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PHYSICS WITH BEAMS OF VIRTUAL PIONS^{*)}

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Abstract

An alternative approach to spin-isospin excitations in various charge exchange reactions is that the projectile is equivalent to a beam of virtual pions. Although equivalent to the standard approaches this view emphasizes the role of the pion and how it is generated inside the nuclear by its source. It clarifies why virtual pions are so much more effective in the exploration of pion modes in nuclei and it gives a natural explanation of coherent pion production as the pionic counterpart to the transition radiation of charged particles in inhomogeneous dielectrics.

There is a very close connection of the physics of spin-isospin excitations to that of nuclear pion physics. This is apparent already from the famous Goldberger-Treiman relation, which relates the axial coupling constant g_A for the spin-isospin operator of the nucleon to the π NN coupling constant to a precision of a few per cent[1]. This connection in nuclei leads naturally to a renormalization and quenching of the axial coupling inside of nuclei as was initially pointed out by Magda Ericson in an approach that now is called the Δ mechanism for quenching, one of the major ways by which the renormalization occurs [2][3]. The understanding of this connection for axial transitions both at low and at high excitations have been much refined since. Once more much of this is due to the Lyon group which has pioneered this. A major reason for this close link is now understood as being to the chiral symmetry constraints, which enter in term of the partially conserved axial current (PCAC) relations in nuclei[4]. Once PCAC is assumed to be a locally exact relation, it follows that the axial current, i.e. the spin-isospin physics, is given by the corresponding matrix elements for a pion even with meson exchange currents included[5]. It is therefore not surprising that the longitudinal spin-isospin response function is a basic tool for exploring the pionic nature of the nucleus and that it should be sensitive to a pionic mode in the nucleus. All of this is incorporated in the standard approaches to this problem, which rely heavily on the Δ hole model for the theoretical description of the response[6] and this approach is now routinely used by many authors. In this sense we know well that spin-isospin response is pion physics. The connection to pions also concerns the probes we use in the forward charge exchange reactions. Just consider the situation for the prototype $np \rightarrow pn$ charge exchange at small angles. If we normalize the 0° cross section (which is nearly constant in the lab. system any way in a huge energy range from a few hundred MeV up to about 10 GeV), the cross sections have a universal forward peaked shape with momentum transfer to percent accuracy, to such an extent that the information from widely different energies undistinguishable[7]. The scale is that of the pion mass. Thus, also the common probes are strongly linked to pion physics. This is all known. Here I ask another question and look at the same physics in a different perspective. Is there an 'easy' way to qualitatively visualize the physics in such processes? Can we readily see that spin-isospin excitations with different charge exchange probes will have universal features and in particular, can one see readily that new features like the recently observed coherent pion production at Saturne[8]? This approach is what I refer to as 'the virtual pion beam', since it has physical features which we do not meet for physical pions[9]. These are hidden features in the usual approach, which is more suitable for quantitative investigations. A plane wave beam e^{iqz} of physical pions has the energy-momentum relation $\omega^2 - q^2 = m_\pi^2$. One can construct a plane wave beam of virtual pions from the pion cloud of a projectile, which virtually dissociates, such as $p \rightarrow p\pi^+$, $^3He \rightarrow ^3H\pi^+$ or $\mu^- \rightarrow \nu\pi^-$. Such processes are dominant in the long range meson cloud, since the pion is light. Now, the energy-momentum relation differs from that for free pions: $\omega^2 - q^2 \neq m_\pi^2$. The role of the projectile is simply that of a source of the virtual pion field, which cannot exist in its absence: the virtual pion 'piggybacks' on the projectile. For the sake of concreteness consider the nucleon pion cloud in the Breit system. The pion is emitted at no cost in energy but with a momentum k . We are only concerned with the case that the pion is emitted along the z -axis, since the transverse momentum transfer vanishes for 0° reactions. The rest frame energy-momentum transfer is $\omega = 0, q = k$. Let us now boost the nucleon into the laboratory frame by giving it a velocity β in the z -direction. The energy-momentum loss in the charge exchange reaction ω, q is now the

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virtual pion energy-momentum: $\omega = \Delta E = \gamma\beta k$; $q = \gamma k$, where $\gamma = (1 - \beta^2)^{-1/2}$. The value for k follows from the relation $k^2 = q^2 - \omega^2 = \omega^2/\gamma^2$. In practice k is rather small. For energy losses corresponding to the region of the Δ isobar excitation k is about 200 MeV/c for a 600 MeV incident nucleon. One notes that for constant energy transfer the rest frame momentum transfer $k \rightarrow 0$ with increasing projectile energy. It is therefore a generic feature of charge exchange reactions that any reduction in the cross sections due to the projectile form factor diminishes and becomes increasingly irrelevant as the projectile energy increases.

