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RELATIVISTIC BEAMING AND GAMMA-RAY BURSTS

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1. Introduction

The Burst and Transient Source Experiment (BATSE) on board the Compton Gamma-Ray Observatory has established that the distribution of gamma-ray bursts is isotropic over the sky and bound in the radial direction (Meegan et al. 1992). This indicates that gamma-ray bursts are either cosmological or they reside in an extended Galactic halo (see Paczyński 1993 for a review). In this paper, we will concentrate on the cosmological model, although our analysis can be applied to the Galactic halo model as well.

The peak luminosity required for a typical gamma-ray burst is

$$L_{\text{peak}} \approx 10^{50} \text{ erg s}^{-1} \frac{F}{10^{-7} \text{ erg cm}^{-2} \text{ s}^{-1}} \left(\frac{D}{10^{28} \text{ cm}} \right)^2 \frac{\Omega}{4\pi} \quad (1)$$

where D is the (luminosity) distance to the burster, F is the observed flux and Ω is the solid angle the emission extends. The required peak luminosity is highly super-Eddington for any compact object that can produce variability down to $\sim 200\mu\text{s}$ (Bhat et al. 1992). A relativistic wind is therefore inevitable for the cosmological model (Paczynski 1990). The lack of pair cutoff around 511 keV (Schaefer et al. 1992) further suggests relativistic expansion (Goodman 1986; Paczyński 1986; Fenimore, Epstein, & Ho 1993). The interaction between the relativistic outflow and the external matter has been proposed as a model of gamma-ray bursts (Rees & Mészáros 1992; Mészáros & Rees 1993).

The statistics of gamma-ray bursts was studied for spherically symmetric sources by a number of authors (Dermer 1992; Mao & Paczyński 1992; Piran 1992). They found that the observations can be well explained by adopting standard candles and almost any cosmology. The maximum redshift of bursts that can be detected is around $z_{\text{max}} \approx 1$, and the expected rate is about 1 event per galaxy per million years if the emission is isotropic. However, since the Lorentz factor γ 's of bulk motion are very large ($\gamma \gtrsim 10^2$, Fenimore, Epstein & Ho 1993), some form of relativistic beaming is likely to occur just as in the

context of extragalactic radio sources (e.g. Begelman, Blandford, & Rees 1984). Yi (1993) studied the influence of relativistic bulk motion on the statistics of gamma-ray bursts. He found that with relativistic bulk motion, the maximum redshift of bursts is increased by a factor of $\gtrsim 2$ compared with the unbeamed case. Due to different orientations of the relativistic motion, a luminosity function is naturally produced. One may expect such a luminosity function smooths out the transition in the source count versus any luminosity indicator (e.g., peak count rate) curve and thus may be in conflict with the observation. Another potential problem associated with the pure relativistic beaming model is the rate of bursts required. As the radiation is beamed into a solid angle of $\pi\gamma^{-2}$, only a fraction of bursts are seen and therefore the intrinsic rate of gamma-ray burst has to be increased by a factor of $\sim \gamma^2$. For $\gamma = 10^3$, the rate expected is one event per year per galaxy. This rate is very high compared with the rates of violent events that have been proposed so far to explain the cosmological gamma-ray bursts. Such frequent violent events are difficult to escape detection in various bands and by neutrino detectors.

In this paper, we explore a simple relativistic beaming model in order to examine the constraints on the non-spherical models from the event rate and also from the shape of source count curve. In §2, we present the observational data we use. In §3, we introduce our model and in §4 present our results. Finally in §5 we give a brief discussion and comment on the effects of general luminosity functions.

