NON-HERMITIAN YUKAWA COUPLINGS?

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Abstract

We investigate the possibility of having a non-Hermitian structure for Yukawa couplings. In particular, we suggest a non-Hermitian ansatz, which is in agreement with all experimental data and is not equivalent to any of the Hermitian Yukawa matrices with texture zeros, recently classified by Ramond, Roberts and Ross. This shows that a complete classification of Yukawa couplings with texture zeros has to include non-Hermitian structures.

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^{+) Work supported in part by Science Project No. SCI-CT91-0729 and by the EC contract No. CHRX-CT93-0132.}

The Yukawa sector of the standard model (SM) consists of three 3×3 complex matrices; it is the source of most of the arbitrary parameters of the SM. With the increasing accuracy reached in the determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one is tempted to extract from the data some clue about the correct structure of the Yukawa couplings. The hope is that finding the phenomenological pattern for Yukawa couplings may in turn provide a hint about the symmetry principle behind the mechanism of fermion mass generation. Most of the previous attempts, including a recent systematic analysis by Ramond, Roberts and Ross [1], assume that the matrices of Yukawa couplings are Hermitian or symmetric. However, in the context of the SM, or even SU(5), Yukawa couplings need not be either symmetric or Hermitian, and therefore it is worth while to complement the previous analyses by looking for patterns of Yukawa couplings that are not necessarily of that kind.

In the present letter, we search for a phenomenologically viable pattern for the Yukawa couplings, without assuming whether the Yukawa matrices are Hermitian or symmetric, as in [1], but working in the framework of a specific weak basis, namely the nearest-neighbour-interaction (NNI) basis [2]. In particular we will suggest an ansatz for the Yukawa couplings of the quark sector, which is in agreement with all the experimental data on quark masses and mixings. We will further show that this non-Hermitian (n.H.) ansatz is not equivalent to any of the Hermitian Yukawa matrices with texture zeros, recently classified in Ref. [1].

The task of extracting the pattern of Yukawa couplings of the SM from the observed values of quark masses and mixings is rendered specially difficult by the fact that Yukawa couplings contain a large number of unphysical parameters. Indeed from the 36 parameters contained in the two 3×3 complex Yukawa matrices of the quark sector, only 10 parameters are physical, corresponding to the six quark masses and the four parameters characterizing the CKM matrix.

Let us write the Yukawa couplings as:

$$L_y = Y_d \bar{Q}_L d_R \Phi + Y_u \bar{Q}_L u_R \tilde{\Phi} + h.c., \quad (1)$$

where we have omitted family indices. In Eq. (1), $(Q_L)_i$ stand for the three left-handed quark doublets, $(u_R)_i$, $(d_R)_i$ denote the right-handed quark singlets. The Yukawa couplings $Y_{d,u}$ contain redundant information since one can make a change of weak basis under which

$$\begin{aligned} Q_L &\rightarrow W_L Q_L \\ d_R &\rightarrow W_R^d d_R \\ u_R &\rightarrow W_R^u u_R, \end{aligned} \quad (2)$$

where W_L, W_R^d, W_R^u are 3×3 unitary matrices acting in family space. The transformations of Eq. (2) leave the rest of the Lagrangian invariant, while the Yukawa couplings transform as:

$$\begin{aligned} Y_d &\rightarrow Y'_d = W_L^\dagger Y_d W_R^d \\ Y_u &\rightarrow Y'_u = W_L^\dagger Y_u W_R^u. \end{aligned} \quad (3)$$

The NNI Basis

We will now address the question of whether there is a phenomenologically viable ansatz, which is not related through a weak-basis transformation of Eq. (3) to any of the

Yukawa structures classified in Ref. [1]. In order to simplify our task, we will work in a specific basis, the NNI basis.

It has been shown [2] that in the context of the SM, starting from arbitrary complex Yukawa couplings Y_d, Y_u , one can make a weak-basis transformation such that in the new basis Y'_d, Y'_u have the form:

$$Y'_d = \begin{bmatrix} o & a' & o \\ a & o & b' \\ o & b & c \end{bmatrix}, \quad Y'_u = \begin{bmatrix} o & d' & o \\ d & o & e' \\ o & e & f \end{bmatrix}. \quad (4)$$

Therefore, the NNI form of Eq. (4) reflects a choice of weak basis and does not imply any loss of generality for the Yukawa couplings. More precisely, the NNI form can accommodate any values that are chosen for quark masses and mixings. On the other hand, if one assumes both hermiticity and the NNI form, one obtains the well-known Fritzsch ansatz [3], which does imply correlations between quark masses and mixings.

