2)

where the two-body Moeller operator $\Omega(E)$ depends on the total energy of the “scattering-system” and follows the equation

$$\Omega(E) = 1 + [E^+ - H_0^=]^{-1}v^=\Omega(E).$$

(5.3)
Equation (5.3) is equivalent to (4.5) when defining the $G$-matrix by $G(E) = v \Omega(E)$.

Within this concept of two different time-scales long-range correlations are included assuming that they are operative essentially on the long time-scale, i.e. in between the collisions. This allows to apply the same integration procedure from [3, 32, 34] to eq. (3.2) which yields

$$C^\pm \approx -\rho_{20} + C^\pm + \Omega(\rho_{20} + C^\pm)\Omega^\dagger.$$  \hfill (5.4)

The Moeller-operators may be expressed in terms of $G$ as

$$\Omega(E) = 1 + g(E)Q^\mp G(E)$$ \hfill (5.5)

with $g(E) = [E^+ - H_0^-]^{-1}$. Insertion of (5.4) into (3.8) provides an equation for the long-range correlations

$$i\dot{C}^\pm = (1 + \mathcal{P}\mathcal{P})H^\pm C^\pm + (1 + \mathcal{P}\mathcal{P})v^\pm([gQ^\mp G, C^\pm]) + gQ^\mp GC^\mp G^\dagger Q^\mp g^\dagger + D(\rho_{20}).$$ \hfill (5.6)

We observe that channel mixing produces a driving term

$$D(\rho_{20}) = (1 + \mathcal{P}\mathcal{P})v^\pm(\Omega\rho_{20}\Omega^\dagger - \rho_{20})$$ \hfill (5.7)

as well as an additional interaction term of mixed structure (second term on the r.h.s.): $C^\pm$ is horizontally connected with $G$ but the resulting expression is vertically connected with $v^\pm$. Both terms are non-hermitean due to the propagator $g(E)$.

Equation (5.6) holds for an arbitrary single-particle basis. Together with (2.3) and (5.3) it represents a closed set of equations for the long-range correlations $C^\pm$, the $G$-matrix $G$ and the one-body density $\rho$. The one-body equation reads, with (3.1) and (5.4),

$$i\dot{\rho} - [\hat{h}, \rho] = I_{coll}(\rho) + tr_2[G, C^\pm] + tr_2[GC^\pm G^\dagger, Q^\mp g^\dagger],$$ \hfill (5.8)

with the on-shell collision term from time-dependent $G$-matrix theory

$$I_{coll} = i tr_2([ImG, \rho_{20}] + [G\rho_{20}G^\dagger, Q^\mp Img])$$ \hfill (5.9)

and the renormalised mean field $\hat{h} = \hat{t} + tr(ReG^*\rho)$. Equation (5.8) was used in [37] to study nuclear damping without channel mixing, i.e. with approximate long-range correlations which follow from (3.8) when dropping the term with $C^\mp$.
Due to the non-hermitean character of (5.6) we expect that the mixing with the \( pp \)-channel leads to damped vibrations in the \( ph \)-channel. To make this more transparent we assume - as in section 3 - that the one-body density appearing in the interaction terms of (5.6) is diagonal and time-independent. This allows for a transformation into the RPA-basis defined in (3.10, 3.11). The result is

\[
i \dot{C}_{\mu\nu} = \Omega_{\mu\nu} C_{\mu\nu} + \sum_{\mu'\nu'} K_{\mu'\nu'\mu\nu}^{\mu\nu} C_{\mu'\nu'} + D_{\mu\nu}(\rho_{20}) \tag{5.10}
\]

with the complex coupling matrix

\[
K_{\mu'\nu'\mu\nu}^{\mu\nu} = N_{\mu'\nu'} N_{\mu\nu} \sum_{1234} K_{1234}^{\mu'\nu'} T_{1234}^{\mu'\nu'} \tag{5.11}
\]

\[
T_{1234}^{\mu'\nu'} = \langle 12 | (\Omega \chi^{\mu'} \circ \chi^{\nu'} \Omega \dagger - \chi^{\mu'} \circ \chi^{\nu'}) | 34 \rangle \tag{5.12}
\]

and the driving term

\[
D_{\mu\nu}(\rho_{20}) = \sum_{1234} K_{1234}^{\mu\nu} (\langle 12 | (\Omega \rho_{20} \Omega \dagger - \rho_{20}) | 34 \rangle). \tag{5.13}
\]