2. Observational Sample

The first BATSE catalog of gamma-ray bursts (Fishman et al. 1993) provides detailed information for more than two hundred gamma-ray bursts that triggered the BATSE detectors between April 1991 and March 1992. The observed intensity distribution of gamma-ray bursts is distorted by various selection effects. These include the decreasing

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ABSTRACT

Gamma-ray bursts are likely to be relativistically beamed and non-spherical. We study the statistics of gamma-ray bursts by assuming gamma-ray radiation is produced in a cone with a large Lorentz factor. Such a geometry naturally introduces a luminosity function due to different orientations of the cone. The width and shape of the luminosity function depend on the opening angle and the Lorentz factor. The luminosity function generally increases the redshift of the most distant bursts. Large Lorentz factors and opening angles provide sharply peaked luminosity functions which are almost indistinguishable statistically from standard candles. We find it impossible to constrain the opening angle of the cone and in general the width of a wide range of luminosity functions. The opening angle is, however, constrained by the observed rate of gamma-ray bursts and the total rate of their progenitors. In the cosmological context, the opening angle must satisfy $\Delta\theta \gtrsim 10^{-3}/R_i^{1/2}$, where R_i is the rate of progenitors per year per galaxy. This requires the emission to be quasi-spherical for the neutron star merger model and $\Delta\theta \gtrsim 5^\circ$ for the failed supernovae model as proposed by Woosley.

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effects have to be taken into account. Namely, the distance in equation (4) must be substituted by the luminosity distance $D_L(z)$, the frequency is increased by a factor of $1+z$ and an extra factor $1+z$ is added due to the change in bandwidth (e.g. Weinberg 1972)

$$F(\nu, \theta) = \frac{(1+z)R^2}{D_L^2(z)} \int_0^{2\pi} d\phi' \int_0^{\Delta\theta} \sin\theta' d\theta' \cos\Theta \Gamma^3 I_0(\nu(1+z)\Gamma^{-1}), \quad (5)$$

where the luminosity distance is given by $D_L(z) = 2c/H_0(1+z-\sqrt{1+z})$, and H_0 is the Hubble constant. In the above formulation, we have ignored the relative time delay of radiation from different parts of the cone. Under this assumption, the peak flux and peak count rate are simply determined by the peak intensity in the comoving frame. To compare with the observation, we have to calculate the peak count rate in a fixed band ($\nu_1 < \nu < \nu_2$), i.e.,

$$P(\theta) = \int_{\nu_1}^{\nu_2} \frac{F(\nu, \theta)}{\nu} d\nu = \frac{(1+z)^{2-\alpha}}{D_L^2(z)} \times \text{const} \int_{\nu_1}^{\nu_2} \nu^{1-\alpha} d\nu \times R^2 \int_0^{2\pi} d\phi' \int_0^{\Delta\theta} \sin\theta' d\theta' \cos\Theta \Gamma^{2+\alpha}, \quad (6)$$

where the normalization constant is related to the absolute intensity in the comoving frame but is irrelevant in our study. Equation (6) is a product of three terms which are determined separately by the cosmology, spectrum, and geometry. Furthermore, the second term can be treated as a constant as we assume the spectra of gamma-ray bursts are identical. As the cosmological information is contained only in the first term, we can loosely define the last two terms as a local peak count

$$P_{\text{loc}}(\theta) = \text{const} \int_{\nu_1}^{\nu_2} \nu^{1-\alpha} d\nu \times R^2 \int_0^{2\pi} d\phi' \int_0^{\Delta\theta} \sin\theta' d\theta' \cos\Theta \Gamma^{2+\alpha}. \quad (7)$$

Obviously the local peak count rate is a function of the cone orientation and its maximum is achieved when the symmetry axis of the cone is along the line of sight ($\theta = 0$). This maximum can be obtained analytically from equation (7)

$$P_{\text{loc, max}} = \text{const} \int_{\nu_1}^{\nu_2} \nu^{1-\alpha} d\nu \times \frac{2\pi R^2}{\beta^2 \gamma^{2+\alpha}} \left[-\frac{1}{1+\alpha} (x_u^{1+\alpha} - x_l^{1+\alpha}) + \frac{1}{\alpha} (x_u^\alpha - x_l^\alpha) \right], \quad (8)$$

where $x_u = 1 - \beta \cos \Delta\theta$, and $x_l = 1 - \beta$. This maximum offers a natural scale for the local peak count rate and hereafter the local peak count rate will be normalized to this scale.