With the help of a projectile as a source it is therefore in principle straightforward to construct a plane wave beam of virtual pions. There are of course some problems. The projectile will in general not be a plane wave. It is usually a strongly interacting particle and it will undergo absorption by the target as well as distortions, which must be included in the description. Let us for the moment neglect this point and concentrate on the special character of a beam of virtual pions. Its characteristic feature is that it cannot exist unless it is constantly replenished by a source. In order to have an incident wave $\phi_{inc}(z, \omega) = \exp(iqz)$ the free pion wave equation must have a source term:

$$(\nabla^2 - m_\pi^2 + \omega^2)\phi(z, \omega) = (-q^2 - m_\pi^2 + \omega^2)\exp(iqz) \quad (1)$$

with $(q^2 + m_\pi^2 - \omega^2) = (k^2 + m_\pi^2)$. As we are used to, the right hand side of the equation vanishes, when the pion is a physical one. This simply means that a physical pion needs not to be sustained by a source in order to propagate like a free wave. Let us next consider what happens in an infinite nuclear medium. We can then describe the propagation of a coherent pion wave in the medium in terms of a uniform optical potential $U(q^2)$ dependent on the incident wave number q . Since we know that various processes attenuate the coherent wave, we will assume the potential to have an absorptive part, which corresponds to the physics. We then solve the inhomogeneous wave equation

$$(\nabla^2 - m_\pi^2 + \omega^2 + 2\omega U(q))\phi(z, \omega) = (-q^2 - m_\pi^2 + \omega^2)\exp(iqz), \quad (2)$$

which has the solution

$$\phi(z, \omega) = \frac{q^2 + m_\pi^2 - \omega^2}{q^2 + m_\pi^2 - \omega^2 - 2\omega U(q^2)} \exp(iqz). \quad (3)$$

In addition there will solutions to the dispersion equation with wave number K corresponding to the spontaneously propagating waves. Since these are damped by absorption, we neglect them for the moment. Instead, there is now a wave with the *incident* wave number q generated throughout the medium. This wave exists only for virtual pions. For real pions it is suppressed by the factor $(q^2 + m_\pi^2 - \omega^2) = 0$, which is the famous Ewald extinction of an incident plane wave within a medium (named after Hans Bethe's father-in-law). Because of the generated wave there now is an absorption probability per unit volume P

$$P = 2ImU(q) \left| \frac{q^2 + m_\pi^2 - \omega^2}{q^2 + m_\pi^2 - \omega^2 - 2\omega U(q)} \right|^2. \quad (4)$$

This means that one can immediately deduce the reaction rate in the medium. Since the absorption processes of the coherent wave of virtual pion is mainly due to either quasi-elastic scattering on nucleons or to real absorption which is a rather short ranged phenomenon for physical pions. We have then excellent reasons to expect that the reaction products will be very similar to those for physical pions as long as the energy transfer stays comparable. Since the denominator will be small if there is a pion mode which approximately fulfills the dispersion relation in the medium we can also conclude at once that there will be a volume response of the medium to the virtual pion beam. This is important, since the strong absorption of physical pions in the Δ region makes them unsuitable for the exploration of the nuclear interior. Here we can by-pass that problem.

Coherent production of physical pions occurs in the interface region between the medium and free space. Consider the virtual pion wave propagating through an infinite half-plane to the left, emerging into free space to the right. We recall once more that the medium is an absorptive one. We must now take into account the reflection of the wave at the surface on the one hand and the fact that real pions will appear in the free region to the right. Matching of the boundary conditions for the problem (to keep notation short, we will not discuss the case of a momentum dependent potentials but this is readily incorporated)

$$\phi_{medium}(z, \omega) = \frac{q^2 + m_\pi^2 - \omega^2}{q^2 + m_\pi^2 - \omega^2 - 2\omega U(q)} (e^{iqz} + \frac{K - q(\omega)}{q + q(\omega)} e^{-ikz}). \quad (5)$$

$$\phi_{out}(z, \omega) = e^{iqz} - \frac{K - q(\omega)}{K - q} e^{iq(\omega)z}, \quad (6)$$

