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Abstract

Gamma-ray bursts may be from jet-like sources in relativistic bulk motion. In this case, emission of gamma-rays is strongly beamed along the direction of the bulk motion. We analyze effects of the relativistic bulk motion and the resulting beamed emission on statistics of the gamma-ray bursts.

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The physical origin of gamma-ray bursts (GRBs) has remained elusive. Major difficulties are mainly due to lack of positive identifications of GRBs with known astrophysical counterparts and to uncertain distance scales. BATSE's relatively low detection threshold ($F_{min} \approx 10^{-7} \text{ erg/s/cm}^2$) and homogeneous detection have facilitated meaningful statistical analyses [1]. The isotropy of GRB sky positions [2] poses a serious difficulty for Galactic or Extended Halo models [3]. Cosmological models [4] are subject to a rather tight energy constraint at the burst sources [5]. In the cosmological scenario, the deviation of the slope of $\log N - \log (C_{max}/C_{min} \equiv F/F_{min} \equiv f)$ from $-3/2$ (for a homogeneous distribution in the Euclidean space) or $\langle V/V_{max} \rangle < 0.5$ is naturally explained [6] while in the Halo scenario some special radial distributions are required [3,7]. The lack of

source identifications with counterparts is puzzling in the cosmological scenario since any sources residing in galaxies could have been identified with their hosts.

If gamma-ray emission is isotropic, the luminosity, L , of a burst from a source at a redshift z

$$L \approx [1.1 \times 10^{51} \text{ erg/s}] \left(\frac{F_{min}}{5 \times 10^{-8} \text{ erg/s/cm}^2} \right) \left(\frac{h}{0.75} \right)^{-2} \left(\frac{\chi}{(1-\chi)\alpha} \right)^2 \quad (1)$$

where $\chi = 1 - (1+z)^{-1/2}$, $H_o = 100 \text{ h km/s/Mpc}$ (for our discussions, $h = 0.5$), and $\alpha \approx 1-2$ is the photon index [6]. Isotropic gamma-ray emission from a typical cosmological burst at $z \sim 1$ roughly requires a total burst energy $\sim 10^{51} \text{ erg}$ or a fluence $> 10^{-7} \text{ erg/cm}^2$. This large energy with short emission time scales ($\sim \text{min}$ [duration, variability time scale]) implies that some physics is required for gamma-ray photons to escape from the emission region at all [5]. However, if emission is from a jet-like source in relativistic bulk motion, this severe energy constraint could be avoided [5,8]. Since no other models convincingly predict relativistically moving sources, plausible candidates could be rare jets with high Lorentz factors from faint unidentified extragalactic objects (QSO or AGN). If extragalactic objects are indeed involved, identification of GRBs with (X-ray, optical, IR, or radio) counterparts is likely. The relativistic bulk motion motivates a physically meaningful GRB luminosity distribution [9], which is otherwise rather arbitrary and unconstrained. We consider effects of relativistic bulk motion and resulting beamed emission on GRB statistics. We assume that sources emit radiation isotropically in their comoving frames. Our statistical analysis is independent of any specific emission mechanisms whose details will be addressed in a separate paper (I. Yi, in preparation).

We assume that sources are described either by a 'standard candle' (a standard luminosity with a spectral index) or by a 'standard source' (a standard

Lorentz factor, a standard intrinsic luminosity in the source frame with a spectral index). Diverse observed types of GRBs may indicate deviation from our simplifying assumption. The photon spectrum is described by the photon index α , $\phi(E) \propto E^{-\alpha}$ being the number of photons emitted per unit time per unit photon energy (E) interval. We take $\alpha = 1, 2$. The observed flux (in the band of energies E_1 and E_2) from a source at redshift z with the luminosity $L = \int_{E_1}^{E_2} dE E \phi(E)$ is then given by

$$F(z; L) = \left(\frac{H_o^2 L}{16\pi c^2} \right) (1+z)^{1-\alpha} (\sqrt{(1+z)} - 1)^{-2} \quad (2)$$

for a flat universe (with zero cosmological constant) and c is the speed of light.