4. Results

The direction of the symmetry axis of the radiation cone is randomly distributed on the sky, i.e., the probability distribution of Θ is given by

$$p(\theta) d\theta = \frac{1}{2} \sin\theta d\theta, \quad 0 \leq \theta \leq \pi \quad (9)$$

where we assume a one-sided cone jet. The probability distribution for two-sided cones (“twin jets”) is modified by a factor of 2. Different orientations of symmetry axis have different local peak count rates. The probability distributions of the local peak count rate are shown in Figure 2 for three Lorentz factors $\gamma = 10, 10^2, 10^3$, each with three opening angles $\Delta\theta = 0.1^\circ, 1^\circ, 5^\circ$. The probability distribution of the local peak count rate, $p_L(P_{\text{loc}})$, is defined as

$$p_L(P_{\text{loc}}) = p(P_{\text{loc}}) P_{\text{loc}} \ln 10, \quad (10)$$

so that the normalizations are

$$\int_0^1 p(P_{\text{loc}}) dP_{\text{loc}} = 1, \quad \int_{-\infty}^0 p_L(P_{\text{loc}}) d\log P_{\text{loc}} = 1. \quad (11)$$

As can be seen, when the opening angle is small, the distribution is a power law of index $-1/4$, consistent with simple analytical result (cf. Yi 1993). When the opening angle increases the probability at the maximum local peak count rate also increases. The reason is simple: those bursts with the line of sight inside the radiation cone essentially have the same peak count rate and when $\Delta\theta$ increases the probability for the line of sight to fall inside the cone also increases. A model with large Lorentz factor ($\gamma \gtrsim 100$) essentially provides a standard candle-like model when the opening angle is large ($\Delta\theta \gg \gamma^{-1}$) because almost all the bursts with the line of sight within the cone achieves the maximum value. These bursts likely dominate the observed sample because of their large luminosity compared with those bursts falling outside the cone. We can also see that for a given opening angle, the

detection efficiency close to the threshold, non-uniform sky coverage of BATSE and also the effects of atmospheric scattering (Meegan 1993). To minimize these effects, we follow the recommendation of Meegan (1993) by choosing the bursts which have the peak count rate, P , in the 1024ms channel above $0.4 \text{ photons cm}^{-2} \text{ s}^{-1}$. We further require that the bursts could be detected in the 1024ms channel, i.e., the bursts must be above the detection threshold in this channel (a burst can be triggered by any of the 64ms, 256ms and 1024ms channels in the BATSE detectors). After imposing these conditions, we are left with 157 bursts. It is possible to form other subsamples from the first BATSE burst catalog and use a luminosity indicator other than the peak count rate, however our results are little changed and our conclusions remain the same.

3. Theoretical Model

In order to isolate the relativistic effects, we make the simplest assumptions for other parameters in our model. We assume a cosmology with $\Omega_0 = 1$ and zero cosmological constant. Different cosmological models have very small effects on our statistical analysis. The rate of bursts is a constant in the cosmological comoving frame, and the spectra of bursts are simple power laws in photon numbers, $n(\nu) d\nu \propto \nu^{-\alpha} d\nu$. We adopt $\alpha = 2$, i.e., equal energy per logarithmic decade (cf. Schaefer et al. 1992).

In Figure 1, we show the geometry of our model. In the model, the ejecta is flowing outwards relativistically in a cone with opening angle $\Delta\theta$. The observed gamma-ray emission is produced when the radiation becomes optically thin at radius R with the ejecta having a Lorentz factor γ (cf. Rees & Mészáros 1992; Mészáros & Rees 1993). For simplicity, we assume the radiation is isotropic in the comoving frame of the ejecta and has no dependence on the angular positions within the cone. The radiation cone is uniquely defined by the angular spherical coordinates (θ, ϕ) of its symmetry axis, here θ is the angle

between the line of sight (along Z axis) and the symmetry axis and ϕ is the azimuthal angle. Because of cylindrical symmetry, we can assume that the symmetry axis of the cone is in the Y - Z plane. In order to see more clearly the geometrical relations, let us establish an auxiliary coordinate system (x', y', z') with the z' axis along the symmetry axis of the cone and the x' parallel to the X axis. The position of a direction within the cone is then specified by its angular spherical coordinates θ' and ϕ' ($0 \leq \theta' \leq \Delta\theta$, $0 \leq \phi' \leq 2\pi$). It can be shown that the angle Θ between a direction (θ', ϕ') within the cone and the line of sight satisfies

$$\cos \Theta = -\sin \theta' \sin \phi' \sin \theta + \cos \theta' \cos \theta. \quad (2)$$

In the following, we will first calculate the flux received by an observer located very close to the source (local observer) where no cosmological correction is needed. We then calculate the flux received by us by taking the cosmological redshift effects into consideration.