The difficulty with the Fritzsch ansatz [F.A.] has to do with the fact that it leads to the relations,

$$|V_{us}| = |(m_d/m_s)^{1/2} + e^{i\phi}(m_u/m_c)^{1/2}| \quad (5a)$$

$$|V_{cb}| = |(m_s/m_b)^{1/2} + e^{i\sigma}(m_c/m_t)^{1/2}|, \quad (5b)$$

which predict a correct value for $|V_{us}|$ but lead to too large a value for $|V_{cb}|$. Indeed in order for Eq. (5b) to predict a correct value for $|V_{cb}|$ a substantial cancellation is required between the two terms contributing to $|V_{cb}|$ in Eq. (5b). This in turn requires [4] $m_t < 50$ GeV, in conflict with the present experimental bound [5] $m_t > 113$ GeV. One concludes that experimental data on quark masses and mixings imply that at least one of the Yukawa matrices Y_d, Y_u is non-Hermitian when written in the NNI basis.

In order to motivate the construction of an alternative ansatz for the Yukawa couplings we will analyse next some special features of this basis. Let us consider the matrices H_u and H_d defined by:

$$H_u = M_u M_u^\dagger, \quad H_d = M_d M_d^\dagger, \quad (6)$$

where $M_{u,d} = \langle \Phi^0 \rangle Y_{u,d}$. The matrices $H_{u,d}$ are diagonalized by the unitary transformations:

$$U_d^\dagger H_d U_d = D_d \\ U_u^\dagger H_u U_u = D_u, \quad (7)$$

where $D_d = (m_d^2, m_s^2, m_b^2), D_u = (m_u^2, m_c^2, m_t^2)$. The CKM matrix is then given by $V = U_u^\dagger U_d$. Note that when the Yukawa couplings are written in the NNI basis, it follows from Eq. (4) that:

$$(H_d)_{12} = (H_u)_{12} = 0, \quad (8)$$

which in view of Eq. (7) give:

$$m_u^2 (U_u)_{11} (U_u)_{21}^* + m_c^2 (U_u)_{12} (U_u)_{22}^* + m_t^2 (U_u)_{13} (U_u)_{23}^* = 0 \quad (9a)$$

$$m_d^2 (U_d)_{11} (U_d)_{21}^* + m_s^2 (U_d)_{12} (U_d)_{22}^* + m_b^2 (U_d)_{13} (U_d)_{23}^* = 0. \quad (9b)$$

Given the hierarchy of quark masses, Eqs. (9a) and (9b) imply:

$$|(U_u)_{13}| \cong \frac{m_c^2}{m_t^2} \frac{m_c}{m_t} \frac{|(U_u)_{22}|}{|(U_u)_{23}|} \quad (10)$$

$$|(U_d)_{13}| \cong \frac{m_s^2}{m_b^2} \frac{m_s}{m_b} \frac{|(U_d)_{22}|}{|(U_d)_{23}|}.$$

In view of the experimental values of $|V_{us}|$ and $|V_{cb}|$, one is tempted to look for an ansatz leading to:

$$(U_d)_{12} = C_d \left(\frac{m_d}{m_s} \right)^{1/2}, \quad (U_d)_{23} = C_d \left(\frac{m_s}{m_b} \right), \quad (11)$$

$$(U_u)_{12} = C_u \left(\frac{m_u}{m_c} \right)^{1/2}, \quad (U_u)_{23} = C_u \left(\frac{m_c}{m_t} \right),$$

where $C_{u,d}$, $C'_{u,d}$ are of order 1. With relations (11) one would keep the successful prediction of Eq. (5a) and avoid the difficulty encountered in Eq. (5b). An ansatz written in the NNI basis, where Eqs. (11) are satisfied, would automatically lead to the relations

$$(U_d)_{13} = \left(\frac{C_d}{C'_d} \right) \frac{m_s}{m_b} \frac{|(U_d)_{22}|}{|(U_d)_{23}|} \quad (12)$$

$$(U_u)_{13} = \left(\frac{C_u}{C'_u} \right) \frac{m_c}{m_t} \frac{|(U_u)_{22}|}{|(U_u)_{23}|}.$$

A new Ansatz

Next we will construct an ansatz where the relations of Eqs. (11) hold. We propose the following non-Hermitian form for the quark mass matrices:

$$M_d = \begin{bmatrix} o & a_d & o \\ a_d & o & b_d \\ o & c_d & c_d \end{bmatrix}, \quad M_u = \begin{bmatrix} o & a_u & o \\ a_u & o & b_u \\ o & c_u & c_u \end{bmatrix}, \quad (13)$$

where $a_{d,u}$, $b_{d,u}$, $c_{d,u}$ are arbitrary complex numbers. At this point, it is worth noting that in the limit where a_u, a_d, b_u, b_d vanish, $m_u = m_d = m_c = 0$, and only m_b, m_t are non-vanishing. In this limit, $V_{CKM} = 1$, which can be seen directly from Eq. (13) by recalling that U_u, U_d are the unitary matrices which diagonalize $H_u = M_u M_u^\dagger$, $H_d = M_d M_d^\dagger$. In the above limit, H_u, H_d are diagonal and have all entries zero, except $(H_u)_{33}, (H_d)_{33}$, corresponding to m_t^2, m_b^2 .