In $\alpha = 1$ case, with

$$x_F(L) \equiv 1 + z_F(L) = \left(1 + \frac{H_o}{4\sqrt{\pi}c} \left(\frac{L}{F} \right)^{1/2} \right)^2 \quad (3)$$

we get the cumulative number of bursts with observed fluxes larger than F

$$N(> F) \propto \int dL \Phi(L) \int_1^{x_F(L)} (x^{-5/2} - 2x^{-3} + x^{-7/2}) dx \quad (4)$$

and $\langle V/V_{max} \rangle$

$$\begin{aligned} \left\langle \frac{V}{V_{max}} \right\rangle &\equiv \left\langle \left(\frac{F_{min}}{F} \right)^{3/2} \right\rangle \\ &= \left(\frac{16\pi c^2 F_{min}}{H_o^2} \right)^{3/2} \frac{\int dL \Phi(L) L^{-3/2} \int_1^{x_F(L)} dx x^{-7/2} (x^{1/2} - 1)^5}{\int dL \Phi(L) \int_1^{x_F(L)} dx x^{-7/2} (x^{1/2} - 1)^2} \end{aligned} \quad (5)$$

where $x(L) = x_F(L)$ with $F = F_{min}$ and $\Phi(L)$ is the luminosity distribution of GRBs (see below). Similarly, for $\alpha = 2$,

$$x_F(L) = 1 + z_F(L) = \left(\frac{1 + \sqrt{1 + (L H_o / \pi c^2 F)^{1/2}}}{2} \right)^2 \quad (6)$$

$$N(> F) \propto \int dL \Phi(L) \int_1^{x_F(L)} (x^{-3/2} - 2x^{-2} + x^{-5/2}) dx \quad (7)$$

$$\begin{aligned} \left\langle \frac{V}{V_{max}} \right\rangle &= \left\langle \left(\frac{F_{min}}{F} \right)^{3/2} \right\rangle \\ &= \left(\frac{16\pi c^2 F_{min}}{H_0^2} \right)^{3/2} \frac{\int dL \Phi(L) L^{-3/2} \int_1^{x(L)} dx x^{-2} (x^{1/2} - 1)^5}{\int dL \Phi(L) \int_1^{x(L)} dx x^{-7/2} (x^{1/2} - 1)^2}. \end{aligned} \quad (8)$$

The observed apparent luminosity from a source in relativistic bulk motion with speed v in the local rest frame is given by $L_{obs} = B^p L_{int}$ where L_{int} is the luminosity in the comoving source frame and $B = [\gamma(1 - \beta \cos \theta)]^{-1}$. θ is the angle between the observer's line of sight and the direction of bulk relativistic motion of the source, $\beta = v/c$, and $\gamma = (1 - \beta^2)^{-1/2}$ [9]. When L 's refer to the monochromatic luminosity at a fixed energy, $p = 2 + \alpha$ accounting for aberration, time contraction, and actual blueshift of photon energy. ($p = 4$ when L 's refer to integrated luminosities over corresponding bands.) Observed fluences are enhanced by a factor B^3 with respect to the corresponding fluences in the comoving source frame. If the bulk motions are randomly oriented in space, the observed fluence is boosted on average by a factor $\int_{-1}^1 B^3 d(\cos \theta) / \int_{-1}^1 d(\cos \theta) = \gamma$. If angle-averaged, at the source, the total emitted energy required for a fixed observed fluence is reduced by a factor γ^{-2} . For randomly oriented directions of bulk motion, B follows the probability distribution $P(B) = (\gamma \beta B^2)^{-1}$ with the maximum $B_{max} = (\gamma(1 - \beta))^{-1}$ and the minimum $B_{min} = \gamma^{-1}$ provided that the bulk motion is directed to two opposite directions ('twin jets') and that emission from the forward-directed bulk motion dominates. If the bulk motion is only in one direction $P(B) = (2\gamma \beta B^2)^{-1}$.

For standard sources with an intrinsic luminosity in the comoving source frame,

$$L_{int} \equiv L_o = \int_{E_1}^{E_2} E \phi(E) dE, \quad (9)$$

the apparent distribution of observed luminosities is

$$\Phi_{obs}(L) = \frac{1}{\beta \gamma^p} L_o^p L^{-\frac{p+1}{p}} \quad (10)$$

with $L_{max} = L_o B_{max}^p$ and $L_{min} = L_o B_{min}^p$. For some distributions of L_o (e.g., a power-law), generalization is straightforward. In order for gamma-ray photons to escape from the emission region, large γ 's (or B 's) are required. For typical GRBs, roughly

$$B > \left(\frac{\sigma_T}{m_e c^2} \right)^{1/4} \left(\frac{L_{obs}}{\Delta t_{obs}} \right)^{1/4} \quad (11)$$

is required in order for the emission region to be transparent to pair-producing photon-photon interactions where L_{obs} is the observed luminosity and Δt_{obs} is the time scale constraining the source size $< c \Delta t_{obs}$. If emission is due to synchrotron-self inverse Compton process, a different constraint exists [8]. For typical bursts, $\gamma \sim O(100)$ can make a source sufficiently transparent. For our analysis, we do not constrain either γ or specific emission mechanisms. We take γ as a model parameter.