The monochromatic flux received by a local observer at a distance D from the source can be calculated as (cf. Rybicki & Lightman 1979, p. 8)

$$F(\nu, \theta) = \int_0^{2\pi} d\phi' \int_0^{\Delta\theta} \sin \theta' d\theta' I(\nu) \frac{R^2 \cos \Theta}{D^2}, \quad (3)$$

where $I(\nu)$ is the intensity along the line of sight measured by a local observer. Since $I(\nu)/\nu^3$ is a Lorentz invariant (e.g. Rybicki & Lightman 1979), equation (3) can be rewritten using the intensity measured in the comoving frame, $I_0(\nu_0)$, here ν_0 is the corresponding frequency in the comoving frame,

$$F(\nu, \theta) = \frac{R^2}{D^2} \int_0^{2\pi} d\phi' \int_0^{\Delta\theta} \sin \theta' d\theta' \cos \Theta \Gamma^3 I_0(\nu \Gamma^{-1}) \quad (4)$$

where $\beta = \sqrt{1 - \gamma^{-2}}$, $\Gamma = [\gamma(1 - \beta \cos \Theta)]^{-1}$ and we have used the relation $\nu = \Gamma \nu_0$. Notice that the intensity $I_0(\nu_0)$ is related to the photon number spectrum by $I_0(\nu_0) \propto \nu_0 n(\nu_0) \propto \nu_0^{1-\alpha}$. For a cosmological source at redshift z , several redshift

is R_i , then the opening angle must satisfy $\Delta\theta \gtrsim 10^{-3}/R_i^{1/2}$. This requires the emission to be quasi-spherical for the neutron star merger model ($R_i \lesssim 10^{-5}$ per year per galaxy, Narayan, Piran & Shemi 1991; Phinney 1991) and $\Delta\theta \gtrsim 5^\circ$ for the failed supernovae model as proposed by Woosley (1993, $R_i \lesssim 10^{-2}$ per year per galaxy). Attempts to use very narrow jets to avoid significant baryon loading can potentially violate the correspondingly more severe rate requirement. Reprocessing of the initial energy along the line of Rees & Mészáros (1992) and Mészáros & Rees (1993) then becomes vitally important.

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probability to achieve small peak count rate generally decreases when the Lorentz factor increases. This is because the local peak count rate is a steeper decreasing function of Θ for larger Lorentz factors due to smaller beaming angle.

Once the probability distribution of the local peak count rate is obtained, we can convolve the probability distribution with the cosmology term in equation (6) to obtain the probability of peak count rate that would be observed by BATSE (cf. Mao & Paczyński 1992). Each model is described by three parameters, the Lorentz factor, γ , the opening angle, $\Delta\theta$, and the maximum redshift, $z_{\max}$, at which a burst can be detected when the radiation cone is perfectly aligned with the line of sight. We use the Kolmogorov-Smirnov (K-S) test to compare the observed distribution and the predicted distribution. For a given set of Lorentz factor and opening angle the best fit $z_{\max}$ is obtained when the K-S probability achieves maximum. We have computed the best fit models for all the models as shown in Figure 2. Surprisingly, all the models are consistent with the observed distribution with very high K-S probabilities ($\approx 80\%$). The only differences between models are the maximum redshifts. For example, for $\gamma = 10$ we have $z_{\max} = 1.3, 1.1$ for $\Delta\theta = 0.1^\circ, 5^\circ$ respectively. And for $\gamma = 10^3$, $z_{\max} = 0.95, 0.75$ for $\Delta\theta = 0.1^\circ, 5^\circ$ respectively. For the limiting case of $\Delta\theta = 0^\circ$, $z_{\max} = 1.3$. As a general rule, for a fixed opening angle $z_{\max}$ increases as γ decreases and for a fixed Lorentz factor $z_{\max}$ also increases as $\Delta\theta$ decreases. The luminosity functions become more sharply peaked for larger γ 's and the statistical significance of peaks at the bright ends of the luminosity functions rises as $\Delta\theta$ increases (Figure 2). That is, bursts behave more like standard candles for larger γ 's and larger $\Delta\theta$'s, and as a result $z_{\max}$ decreases. The case for $\gamma = 10^3$ and $\Delta\theta = 5^\circ$ has the same $z_{\max}$ ($= 0.75$) as that for a standard candle case. In Figure 3, four theoretical cumulative probability distributions of the peak count rate are shown along with the observed distribution. The theoretical models are almost identical and are all consistent with the observations. This means the observed distribution of peak count rate can not be used to limit the opening