The contributions to the sums in (5.11) and (5.13) factorize into a term \( K_{1234}^{\mu\nu} \) which is, according to (3.18), a linear combination of phonon-particle vertices, multiplied by a term which comprises the short-range correlations in form of Moeller operators \( \Omega(E) \). In the limit \( \Omega = 1 \), i.e. without collisions, both coupling matrix and driving term vanish. The source for damping is the imaginary part of the coupling matrix. Using (5.5) we obtain

\[
Im K_{\mu'\nu'\mu\nu}^{\mu\nu} = -N_{\mu'\nu'} N_{\mu\nu} \sum_{1234} K_{1234}^{\mu'\nu'} \langle 12 | (1 + R) \chi^{\mu'} \circ \chi^{\nu'} \dagger S | 34 \rangle - \langle 12 | (1 + R \dagger) \chi^{\mu'} \circ \chi^{\nu'} S \dagger | 34 \rangle, \tag{5.14}
\]

with

\[
R \approx -Im g Q = Im G \tag{5.15}
\]

\[
S \approx Im g Q = Re G. \tag{5.16}
\]

Here we have adopted the on-shell approximation

\[
g(E) \approx i Im g(E) = -i \pi \delta(E - h(1) - h(2)). \tag{5.17}
\]
A further evaluation of (5.10) is achieved by solving the non-hermitean eigenvalue-problem

$$\omega_\eta \xi_\eta = A\xi_\eta$$  \hspace{1cm} (5.18)

$$A^\dagger \psi_\eta = \omega_\eta^* \psi_\eta$$  \hspace{1cm} (5.19)

with $A_{\mu\nu,\mu'\nu'} = (\Omega_\mu - \Omega_\nu)\delta_{\mu\mu'}\delta_{\nu\nu'} + K_{\mu\nu}^{\mu'\nu'}$. The eigenstates $\xi_\mu$, $\psi_\mu^*$ form a biorthogonal system which follows the orthogonality relation

$$\sum_{\mu'} \psi_\mu^* \psi_\mu' = \delta_{\eta\eta'}.$$  \hspace{1cm} (5.20)

An expansion $C_{\mu\nu}(t) = \sum_\eta C_\eta(t)\xi_\mu$ yields, together with (5.10), an equation for the components $C_\eta(t)$, the solution of which is given by

$$C_\eta(t) = e^{-\omega_\eta t}C_\eta(0) + \frac{1}{i} \int_0^t dt' e^{-i\omega_\eta(t-t')} \sum_{\mu\nu} \psi_\mu^* \xi_\mu \frac{\partial}{\partial \rho_\nu}\rho_\nu(t').$$  \hspace{1cm} (5.21)

A transformation back into the single-particle basis provides for the long-range correlations

$$C_{\alpha\beta\alpha'\beta'}^{\perp} \approx \sum_\eta J_\eta^{\beta_\alpha\beta'\alpha'} e^{-i\omega_\eta t}C_\eta(0) +$$

$$\frac{1}{i} \int_0^t dt' \sum_{\eta,1234} J_\eta^{\beta_\alpha\beta'\alpha'} K_{1234}^\eta e^{-i\omega_\eta(t-t')} (\Omega\rho_\nu(t'))^{\eta\eta'} - \rho_\nu(t'))_{1234}.$$  \hspace{1cm} (5.22)

with

$$J_{1234}^\eta = \sum_{\mu\nu} N_\nu \chi_\nu^\mu \chi_\nu \chi_\nu \xi_\mu^*$$  \hspace{1cm} (5.23)

$$K_{1234}^\eta = \sum_{\mu\nu} \psi_\mu^* K_{1234}^\nu.$$  \hspace{1cm} (5.24)

Expression (5.22) is our central result. The long-range correlations are superpositions of vibrational modes which are damped due to the coupling with short-range correlations. This collisional damping is non-perturbative (i) with respect to the phonon-particle vertices due to the diagonalisation of $A$ (5.18, 5.19) and (ii) with respect to the two-body collisions due to the $G$-matrix appearing in (5.14). Furthermore, after insertion of (5.22) into (5.8), we observe that long-range correlations produce vibrational terms in the one-body equation.
6 Summary

In this paper we have presented a reformulation of two-body dynamics in which the equation for the two-body correlation function – as obtained from the time-dependent density matrix hierarchy – is replaced by two coupled equations for correlations in the \( pp \) and the \( ph \)-channel. These correlations are addressed as short- and long-range correlations and associated with in-medium collisions and vibrational motion, respectively. Their mutual influence (termed channel-mixing) is analysed by means of an integral equation for the corresponding mixing correlations which exhibits the iterative structure of two-body dynamics: After a resummation of ladders accounting for in-medium collisions a resummation of loops in terms of vibrational RPA-modes is performed which accounts for the particle-phonon coupling. In this way the complete two-body correlation function is traced back to the uncorrelated two-body density and the initial vibrations in the \( ph \)-channel. We stress that our approach is non-perturbative also with respect to mixed diagrams and that double-counting of interactions is avoided.