where K is the wave number in the medium and $q(\omega)$ is the vacuum wave number at energy ω of a physical pion. In addition to the wave of virtual pions there is now an additional wave of physical pions with wave number $q(\omega)$. Thus, similarly to the electric transition radiation generated when a charged particle traverses the boundary between two media of different refractive indices, a coherent pion transition radiation is generated in the present case and the physical reason is basically the same one. The missing momentum is furnished by the boundary, i.e., it is furnished by the nucleus and not by the pion momentum distribution, such that it does not reflect in the energy-momentum loss of the projectile carrying the virtual pion beam. Since the virtual beam is physically distinguished from the generated wave, we note that this wave has the amplitude $-(K - q(\omega))/(K - q)$. This means that in the limit of a physical pion the amplitude is -1, i.e., there is then complete shadowing. The emerging beam in the present case will instead correspond to an emission with the same amplitude but the diffractive effects are closely related: according to Babine's principle an absorbing disk and an emitting disk produce the same diffraction pattern. We also note from the expression above that the amplitude for emission varies rather weakly with off-shell nature of the scattering as is also found in detailed calculations[10].

Suppose now that there is a mode for the pion in the medium corresponding to the solution of the pion dispersion relation, i.e., that the energy-momentum transfer corresponds to that of a

pion propagating spontaneously in the medium. Such a collective pion mode in the medium was predicted already some time ago[6]; it has been clearly realized that it is unsuitable to investigate such a mode using physical pions but that it would show up in investigations of response functions using probes such as neutrinos, charge-exchange reactions etc. We may visualize this mode as the solution of the momentum-dependent optical potential dispersion relation, but the concept is far wider. The wave number is in general a complex one $K = K_r + iK_i$, since the wave will be attenuated. From Eq. (4) it follows that the unphysical pion will have the interaction probability enhanced by a factor $|(q^2 - q(\omega)^2)/(q^2 - K^2)|^2$. This pionic mode will show up as a peak in the energy loss spectrum of the projectile which generates the virtual pion beam. Such a peak has indeed been observed [11],[12] and been interpreted in this fashion[13],[14]. It is clear in the approach we have used here, that the wave function is generated within the medium with an enhancement factor such that the unphysical pion in general will be absorbed or scattered. This means that not only the energy loss spectrum, but also the spectrum of those reaction products which escape out of the nucleus should be sensitive to the collective mode and show a corresponding enhancement. We may then ask whether the coherently produced pions will also be enhanced by the mode. The answer to this question is affirmative. We have already seen that physical pions in scattering processes do not probe the pion mode due to the mismatch in energy-momentum. For coherently produced pions in our simple model the coherent forward amplitude is $-(K - q(\omega))/(K - q)$. This amplitude is strongly enhanced as the value of q satisfies the condition for the pionic mode even though the physical pion itself cannot fulfill the resonant condition. The reason is that the enhancement of the wave in the medium survives the mismatch at the surface. This indicates that the variation of the forward cross section for coherent pions can be used as an indicator for the pion mode. Indeed, detailed calculations show such an enhancement [10]. Such calculations must of course incorporate the attenuation and the distortion of the incident beam in the nucleus, i.e., of the pion source.

The physics of a virtual pion beam stands out particularly clearly in the case of forward neutrino reactions. Consider a neutrino induced reaction $\nu + N \rightarrow l + X$, where N is a nuclear or a hadronic target, l is a lepton (e or μ) and X is the specific final reaction products of the reaction, which can be anything from individual nuclear states, complicated emissions of nucleons or states with pions whether coherently or incoherently produced. Any one of these specific processes can be related to the corresponding reactions $\pi(q, \omega) + N \rightarrow X$ induced by a virtual plane pion wave carrying the same energy and momentum as the energy-momentum transfer. More precisely: Adler's theorem[15] states that the weak interaction matrix element M for such a forward neutrino reaction is exactly proportional to the corresponding T-matrix element for the virtual pion-induced reaction.