For the present ansatz, the matrices U_u and U_d , defined by Eq. (7), can be written,

$$U_u = K_u O_u, \quad U_d = K_d O_d; \quad (14)$$

K_u, K_d are diagonal unitary matrices, while O_d, O_u are orthogonal matrices, which can be expressed in terms of quark masses as:

$$O_d \cong \begin{bmatrix} 1 & -2^{-1/4}(m_d/m_s)^{1/2} & 2^{-1/4}(\frac{m_d m_s}{m_b^2})^{1/2} \\ 2^{-1/4}(m_d/m_s)^{1/2} & 1 & m_s/m_b \\ -2^{3/4}(\frac{m_d m_s}{m_b^2})^{1/2} & -m_s/m_b & 1 \end{bmatrix}, \quad (15)$$

with an analogous expression for O_u . The CKM matrix is then given by

$$V_{CKM} = O_u^\dagger K O_d, \quad (16)$$

with $K = K_u^\dagger K_d = \text{diag. } (1, e^{ip}, e^{iq})$. The CKM matrix elements are thus expressible in terms of quark mass ratios and the two phases p, q . Using Eqs. (15), (16) one obtains:

$$\begin{aligned} |V_{12}| &\cong \left| 2^{-1/4} \left(\frac{m_d}{m_s} \right)^{1/2} - 2^{-1/4} \left(\frac{m_u}{m_c} \right)^{1/2} e^{ip} - 2^{3/4} \left(\frac{m_u m_c}{m_b^2} \right)^{1/2} \left(\frac{m_s}{m_b} \right)^{1/2} e^{iq} \right| \\ |V_{21}| &\cong \left| -2^{-1/4} \left(\frac{m_d}{m_s} \right)^{1/2} + 2^{-1/4} \left(\frac{m_u}{m_c} \right)^{1/2} e^{-ip} - 2^{3/4} \left(\frac{m_d m_s}{m_b^2} \right)^{1/2} e^{i(q-p)} \right| \\ |V_{23}| &\cong \left| \frac{m_s}{m_b} - \frac{m_c}{m_t} e^{i(q-p)} - 2^{-1/2} \left(\frac{m_u}{m_c} \right)^{1/2} \left(\frac{m_d m_s}{m_b^2} \right)^{1/2} e^{-ip} \right| \\ |V_{32}| &\cong \left| -\frac{m_s}{m_b} + \frac{m_c}{m_t} e^{i(p-q)} - 2^{-1/2} \left(\frac{m_u m_c}{m_b^2} \right)^{1/2} \left(\frac{m_d}{m_s} \right)^{1/2} e^{-iq} \right|. \end{aligned} \quad (17)$$

We have not included the corresponding approximate expressions for $|V_{13}|$ and $|V_{31}|$ since the uncertainties they contain are too large. In any case, in the numerical evaluation of the CKM matrix elements, we have used the exact expressions for O_d, O_u and not the approximate expressions of Eq. (15).

Next we present a representative example of the numerical predictions of the ansatz:

$$|V_{CKM}| = \begin{bmatrix} 0.9748 & 0.2230 & 0.0046 \\ 0.2227 & 0.9741 & 0.0387 \\ 0.0130 & 0.0368 & 0.9992 \end{bmatrix}, \quad (18)$$

Input: $m_u = 5.2 \text{ MeV}, m_c = 1.35 \text{ GeV}, m_t = 162 \text{ GeV}, m_d = 8.9 \text{ MeV}, m_s = 187 \text{ MeV}, m_b = 5.3 \text{ GeV}$.