In the stationary limit the channel-mixing can be traced back to an effective interaction which combines the usual \( G \)-matrix with the polarisation matrix. The resulting equations may be viewed as basis for a more sophisticated treatment of the true (correlated) ground state in the sense that channel mixing and hence the mutual influence of long- and short-range correlations is nonperturbatively included.

Assuming that short- and long-range correlations evolve on two different time-scales the short-range correlations can be integrated in terms of the \( G \)-matrix. In this in-medium scattering approach (termed time-dependent \( G \)-matrix theory when neglecting long-range correlations) channel-mixing shows up in the damping of the vibrational modes which constitute the long-range correlations. In the one-body equation in-medium collisions care for both a renormalisation of the mean field and a collision term, whereas long-range correlations produce collective effects on the one-body level in terms of vibrations which are damped due to channel mixing.

The importance of channel-mixing has been, furthermore, demonstrated by numerical results for the total energy which approaches the true ground state energy much closer than without this mixing. In addition, the relative importance of higher order correlations is found to decrease substantially with increasing order of the correlations thus indicating the convergence of the cluster expansion at least for configurations close to the ground
state, i.e. for temperatures up to a few MeV.

References


Appendix A

The cluster expansion for the density matrices \( \rho_n \), that is needed for the derivation of (2.1) and (2.2) as well as for the trace relations (2.6) - (2.8) up to the four-body level is given by

\[
\rho_1(11') = \rho(11'),
\]

\[\text{(A1)}\]

\[
\rho_2(12, 1'2') = A_{12'} \rho(11') \rho(22') + C_2(12, 1'2') = \rho_{20}(12, 1'2') + C_2(12, 1'2'),
\]

\[\text{(A2)}\]

\[
\rho_3(123, 1'2'3') = S_{12'} A_{23'} \rho(11') \rho(22') \rho(33') + S_{12'} \rho(11') C_2(23, 2'3') \\
+ S_{2'} \rho(22') C_2(13, 1'3') + S_{3'} \rho(33') C_2(12, 1'2') + C_3(123, 1'2'3'),
\]

\[\text{(A3)}\]

\[
\rho_4(1234, 1'2'3'4') = A_{125'} S_{34'} A_{4'} \rho(11') \rho(22') \rho(33') \rho(44') \\
+ \Lambda_{14'} (1 - P_{25'} - P_{24'}) \rho(11') \rho(22') C_2(34, 3'4') \\
+ \Lambda_{14'} (1 - P_{34'} - P_{35'}) \rho(11') \rho(44') C_2(23, 2'3') \\
+ \Lambda_{24'} (1 - P_{13'} - P_{35'}) \rho(22') \rho(33') C_2(14, 1'4') \\
+ \Lambda_{24'} (1 - P_{14'} - P_{34'}) \rho(22') \rho(44') C_2(13, 1'3') \\
+ \Lambda_{34'} (1 - P_{13'} - P_{24'}) \rho(33') \rho(44') C_2(12, 1'2') \\
+ \Lambda_{34'} (1 - P_{23'} - P_{34'}) \rho(33') \rho(11') C_2(24, 2'4') \\
+ \Gamma_{2'} C_2(12, 1'2') C_2(34, 3'4') + \Gamma_{3'} C_2(13, 1'3') C_2(24, 2'4') + \Gamma_{4'} C_2(14, 1'4') C_2(23, 2'3') \\
+ \Lambda_{14'} \rho(11') C_3(234, 2'3'4') + \Lambda_{24'} \rho(22') C_3(134, 1'3'4') \\
+ \Lambda_{34'} \rho(33') C_3(124, 1'2'4') + \Lambda_{4'} \rho(44') C_3(123, 1'2'3') \\
+ C_4(1234, 1'2'3'4'),
\]

\[\text{(A4)}\]
with the two- and three-body antisymmetrization operators

\[ A_{ij} = 1 - \mathcal{P}_{ij}; \quad A_{i'j'} = 1 - \mathcal{P}_{i'j'}, \]

(A.5)

\[ S_1' = 1 - \mathcal{P}_{1'2'\cdots} - \mathcal{P}_{1'3'\cdots}; \quad S_1 = 1 - \mathcal{P}_{12} - \mathcal{P}_{13}, \]
\[ S_2' = 1 - \mathcal{P}_{1'2'\cdots} - \mathcal{P}_{2'3'\cdots}; \quad S_2 = 1 - \mathcal{P}_{12} - \mathcal{P}_{23}, \]
\[ S_3' = 1 - \mathcal{P}_{1'3'\cdots} - \mathcal{P}_{2'3'\cdots}; \quad S_3 = 1 - \mathcal{P}_{13} - \mathcal{P}_{23}. \]

(A.6)