$$M \propto \frac{G_W}{q_0} T(\pi(q) + N \rightarrow X), \quad (7)$$

where $G_W \simeq 10^{-5} M^{-2}$ is the usual weak interaction coupling constant. Only weak and general assumptions go into this theorem. It assumes on the one hand that the divergence of the axial current is proportional to the pion field (PCAC)[16], which is natural in chiral models also in nuclear physics, and on the other hand that the lepton mass is negligible, an approximation

that is increasingly accurate with increasing neutrino energy. It is therefore evident that O^+ lepton charge exchange is exactly equivalent to the physics of a virtual pion beam with $\omega = q$. In this sense these neutrino reactions are the best possible example of how such virtual beams arise: the neutrino can be viewed as virtually decomposing into a pion and lepton and it is this pion that is responsible for the virtual pion beam. The trouble with the neutrino reactions is the very small cross sections on the one hand and the poor resolution on the other one which presently make the neutrino a very exotic tool for exploring the physics of virtual pions. This situation may change in the future with the advent of kaon factories. There are still several aspects of the situation for neutrinos that merit further reflection. First, there is the surprising situation that neutrino reactions can be proportional to the physics of a strongly interacting particle, which in the physical region has a cross section proportional to $A^{2/3}$, while neutrinos freely penetrate even a heavy nucleus and has interactions proportional to the nuclear volume or A . The explanation was given by Bell[18]. As we have seen above the virtual beam generates a non-vanishing wave throughout a large uniform medium and this wave produces pionic processes proportional to the volume. This means that there is no conflict between weakness of neutrino interactions and their pionic nature. The second interesting statement of the theorem is that the neutrino processes at small energy transfers are also described explicitly by a pion off-the-mass-shell amplitude and that therefore those reactions are pure pion physics. For small space-momentum transfers in such reactions the allowed impulse approximation is a good one. The axial transition interaction is

$$< 0 | M^{(+,-)} | n > \propto \sigma_{lepton} < 0 | A^{(+,-)} | n > \simeq \sigma_{\tau_1 \tau_1^{(+,-)}} < 0 | \sigma_1 \tau_1^{(+,-)} | n >, \quad (8)$$

i.e., the nuclear transition is given by the unretarded Gamow-Teller operator. It is of course well-known that the axial transition operator in nuclei is intimately linked to the nuclear pion source function via the hypothesis of axial locality[5], but we see here this statement realized very transparently in a special case.

While the Adler theorem and its physics applies to high energy neutrino reactions, a somewhat reminiscent situation is associated with the physics of the nuclear capture of stopped muons from the $1s$ orbit of a muonic atom. This is the muonic counterpart of the K-capture of an electron, although for muons the energy available is 106 MeV. The muonic wave function is determined by the extended Coulomb potential. In practice that means that it is nearly constant over the nucleus, which is the only point of relevance for the present discussion. The muon lifetime in this state is long and it is in a heavy nucleus mostly determined by the absorption by the weak interaction processes. These are dominated by capture due to the axial current coupling, strongly related to pion physics and the excitation of nuclear Gamow-Teller states as has been known for a long time. The muon being a weakly interacting particle penetrates freely throughout the nucleus and its wave function is not distorted by strong interactions. However, the muon surrounds itself for a small fraction of the time by a cloud of virtual pions for which it acts as a source. We can view this approximately as produced by the virtual decay of the muon $\mu^- \rightarrow \nu \pi^-$, where the pion has an energy ω corresponding to the energy transfer to the final nucleus. Since the pion wave function is now generated by the constant muon wave function as a source without any attenuation, we have here a particularly clear case of off-the-mass shell

physics. The situation resembles the previous one for neutrinos, but there are some important differences. In order to be precise we recall that the muon capture per unit energy in the limit of vanishing neutrino energy is proportional to the imaginary part of the off-mass-shell π -nuclear scattering length $a_s(\omega)$ and scattering volume $a_p(\omega)$. The s wave capture is associated with the time component of the axial current, the p wave capture with its space component. At this particular kinematical point the free pion wave number is imaginary with $\kappa = (m_\pi^2 - \omega^2)^{1/2}$ and $\omega = m_\mu$ and there is no external space momentum transfer. As before the incident wave generated in the off-mass-shell scattering is normalized as $\exp(i\mathbf{q}\cdot\mathbf{r})$, i.e., to unity in the present case. Consider first the case of a point nucleus. Therefore in this case the value for the off-shell s wave scattering length is

$$a_s(\omega) = \frac{a_s}{1 + \kappa a_s}; \quad \text{Im} a_s(\omega) = \frac{\text{Im} a_s}{|1 + \kappa a_s|^2}, \quad (9)$$

which is a well known result, when the finite size of the nucleus can be neglected. Let us now see what happens for a large nucleus, the interior of which can be treated as an infinite nuclear medium[19]. Although it is not really essential, I will for specificity and transparency of argument assume that the π -nucleus interaction is sufficiently well described in terms of a standard low energy optical potential of the schematic type:

$$2uU = -\nabla \cdot \chi \nabla + Q \quad (10)$$

with the corresponding wave equation including the source term:

$$\nabla \cdot (1 - \chi) \nabla \phi + Q\phi + (\omega^2 - m_\pi^2)\phi = \omega^2 - m_\pi^2. \quad (11)$$