For the quark masses, we have chosen values within the range allowed by the analysis of Gasser and Leutwyler [6]:

$$\begin{aligned} m_u(1 \text{ GeV}) &\cong 5.1 \pm 1.5 \text{ MeV} \\ m_d(1 \text{ GeV}) &\cong 8.9 \pm 2.6 \text{ MeV} \\ m_s(1 \text{ GeV}) &\cong 175 \pm 55 \text{ MeV} \\ m_c(1 \text{ GeV}) &\cong 1.35 \pm 0.05 \text{ GeV} \\ m_b(1 \text{ GeV}) &\cong 5.3 \pm 0.1 \text{ GeV} \end{aligned} \quad (19)$$

and also in agreement with the predictions for quark masses derived from charged meson and baryon masses:

$$\frac{m_d}{m_u} = 1.76 \pm 0.13, \quad \frac{m_s}{m_u} = 34.5 \pm 5.1, \quad \frac{m_s}{m_d} = 19.6 \pm 1.6 \quad (20)$$

The numerical predictions for $|V_{CKM}|$ given in Eq. (18) are in agreement with the present experimental data and indeed one obtains values that are close to the central values given by experiment. A possible exception is the value of $|V_{ub}/V_{cb}|$, which is near the present upper bound [7]. However, one should keep in mind the theoretical uncertainties [8] in extracting the value of $|V_{ub}/V_{cb}|$ from the measured spectra.

At this point, we should comment on the relationship between the present ansatz and the one proposed by Fritzsch. If one considers the F.A. as the result of a discrete family symmetry imposed at the Lagrangian level, but broken by the vacuum, then there will be finite radiative corrections to the zero-order relations of Eqs. (5a), (5b). By an appropriate weak-basis transformation, these higher-order corrections can always be viewed as small deviations from hermiticity. It has been shown [9] that a heavy top can only be accommodated by the F.A. if radiative corrections are at the 10% level. On the other hand, the present ansatz has the same predictive power as the F.A., but assumes, ab initio, a strong deviation from hermiticity.

Next we show that the n.H. ansatz is not equivalent to any of the Hermitian Yukawa structures considered in Ref. [1]. More precisely, we have to make sure that the Yukawa matrices of Eq. (13) are not related to any of the Hermitian matrices classified in Ref. [1] through a weak-basis transformation defined in Eq. (3). It can be easily seen that this is not possible since the existence of such a weak-basis transformation would imply that, the two ansätze, for a given set of quark mass values, would lead to the same CKM matrix:

$$V_{CKM}^H = V_{CKM}^{n.H.} \quad (21),$$

where H. and n.H. stand for Hermitian and non-Hermitian, respectively. Note that for the n.H. ansatz and for any of the Hermitian ansätze considered in Ref. [1], the CKM matrix is given by:

$$\begin{aligned} V_{CKM}^{n.H.} &= [O_u]^T K O_d \\ V_{CKM}^H &= [O'_u]^T K' O'_d, \end{aligned} \quad (22)$$

where O_u, O_d, O'_u, O'_d are orthogonal matrices and K, K' diagonal unitary matrices. From Eqs. (20), (21), (22), one obtains:

$$|O_u O_u'^T|_{ij} = |O_d O_d'^T|_{ij}. \quad (23)$$

The orthogonal matrices O_u, O_d corresponding to the n.H. ansatz can be expressed in terms of quark mass ratios, indicated in Eq. (15). Similarly, for all the Yukawa textures classified in Ref. [1], O'_u, O'_d can be expressed in terms of quark mass ratios and at most one extra free parameter. This implies that the nine relations of Eq. (23) have to be fulfilled with the help of one free parameter. It can be readily verified that Eq. (23) has no solution, thus showing that the n.H. ansatz is not equivalent to any of the Hermitian Yukawa structures considered in [1].

We have seen that if one chooses to work in the NNI basis, one is naturally led to a non-Hermitian structure for Yukawa couplings. It is worth while recalling that there is at least another example where non-Hermiticity of the Yukawa couplings is a requirement in order to conform to experimental data on quark masses and mixings. This is the case of the universality hypothesis [10] for the strength of Yukawa couplings. It is a remarkable fact that all data on quark masses and mixings are consistent with pure-phase Yukawa couplings [9]:

$$\begin{aligned} (Y_d)_{ij} &= C_d \exp(i\alpha_d^{ij}) \\ (Y_u)_{ij} &= \bar{C}_u \exp(i\alpha_u^{ij}), \end{aligned} \quad (24)$$

with $C_d, \bar{C}_u$ real constants. For this hypothesis, only non-Hermitian $Y_{d,u}$ can lead to a realistic quark mass spectrum. Indeed for Hermitian couplings, Eqs.(24) lead to the sum rule

$$m_1 m_2 + m_2 m_3 + m_1 m_3 = 0, \quad (25)$$

which is obviously violated by experiment. If one allows for non-Hermitian couplings, it has been shown that one can have a realistic quark spectrum and a correct CKM matrix for a large region of the phase parameters [10], [11].

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