Using the luminosity distributions, Eq. (10), we plot $\langle V/V_{max} \rangle$ as a function of L_o for $\gamma \leq 10^3$ with $\alpha = 1, 2$ (Fig. 1). In the beamed cases, the approximate observed value $\langle V/V_{max} \rangle \approx 0.33$ [1] is easily satisfied as in the non-beamed cases. Obviously, larger γ 's require smaller L_o 's. As in the non-beamed case, $\alpha = 1$ requires a larger L_o (i.e. a larger distance to the most distant sources) than that in $\alpha = 2$ case. In $\alpha = 1$ non-beamed case, the most distant bursts have a redshift $z_{max} \approx 1.5$ in order to satisfy the observed $\langle V/V_{max} \rangle$. However, in the beamed case, $z_{max} \approx 5.5$, which means that if the observed $\langle V/V_{max} \rangle \approx 0.33$ has to be satisfied, the most distant bursts could be much farther than those in the non-beamed case (Eq. (1)). Similarly, in $\alpha = 2$ case, we get $z_{max} \approx 1$ (non-beamed) and $z_{max} \approx 2$ (beamed). In either case, z_{max} increases by a factor ≥ 1 by beaming. (z_{max} could be slightly affected by $h \approx 0.5 - 0.75$.) For redshifts $z_{max} > 2$, if the beamed bursts are from known extragalactic populations such as QSO and AGN in general, significant source evolution seems unavoidable [10]. In this case, luminosity distributions combined with strong source evolution

could make simple statistical analyses quite unclear unless both the luminosity distributions and the source evolution are well constrained (I. Yi, in preparation). The increase of z_{max} due to relativistic beaming may be interpreted simply as due to effective luminosity distributions. The increase of z_{max} could also be due to some intrinsic distributions of GRB luminosity (not due to beaming). If $z_{max} > 5$ is indeed the case, effects of redshifts on other physically interesting quantities such as time scales (*e.g.*, duration) and energy scales (*e.g.*, spectral breaks) of GRBs could be more significant as cosmological redshift can modify them almost by an order of magnitude ($\alpha = 1$). However, the observed $\log N - \log f$ relation does not suggest strong cosmological source evolution, which in turn may suggest that beamed emission is from sources whose source evolution is not strong even at $z > 2$.

In Fig. 2, we plot $\log N - \log f (= F/F_{min})$ for the same parameters as in Fig. 1 together with the observed distribution. We normalize the total number of bursts at 800yr^{-1} [1]. The effect of the luminosity distributions tends to smooth out transition in the slope, roughly from $N \propto f^{-1.0}$ to $f^{-1.5}$, in the observed range $\log f \leq 2.5$. The effects of beaming are rather small (slightly larger in $\alpha = 2$ case than in $\alpha = 1$). More noticeable deviation from the non-beamed (or standard candle) case requires either much wider or a far steeper luminosity distributions than Eq. (10). Currently, the beamed emission is not ruled out based on these simple statistics. Steeper or wider distributions would almost erase the transition in slope from $\propto f^{-1.5}$ to $\propto f^{-1.0}$. In the current cases, the calculated slopes are slightly affected by the beamed luminosity distributions. One noticeable result is that in $\alpha = 2$ case, the curves for beamed cases at the bright end ($\log f > 1.5$) do not fit the observed curve due to relative enhancement (by beaming) of the number of nearby bursts compared with that of distant bursts. Although a large fraction of GRBs may have $\alpha \sim 2$, since the number of bursts at the bright end is

small (hence subject to large statistical errors), this result may not be interpreted as too significant. Moreover, the comparison based on fitting at the faint end ($f \sim 1$) is rather uncertain given the fact that burst detection at the faint end may be strongly affected by some threshold effects. In the strong burst limit, $H_0(L/F)^{1/2}/4\sqrt{\pi}c \ll 1$,