angle of the radiation cone.

5. Discussion

The model presented here is highly simplified. However the inability to constrain the parameters for even such a simple model excludes the necessity to explore more complicated models.

The luminosity functions studied in this paper are naturally produced in our model by different orientations of the radiation cone. We find using the current sample it is impossible to constrain the luminosity functions and hence the parameters responsible for such luminosity functions. This prompts us to check whether it is possible to constrain any luminosity function at all. We have tried different luminosity functions (e.g., power-law and Gaussian) and find that the only effect of the luminosity functions have is to change the maximum redshift of the bursts that can be detected. Source evolution, different spectral index etc. have a similar effect. The shape of the source count curve and a luminosity indicator (e.g. peak count rate) can usually be made in agreement with the current observed distribution. This result is not only because the the observed sample size is small and the observed flux range is narrow, but also because the probed redshift ($z_{\max} \approx 1$) is too small to allow significant cosmological volume effects to take place. The ambiguity may remain even when the BATSE sample is substantially increased (as it will be) unless we can probe a larger redshift range.

However, the opening angle is constrained by the observed rate of gamma-ray bursts and the total rate of their progenitors. A rate of 10^{-6} per year per galaxy is typically required by assuming spherical symmetry in the cosmological scenario (e.g., Mao & Paczyński 1992). Therefore if the radiation is beamed into an opening angle of $\Delta\theta$, only a fraction $\pi\Delta\theta^2$ of bursts can be observed. If the intrinsic rate of bursters in a galaxy

Fig. 1

Fig. 2

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Fig. 1.— Geometry of the model. The gamma-ray radiation is produced by the relativistically expanding material with Lorentz factor γ at radius R and in a cone with opening angle $\Delta\theta$. The line of sight is along the Z axis. The angle between the line of sight and the symmetry axis of the cone is θ . Without losing generality, the symmetry axis of the cone is assumed to be in the Y-Z plane.

Fig. 2.— Logarithm of the probability distribution $p_L(P_{\text{loc}})$ versus logarithm of P_{loc} . The local peak count rate is defined in equation (7), and is normalized to its maximum (cf. equation [8]). The bottom panel is for the Lorentz factor $\gamma = 10$, the middle panel is for $\gamma = 10^2$, and the top panel for $\gamma = 10^3$. Three opening angles are plotted for each Lorentz factor (from bottom to top), $\Delta\theta = 0.1^\circ, 1^\circ, 5^\circ$. The photon number index is assumed to be $\alpha = 2$ (equal energy per logarithmic decade). A dashed line with the analytical power law index of $-1/4$ is shown for $\gamma = 10$. Note that for $\gamma = 10$, the curves for $\Delta\theta = 0.1^\circ$ and $\Delta\theta = 1^\circ$ are almost identical.

Fig. 3.— The cumulative probability distribution of peak count rate versus logarithm of peak count rate. The solid line is the observed distribution. The dashed lines are for four theoretical models with Lorentz factors $\gamma = 10, 10^3$ and each with two opening angles of 0.1° and 5° . They are indistinguishable from each other. The Kolmogorov-Smirnov test shows that the theoretical models are consistent with the observation at a confidence level of $\approx 80\%$. The peak count rate has a unit of photons $\text{cm}^{-2} \text{s}^{-1}$.