CERN-TH.7209/94

High Energy Gamow-Teller Strength in Double Beta Decay

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ABSTRACT

The contribution of high-lying intermediate states to the two-neutrino double beta decay is investigated in several approximations. Even though there is an appreciable β^- strength at high energy connecting the initial and intermediate nuclei, and a similar β^+ strength connecting the intermediate and final nuclei, we conclude that the contribution of high-lying states to the $\beta\beta$ decay is strongly suppressed due to the random nature of the corresponding β^- and β^+ amplitudes. At the same time, our approach suggests that in general the convergence of the double beta decay matrix elements as a function of the excitation energy in the intermediate nucleus is relatively fast. Thus, one expects that only one or few low-lying states are responsible for the bulk of the decay, in agreement with the available data.

PACS numbers: 21.10.Pc, 23.40.Hc, 24.80.-x

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CERN-TH.7209/94

March 1994

The evaluation of double beta decay matrix elements is an important application of the general study of the Gamow-Teller (GT) strength in nuclei. In the two-neutrino mode of $\beta\beta$ decay both the β^- and the β^+ GT strengths are involved, the former in the virtual transition from the ground state of the initial nucleus and the latter in the transition connecting the intermediate nucleus with the ground state of the final nucleus.

The matrix elements of $2\nu\beta\beta$ decay are given by

$$M_{GT}^{2\nu} = \sum_m \frac{\langle f || \sigma \tau_+ || m \rangle \langle m || \sigma \tau_+ || i \rangle}{E_m - E_0}, \quad (1)$$

where $E_0 = (M_i + M_f)/2$, $|f\rangle(|i\rangle)$ are the 0^+ ground states of the final (initial) even-even nuclei, and $|m\rangle$ are the 1^+ states in the intermediate odd-odd nucleus. The last factor in the numerator of Eq. (1) represents the amplitude of the β^- decay (or of the forward angle (p, n) reaction) of the initial nucleus, while the first factor represents the amplitude of the β^+ decay (or of the (n, p) reaction) of the final nucleus.

In the evaluation [1, 2, 3] of Eq.(1), one usually takes into account only the GT strength involving configurations of valence nucleons. Such strength is significant at relatively low excitation energies ($E_{exc} \leq 20$ MeV). However, the GT strength extends to quite high energies, even above the pion threshold [4, 5, 6]. The high-energy strength originates in the nucleon-nucleon tensor force, and corresponds to the two-particle two-hole excitations. It is expected to be proportional to the nucleon number A [5, 7], and to be as important in the β^- as in the β^+ channels. Thus, unlike the low-energy region where the β^- strength dominates (in nuclei with a neutron excess), there is a large energy region where both strengths are comparable. One may therefore wonder whether this high-energy strength, usually not included explicitly, might affect the $\beta\beta$ decay matrix elements.

To see that this might in fact happen, let us consider an extreme case of coherence, where the signs of the β^- and β^+ amplitudes are fully correlated. The strengths per unit energy, $s^\pm(E)$, are approximately constant in a region of width m_π around the pion threshold. The PCAC relation can be exploited to relate the axial current matrix element to the corresponding matrix element for the (virtual) pion absorption. The axial strength can therefore be linked [5] to the absorptive part of the p -wave optical potential, which leads to $s^\pm(m_\pi) \simeq 3 \times 10^{-3} A / \text{MeV}$, where A is the nuclear mass number. The corresponding contribution to the matrix element, Eq. (1), is then

$$|M_{GT}^{2\nu}| = \int_{m_\pi/2}^{3m_\pi/2} s^+(E) \frac{dE}{E} \simeq s^+(m_\pi) \ln(3), \quad (2)$$

Taking ^{48}Ca as an example, we obtain a matrix element which is as large as the matrix element calculated from the shell model [8, 9, 10], and also as large as the experimental upper limit derived from the lower limit of the $\beta\beta$ decay half-life [11]. Clearly, the high-energy strength

would have to be included under the complete coherence hypothesis. Let us therefore examine the opposite extreme, where the two amplitudes have random and totally uncorrelated signs.

Consider a situation in which the residual interaction causes a complex statistical mixing of the intermediate states, characterized by a (slowly) varying average strength at any given energy, with a random sign of the corresponding amplitudes. The quantities $s^\pm(E)$ are related to the corresponding amplitudes $\alpha^\pm(E)$ by

$$s^\pm(E) = |\alpha^\pm(E)|^2 \rho(E), \quad (3)$$

where $\rho(E)$ is the density of 1^+ states at energy E .

The 2ν $\beta\beta$ matrix element is then

$$M_{GT}^{2\nu} = \sum_m \frac{\alpha_m^+ \alpha_m^-}{(E_m - E_0)}, \quad (4)$$

where we now take α_m^+ and α_m^- to be of random sign. The average of the squared matrix element is then obtained by squaring each term first, and adding them afterwards. This is analogous to the random matrix element assumption which leads to the Porter-Thomas distribution for neutron widths (see Ref.[12]). The present assumption is somewhat weaker in that we only take the signs of the matrix elements to be random. Thus, we obtain

$$\Delta\langle (M_{GT}^{2\nu})^2 \rangle = \sum_m \frac{(\alpha_m^+)^2 (\alpha_m^-)^2}{(E_m - E_0)^2} = \int_{E_1}^{\infty} \frac{s^+(E)s^-(E)}{(E - E_0)^2} D(E) dE, \quad (5)$$

where $D(E) = 1/\rho(E)$ is the level spacing at energy E , and E_1 is the lowest energy where our approach is applicable.