$$N(> F) \propto \langle L^{3/2} \rangle F^{-3/2} \quad (12)$$

where $\langle L^{3/2} \rangle = \int dL \Phi(L) L^{3/2}$. Obviously, in this limit, the luminosity distribution does not affect $F^{-3/2}$ dependence regardless of α . In the Euclidean space, with a radial source density distribution $\propto r^{-n}$, $\langle V/V_{max} \rangle = [(3-n)/(6-n)] \int dL \Phi(L)$ and $N(> F) \propto \langle L^{3/2} \rangle F^{3/2-n}$. That is, in the Euclidean limit (equivalent to the strong burst limit), the luminosity distribution does not affect the simple statistics $\langle V/V_{max} \rangle$ and $\log N - \log F$. In Fig. 2, at the bright end, the slopes do approach $-3/2$ in all cases as shown above. Hence, between the faint end ($N \propto f^{-1.0}$) the bright end ($N \propto f^{-1.5}$), with an average slope $-1.5 < m < -0.8$ for $N(> f) \propto f^{-m}$, $0.3 < \langle V/V_{max} \rangle = 2m/(2m+3) < 0.5$ [11] should be easily satisfied (as shown in Fig. 1) in all cases of beaming. The effects of luminosity distributions due to relativistic bulk motion is small and expected to be virtually indistinguishable from mild source evolution (more bursts at lower redshifts than at higher redshifts). Since known extragalactic objects are believed to rapidly evolve in their comoving densities or luminosities at $z > 2$, simple statistics such as $\langle V/V_{max} \rangle$ and $\log N - \log f$ may not provide any meaningful clues as to the nature of the cosmological GRB sources. As the strong beaming effect is likely to be related to AGN or QSO, and the maximum redshifts obtained is rather large, the possibility of beaming may be related to significant source evolution.

Large γ 's and distributions of B provide wide distributions of observed time scales such as GRB durations even if the corresponding intrinsic time

scale in the comoving frame is a standard value for all sources. In the non-beamed cosmological model with a single intrinsic duration and a standard luminosity, the duration and the observed flux is related simply by duration $D \propto (1 + g^{-1/2})^2$ ($\alpha = 1$) or $D \propto \left(1 + \sqrt{1 + 4g^{-1/2}}\right)^2$ ($\alpha = 2$) with $g \equiv F/(L_o H_o^2/16\pi c^2)$. The number of bursts with duration D is $\propto D^{-7/2}(D^{1/2} - 1)^2$. However, in the beamed case, the apparent time contraction due to relativistic motion provides a wide distribution of observed durations. If all sources are at a same distance, the duration will be related to the flux (f) by $D \propto f^{-1/p}$ and the duration will have a distribution which is flat within the interval $D_{max}/D_{min} = B_{max}/B_{min} \sim 2\gamma^2$. For a large z_{max} , $D_{max}/D_{min} = (1 + z_{max})B_{max}/B_{min} \sim 2\gamma^2(1 + z_{max}) > 10^4$ for $\gamma > 100$, hence providing a wide distribution of durations. However, in the realistic observations, durations are expected to be within a smaller interval than the above estimate. This is so since the number of (actually observed apparent) less luminous bursts will be much smaller (Fig. 3) as they occupy a very small comoving cosmological volume despite the fact that they are more abundant in their comoving densities (Eq. (10)). In Fig. 3, using Monte-Carlo realizations, we show the expected distributions of the apparent observed durations for $\alpha = 1$ and $\alpha = 2$ with $\gamma = 10^{2.5}$. The simulated distributions are subject to the same condition $\langle V/V_{max} \rangle = 0.33$ as in the previous figures. Since the duration is in arbitrary units of intrinsic duration (a single value) which is not currently constrained, not the absolute values but the widths of the distributions are meaningful ($D_{max}/D_{min} \approx 10$ in both cases). The distributions are negatively skewed (more shorter durations) despite rather extended tails toward long durations. Longer durations are rare since these bursts are observable only at smaller distances from an observer (corresponding to much smaller comoving

volumes). For $\alpha = 1$ and $\gamma = 10^{2.5}$, the mean duration is shorter than the intrinsic one by a factor $\sim 6.2 \times 10^{-3}$ and for $\alpha = 2$, by a factor $\sim 4.2 \times 10^{-3}$.

In Fig. 4, we plot the relation between the duration and the flux using a large number of randomly selected B 's (following $P(B) \propto B^{-2}$). The flux – duration relation gives a definite prediction in the beamed cases. Fig. 4 is a unique prediction of the beamed bursts as long as they are described by a standard source. Bursts with longer durations will have relatively smaller scatter around smaller fluxes than those with shorter durations. However, if the bursts are composed of several types of standard bursts, the observed flux – duration relation should be a superposition of the types with different spectral indices and Lorentz factors. A large scatter in the flux for bursts with long durations would imply that the bursts may not be described by a single standard burst type. A similar argument may also be applied to the non-beamed cosmological bursts. Two spectral indices show qualitatively similar relations. Any direct comparison with observations does not seem meaningful as intrinsic durations are likely to be distributed with some finite (unspecified) widths.