The four-body antisymmetrization operators in (A4), furthermore, read explicitly

\[ \Gamma_2' = (1 - \mathcal{P}_{1'2'} - \mathcal{P}_{1'4'} - \mathcal{P}_{2'3'} - \mathcal{P}_{2'4'} + \mathcal{P}_{1'3'\cdots} + \mathcal{P}_{1'4'\cdots}), \]
\[ \Gamma_2 = (1 - \mathcal{P}_{13} - \mathcal{P}_{14} - \mathcal{P}_{23} - \mathcal{P}_{24} + \mathcal{P}_{13\cdots} + \mathcal{P}_{14\cdots}), \]
\[ \Gamma_3' = (1 - \mathcal{P}_{1'3'} - \mathcal{P}_{1'4'} - \mathcal{P}_{2'3'} - \mathcal{P}_{2'4'} + \mathcal{P}_{1'2'\cdots} + \mathcal{P}_{1'4'\cdots}), \]
\[ \Gamma_3 = (1 - \mathcal{P}_{12} - \mathcal{P}_{14} - \mathcal{P}_{23} - \mathcal{P}_{34} + \mathcal{P}_{12\cdots} + \mathcal{P}_{14\cdots}), \]
\[ \Gamma_4' = (1 - \mathcal{P}_{1'2'} - \mathcal{P}_{1'3'} - \mathcal{P}_{2'4'} - \mathcal{P}_{3'4'} + \mathcal{P}_{1'2'\cdots} + \mathcal{P}_{1'3'\cdots} + \mathcal{P}_{1'4'\cdots}), \]
\[ \Gamma_4 = (1 - \mathcal{P}_{12} - \mathcal{P}_{13} - \mathcal{P}_{24} - \mathcal{P}_{34} + \mathcal{P}_{12\cdots} + \mathcal{P}_{13\cdots} + \mathcal{P}_{24\cdots}), \]
\[ \Lambda_1' = (1 - \mathcal{P}_{1'2'} - \mathcal{P}_{1'3'} - \mathcal{P}_{1'4'}); \quad \Lambda_1 = (1 - \mathcal{P}_{12} - \mathcal{P}_{13} - \mathcal{P}_{14}), \]
\[ \Lambda_2' = (1 - \mathcal{P}_{2'3'} - \mathcal{P}_{2'4'} - \mathcal{P}_{2'4'}); \quad \Lambda_2 = (1 - \mathcal{P}_{21} - \mathcal{P}_{23} - \mathcal{P}_{24}), \]
\[ \Lambda_3' = (1 - \mathcal{P}_{3'4'} - \mathcal{P}_{3'2'} - \mathcal{P}_{3'4'}); \quad \Lambda_3 = (1 - \mathcal{P}_{31} - \mathcal{P}_{32} - \mathcal{P}_{34}), \]
\[ \Lambda_4' = (1 - \mathcal{P}_{4'1'} - \mathcal{P}_{4'2'} - \mathcal{P}_{4'3'}); \quad \Lambda_4 = (1 - \mathcal{P}_{41} - \mathcal{P}_{42} - \mathcal{P}_{43}), \]

(A.7)

while \( \mathcal{P}_{ij} \) denotes the exchange operator between particle \( i \) and \( j \).
Figure captions

Fig. 1: Time average of the total traces $tr\ C_2$, $tr\ C_3$ and $tr\ C_4$ as a function of the initialization temperature $T$ within the trace-conserving SCD-approach for a model $^{16}$O-nucleus (c.f. [1]). The time-average was performed over a time period of $80 \times 10^{-23}$ s after the in-medium interactions have been switched on adiabatically with $t_s = 100 \times 10^{-23}$ s (see section 2.2). For a better comparison $tr\ C_3$ and $tr\ C_4$ are multiplied with constant factors 10 and 100, respectively.

Fig. 2: Total energy of the model $^{16}$O-nucleus – initialized with $T = 0$ MeV temperature – as a function of the switch-on time $t_s$ in the limits BORN, RPA, TDGTM, NQCD and SCD (see text).

Fig. 3: Monopole response of the $^{16}$O model nucleus in the occupation numbers. The calculation was performed within the NQCD approximation with $T = 1$ MeV initialization temperature. The $^{16}$O-nucleus was excited by a 15 MeV monopole boost at $t_b = 120 \times 10^{-23}$ s after switching on the in-medium interaction with $t_s = 100 \times 10^{-23}$ s.

Fig. 4: Monopole response in the two-body correlation matrix elements. The calculation is the same as in Fig. 3. The four displayed curves correspond to $\beta = 1$ (solid line), 4 (dashed line), 5 (dashed-dotted line) and 7 (dotted line).

Fig. 5: Integral equation for the $G$-matrix.

Fig. 6: Integral equation for the polarisation matrix.

Fig. 7: Integral equation for $G^{tot}$; the operators $G$ and $\Pi$ in the second diagram on the r.h.s. are connected by one line forming the 6-label operator $\Gamma$.

Fig. 8: Second-order contribution in $\Gamma$. 