It is a well-known feature of systems with many degrees of freedom, such as the nucleus, that the level spacing $D(E)$ decreases rapidly and approximately exponentially with energy (see Ref.[13]). The integral is then dominated by the contributions near the lower limit and slowly varying factors can be considered to be constant and taken out of the integral. In terms of the nuclear temperature

$$1/T(E) = \rho(E)^{-1} \frac{d\rho}{dE} \quad (6)$$

the integral can be approximated as

$$\Delta\langle (M_{GT}^{2\nu})^2 \rangle = \frac{s^+(E_1)s^-(E_1)}{(E_1 - E_0)^2} \int_{E_1}^{\infty} D(E) dE \simeq \frac{s^+(E_1)s^-(E_1)}{(E_1 - E_0)^2} D(E_1)T. \quad (7)$$

We evaluate this expression for ^{48}Ca , and choose $E_1 - E_0$ as 20 MeV. (We choose this value for E_1 since an estimate of $s^-(E)$ is available [6] beginning with that energy, and since for smaller energies the GT strength connected to the valence nucleons becomes dominant.) For the β^-

strength we follow Ref. [6], $s^-(E) \simeq 3 \times 10^{-1}/\text{MeV}$ for E between 20 and 40 MeV. For the β^+ strength we take the same, which is an overestimate.

The crucial term is the level spacing $D(E_1)$ which we obtain by extrapolating the Fermi gas expression for the level density

$$\rho(E^*) = \text{const} \frac{1}{E^{5/2}} \exp(2(aE^*)^{1/2}), \quad (8)$$

where E^* is the excitation energy in the odd-odd nucleus, shifted with respect to E_0 in Eq. (1) by $\Delta M = M_{\text{odd-odd}} - (M_i + M_f)/2$; we neglect this shift in the present estimate. We normalize $\rho(E^*)$ to neutron-capture resonance spacing and extrapolate to 20 MeV excitation. With $a = A/15 \simeq 5 \text{ MeV}^{-1}$ we have $D(20 \text{ MeV}) \simeq 5 \text{ eV}$ and $T = 2 \text{ MeV}$. Finally we obtain for the contribution of the energies above 20 MeV

$$\Delta\langle (M_{GT}^{2\nu})^2 \rangle \simeq 10^{-5} / \text{MeV}^2, \quad (9)$$

which is much smaller than the shell model value $(2 - 6 \times 10^{-3}/\text{MeV}^2)$ [8, 9, 10]. Even though our estimate is rather crude, it clearly shows that the high-lying strength is totally negligible, provided our assumption of random signs is a valid one.

Let us comment on the validity of the assumption that the β^- and β^+ amplitudes to the same intermediate state have uncorrelated signs. Actually, again in the simplest approximation, illustrated schematically in Fig. 1, any given intermediate state 1^+ will have either a non-vanishing β^- or β^+ amplitude, but *not both of them* at the same time. If the high-lying strength is associated with the two-particle two-hole states it is easy to see that β^- populates states with two proton particles (or two neutron holes), and β^+ populates states with two neutron particles (or two proton holes). In this extreme there is no contribution to the $\beta\beta$ decay whatsoever.

In reality, however, the states will mix with each other, and with other more complex states nearby. The lack of coupling will still tend to survive in spite of the mixing. The prototype situation is that of only two states $|n_1\rangle$ (populated by β^-) and $|n_2\rangle$ (populated by β^+) as in the paragraph above. The mixed states will be

$$\begin{aligned} | \nu_1 \rangle &= \alpha | n_1 \rangle + \beta | n_2 \rangle, \\ | \nu_2 \rangle &= -\beta | n_1 \rangle + \alpha | n_2 \rangle, \\ \langle \nu_1 | \nu_2 \rangle &= 0 \quad ; \quad |\alpha|^2 + |\beta|^2 = 1. \end{aligned}$$

The contribution of these two mixed states will vanish as long as their splitting is negligible compared to the corresponding energy denominator. That is indeed the case for the high-lying states that we are considering. The splitting is of the order of the spreading width, small compared to the excitation energy.

The argument is essentially identical when the two states are mixed with N other states $n_i, i = 3 \dots (N+2)$, once more provided the energy spread is negligible compared to the denominators. The mixing leads to new states

$$|\nu_i\rangle = \sum_k \beta_{i,k} |n_k\rangle. \quad (10)$$

However, one easily sees that the states are just reshuffled such that

$$\sum_i |\nu_i\rangle \langle \nu_i| = \sum_k |n_k\rangle \langle n_k|, \quad (11)$$

which means that there is still no coupling.