There exist some interesting consequences of the beaming (due to relativistic bulk motion). (i) Spectral break energy could be distributed widely compared to a single energy in the comoving frame (due to relativistic blueshift and cosmological redshift). If the spectral break is related to the electron-positron annihilation line, the break energy could be blueshifted to too large a value to explain the observed break (for $\gamma > 100$) even if it is cosmologically redshifted by $(1 + z) < 10$. (ii) No observed jets are known to have $\gamma \geq 100$. $3 \leq \gamma \leq 20$ have been observed for extragalactic VLBI jets associated with QSO or AGN [9,12]. Although $\gamma > 100$ could be realized in cosmological fireballs [4,5], it is not clear whether such fireballs may lead to jet-like relativistic bulk motions of gamma-ray emitters [13]. It is also unclear whether γ , L_o , and duration are all described

by standard values (i.e. standard sources) rather than potentially complicating distributions of them. (iii) Strong beaming requires a larger number of bursts (by a factor $\sim \gamma^2$) due to forward bunching of emitted photons. (iv) The probability of observing lensed bursts may be enhanced (especially for $\alpha = 1$ with $z_{max} > 5$). As an example, if galaxies are modeled by singular isothermal spheres and if the lensing due to galaxies are considered in a flat universe, the lensing time delay is expected to be days to months with lensing probability $\propto (1 - (1+z)^{-1/2})^3$ for a burst source at redshift z [14]. Hence if z_{max} increases from ~ 2 to ~ 5 in $\alpha = 1$ case, the lensing probability for a source at $z = z_{max}$ increases roughly by a factor ~ 3 . Lensed bursts will be one of the only few unequivocal evidences of a cosmological origin although details of detectability of a lensing event is rather complicated [14]. (v) If the bursts have two or more different spectral indices and intrinsic luminosities, some bimodal distributions (in duration, spectra, and variability) are likely [15]. It is, however, not clear whether diverse source types could be described in sufficient detail to constrain the observed bimodal features. If beamed and unbeamed bursts coexist, they would show two very different sets of physical characteristics such as distributions of durations and spectral breaks. For beamed bursts, larger $z_{max} > 5$ (for a same constraint $\langle V/V_{max} \rangle \approx 0.33$) could somewhat smooth out any prominent intrinsic features in GRB energy scales or time scales (such as durations and spectral breaks).

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Figure Captions

- Fig. 1. — $\langle V/V_{max} \rangle$ vs. L_0 for Lorentz factors, $\gamma = 10^{0.5-3.0}$. The solid lines are for $\alpha = 1$ (from left to right, $\gamma = 10^{3,2.5,2.1,1.5,1,0.5}$ and isotropic case) and the dashed lines correspond to $\alpha = 2$ (from left to right, $\gamma = 10^{2.5,2.1,1.5,1,0.5}$ and isotropic case). The dotted horizontal line corresponds to the approximate observed value $\langle V/V_{max} \rangle = 0.33$ which is satisfied in all cases with different L_0 for the γ 's adopted. The results are almost unaffected by a change in the observed $\langle V/V_{max} \rangle$ from 0.33 to 0.32.
- Fig. 2. — $\log N - \log f$ for various Lorentz factors for $\alpha = 1$ (solid lines; the bottom line corresponds to the isotropic case) and $\alpha = 2$ (dotted lines; the bottom line corresponds to the isotropic case). Two dashed straight lines correspond to slopes of -0.8 (left) and -1.5 (right). Although the deviations from the isotropic cases are rather small, the effect of beaming tends to erase the change of slope from -0.8 to -1.5 in the cosmological model. For $\alpha = 2$, the effects are more important in the observed flux range $\log f < 2.5$.
- Fig. 3. — Distributions of durations for cosmological gamma-ray bursts for $\gamma = 10^{2.5}$. The distributions are subject to the constraint $\langle V/V_{max} \rangle = 0.33$. The duration is in (arbitrary) unit of the intrinsic duration in the jet frame. The solid histogram is for $\alpha = 1$ and the dotted histogram for $\alpha = 2$. The width

of the distribution in the standard source case gives the ratio between the maximum duration and the /minimum duration ~ 10 .

Fig. 4. — Duration vs. flux relation for $\gamma = 10^{2.5}$. (a) for $\alpha = 1$ (b) for $\alpha = 2$.

In both cases, GRBs with longer durations are expected to show smaller scatter in their fluxes than those with shorter durations. This trend is unique and valid only if GRBs may be described by standard sources. Two spectral indices show qualitatively similar relations.

Figure 1.

Figure 2.

Figure 3.

Figure 4A

Figure 4b