The basis for this potential is that the interaction in the medium, whether for elastic scattering or for absorptive processes, are local s and p wave processes and the absorption is described by the imaginary part of χ and Q . As long as the unphysical pion has an energy ω not too far below the physical pion mass and for momenta that are not large this description is a good approximation. We recall that the local interaction Q is repulsive, while the velocity dependent one in χ is attractive. In nuclear matter the generated wave function is a constant. Since $\nabla(\text{constant}) = 0$, the generated wave is

$$\phi_0 = \frac{\omega^2 - m_\pi^2}{\omega^2 - m_\pi^2 + Q}. \quad (12)$$

The remarkable result is therefore that the momentum-dependent terms are irrelevant and that in this case there is *no* local p wave interaction in the medium. The local absorption rate in the medium per unit volume is

$$P = 2\text{Im}U(q^2 = 0) |\phi_0|^2 = \frac{\text{Im}Q}{\omega} \left| \frac{\omega^2 - m_\pi^2}{\omega^2 - m_\pi^2 + Q} \right|^2. \quad (13)$$

The corresponding discussion for the p wave scattering volume a_p is immediate and follows nearly identical reasoning. From these expressions we see at once that the Born approximation holds when the interaction Q is small compared to the characteristic squared wave number $m_\pi^2 - \omega^2$. The reason is that then the external driving force imposes itself completely. The condition for the constant wave in the medium is that the characteristic distance of the wave number in the medium should be smaller than the radius. This means that the range of the Yukawa field about a point source in the medium should stay inside the medium.

We therefore see clearly in the situation that the energy transfer to the nucleus is maximal that the problem of the muon capture in a large nucleus is exactly described by the strong interaction physics of off-mass-shell pions. What about μ -capture into excited nuclear states in the first 10-20 MeV above the ground state? This is the region to which the bulk of the capture takes place and the momentum transfer by the neutrino is typically of the order of 100 MeV. The reaction is dominated by the axial current capture. The description of these processes is in a sense 'trivial' in so far that an impulse approximation on the individual nucleons is an excellent first approximation. The axial current is therefore described by the standard spin-isospin operator with momentum transfer and the predominant excitations are the spin-isospin analogs of the giant dipole resonance. On the other hand, as for neutrino reactions, the axial current can also here be linked directly to the pion source function. The impulse approximation is just the approximation to virtual pion physics of very low energy ω in which the individual nucleons have the usual $\sigma \cdot \nabla$ coupling. The more sophisticated description in terms of axial locality to this situation generates naturally higher order many body terms to the nuclear axial current due to pion propagation in the nuclear environment[20]. These are the terms usually referred to as meson exchange currents, which become predominant in situations of large energy transfer.

It is a generic feature of forward charge exchange reactions that their physics can be viewed as produced by a beam of virtual pions. The characteristic feature of such 'unphysical' pions is that a pion wave is generated by the projectile also inside the nuclear medium. This is a region that is poorly accessible to physical pions for large nuclei. Contrary to physical pions virtual pions are sensitive to the nuclear collective pion mode both via the energy loss spectrum of the projectile and via the yield of reaction products. These will be enhanced when energy-momentum condition of the pionic dispersion equation in the medium is fulfilled. Coherently produced physical pions are a natural feature of such virtual pion beams. They are generated by a mechanism similar to the one which gives rise to the transition radiation which occurs when a charged particle traverses the boundary between two media of different refractive index. In the same spirit, it is unnatural to treat the nuclear spin-isospin states excited by forward charge exchange in a different perspective from the higher excitations in the Δ region with the pion collective state and pion production as well as the associated disintegration phenomena. The natural way of viewing all these exchange reactions, whether for nucleons, neutrinos or heavy ions, is in terms of pion physics. The special approaches in the literature should be viewed as approximations to this general picture. In particular, while one can defend the use of DWBA in the analysis of of nuclear spin-isospin excitations, such an approach obscures the beauty and the generality of the physics. The physics of this situation stands out with special clarity for the neutrino induced reactions. Consequently, such reactions are of great fundamental interest

in spite of the intrinsic experimental difficulties.

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