It is only at this point that we need the statistical argument considered above. Once the mixing is very intense the lack of coupling becomes a delicate issue easily removed by small residual effects, such as the variation of the energy denominators. The situation is then characteristic of a chaotic system. Hence there will be no exact cancellation, but rather a statistical one as described by Eq. (5).

We therefore see from this argument that the statistical picture *enhances* the coupling as compared to the extreme picture described above, since it removes the restriction of strict no-coupling. At the same time one notes that this situation is particularly favourable to a statistical argument. The principal danger in applying this argument to a coherent sum such as a second-order matrix element is that small coherent terms add up in the total matrix element. There is now no obvious mechanism for such an effect.

The second effect that may restrict the full statistical argument is that groups of states over limited energy regions may act coherently even though the process is statistical otherwise, akin to the no-coupling sum rule of Eq. (11). The net effect in this case is that the level density would be lower than in the full scale model. However, the suppression factor in the statistical approach is huge, so that even a very conservative approach of increasing the effective level spacing by a factor of 10^3 still gives a small contribution to the $2\nu\beta\beta$ matrix element.

Having shown that the high-lying GT strength does not contribute to the $\beta\beta$ decay, we wish to examine whether one can make a connection to the shell model states usually included in the evaluation of $\beta\beta$ decay. In the context of such an evaluation, Zhao *et al.* [8] argued that only low-lying states contribute to the $\beta\beta$ matrix element for ^{48}Ca . There, after diagonalization of the shell model Hamiltonian with about 5000 intermediate states, it was found that just a handful of the lowest-lying states account for much of the resulting matrix element. The argument of Zhao *et al.* is in fact rather general and can be reformulated as follows. Since the Gamow-Teller operator cannot change the orbital quantum number, it is sufficient to consider a transition of a neutron pair $(\nu j, l)^2$ into a proton pair $(\pi j', l')^2$. Now, the β^- operator will populate two states

in the intermediate odd-odd nucleus

$$\beta^- : (\nu j, l)(\pi j', l') \text{ and } (\nu j, l)(\pi j'', l'), \quad (12)$$

where j_c is the "complement" of j , i.e. $j_c = l - 1/2$ if $j = l + 1/2$, etc. These two states are separated by the spin-orbit splitting ΔE_{ls} . Similarly, the β^+ operator will also populate two states

$$\beta^+ : (\nu j', l)(\pi j', l') \text{ and } (\nu j'', l)(\pi j', l'), \quad (13)$$

again separated by the spin-orbit splitting. It is easy to see that for both possibilities, $j' = j$ and $j' = j_c$, the lower state can be reached by both transitions, but there are two higher-lying states, one with the β^- and another one with the β^+ strength. The upper-lying states will mix with each other, but their contributions have a tendency to cancel, as we saw previously. Now we have, however, a common low lying state that is expected to dominate the resulting $2\nu\beta\beta$ matrix element.

In nuclei with many valence nucleons there will be several ($\sim A^{1/3}$) combinations $\nu j, l \leftrightarrow \pi j', l'$ that can contribute to the $2\nu\beta\beta$ decay matrix element. The above argument still holds in that for each of them the two higher-lying mixed intermediate states will have a tendency to cancel. The $\sim A^{1/3}$ "allowed" intermediate states will mix through the residual interaction. Only detailed calculation can tell whether this relatively small number of significant states will be further reduced.

It should be stressed, however, that our arguments deal only with the rate of convergence in Eq. (1). Even if only a few low-lying 1^+ states contribute significantly to the matrix element, this does not mean that these states are simple configurations like those in Eqs. (12) and (13). Indeed, these states, as well as the ground states of the initial and final nuclei, are complex superpositions of the states of valence nucleons. In addition, the correlations which create the high-energy GT strength also influence the low energy transitions, since they provide one of the mechanisms for renormalization of the axial vector coupling constant in nuclei.

In conclusion, we have presented qualitative arguments suggesting that the considerable β^- and β^+ GT strength at high energies will not contribute noticeably to the double beta decay. Moreover, our arguments carry over to the low-lying states of valence nucleons and explain, albeit only qualitatively, why the matrix element in the extensive shell model calculation of ^{48}Ca gets an overwhelming contribution from a small fraction of low-lying states, and perhaps also why in the decays of ^{100}Mo and ^{128}Te the ground state of the intermediate odd-odd nucleus (which happens to be, by coincidence, 1^+) accounts for the bulk of the matrix element, as recognized a long time ago [14].

ACKNOWLEDGMENTS

This work was done while two of us, M. E. and T. E., were visiting Caltech. We would like to thank Professor Felix Boehm, and Caltech, for their hospitality. This work was supported in part by the U.S. Department of Energy under grant DE-FG03-88ER40397.

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FIGURE CAPTION

Fig. 1. Schematic illustration of the β^- and β^+ transitions involving the two-particle two-hole admixtures in the initial (leftmost panels) and final (rightmost panels) nuclei. The arrows indicate the transitions to the corresponding states in the odd-odd intermediate nucleus. Part a) is for the transitions involving particle states, part b) is for the transitions involving hole states.