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Detecting the tail effect in gravitational-wave experiments

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Detecting the *tail* effect in the gravitational radiation generated by inspiralling compact binaries is shown to be achievable in future laser interferometric gravitational-wave experiments. In a further analysis of data once a binary signal has been identified, we shall measure the total mass M of the binary, which enters in factor of the wave tail, by means of optimal signal processing. Detecting the tail will then consist in showing the compatibility of the measured values of M and of the other parameters depending on the two masses of the binary. We find that detecting the tail at the $1 - \sigma$ level necessitates a minimal signal-to-noise ratio ~ 40 -50 in the case of neutron-star binaries, and ~ 15 or less in the case of massive black-hole binaries. This could be achieved in a second generation of detectors next to the LIGO and VIRGO detectors (case of neutron-star binaries), or even in LIGO and VIRGO themselves (case of massive black-holes binaries).

The first *direct* detection of gravitational radiation will probably take place in future gravitational-wave experiments such as LIGO and VIRGO [1,2]. For the moment, the detection of gravitational radiation has only been indirect, thanks to the very precise timing observations of the binary pulsar PSR 1913+16 [3]. Among the best candidate sources for a direct detection of gravitational radiation are binary systems of compact objects (neutron stars or black holes) in their late inspiralling stages of evolution [4]. The number of neutron-star coalescences is expected to be a few per year out to a distance of 100 Mpc [5,6], at which distance LIGO and VIRGO might observe the waves with a signal-to-noise ratio (SNR) ~ 10 [7]. Such a *première* will open a totally new field in astronomy, and will permit to verify some fundamental predictions of general relativity. Often quoted is the possibility of verifying that the waves are of pure helicity two, with no admixture of other spin states.

The purpose of our work (this Letter and the detailed account [8]) is to show that the observations of inspiralling compact binaries will permit also to verify another fundamental prediction of general relativity, which is related with the so-called gravitational-wave *tail* effect. This effect is essentially due to the fact that gravitational radiation propagates on the *curved* background space-time generated by its own source. More specifically, the tail of the radiation results, at lowest order, from the nonlinear interaction between the time-varying quadrupole moment of the source (which generates the linear radiation) and its monopole moment, or total mass M (which generates the background). The tail radiation has the distinctive property ("non-locality" in time) of depending on the

source's dynamics at arbitrary remote instants in the past, anterior to the simply retarded time $t - r/c$. This reflects the fact that gravity propagates not only on the light cone (direct propagation with the speed of light c), but also *within* the light cone (propagation with all velocities less than c). The possibility that the tail radiation could play an important role in the dynamics of inspiralling binaries has been recently investigated in [9], after earlier suggestions in [10,11]. (See [9–11] for references on tails and related nonlinear effects.)

Thus, detecting the tail effect (or its immediate consequences) in future gravitational-wave experiments will provide *direct* evidence that gravity propagates on a curved space-time – that generated by its own source. (Note that an indirect evidence from the observations of the binary pulsar is probably out of reach [9].) Such detection will yield an interesting test of the non-linearity of general relativity in the “gravitodynamics” regime of the theory, involving rapidly varying and strong gravitational fields. This will also provide, as we shall see, an independent measurement of the total mass M of the source.

The tail effect arises at the post-Newtonian order c^{-3} beyond the usual quadrupole radiation. Let us consider the gravitational wave emitted by a general isolated source, at a large distance r from the source (neglecting terms that die out like $1/r^2$). More precisely, we denote by $h(t)$ that linear combination of the components of the wave which is directly felt by some detector (e.g., $h(t)$ is the relative variation of the arm's length of a laser interferometric detector). Then the expression of $h(t)$, including all terms in the post-Newtonian expansion up to the order c^{-3} , can be written as [11]

$$h(t) = h_0(t) + \frac{2GM}{c^3} \int_{-\infty}^t dt' \left[\ln \left(\frac{t-t'}{2b} \right) + \frac{11}{12} \right] \frac{d^2 h_0}{dt'^2}(t'). \quad (1)$$

This expression is valid for any slowly-moving source, independently of the strength of its internal gravity [11]. The first term $h_0(t)$ denotes the usual multipolar radiation up to the order c^{-3} , and can be referred to as the “front” of the wave. The second term in (1) is the tail of the wave, which is the first purely nonlinear contribution in the wave. The tail depends on two constants: the total mass-energy M of the source, and a constant b having the dimension of time. The constant b can be chosen at will; it is defined by the relation $t = t_H - (2GM/c^3) \ln(r/cb)$ linking the time t used by the experimenters at distance r from the source and the time t_H of an harmonic coordinate system covering the source.

The Fourier transform $\tilde{h}(\omega)$ of $h(t)$, where $\omega = 2\pi f$ denotes the angular frequency, can be computed in terms of the Fourier transform $\tilde{h}_0(\omega)$ of $h_0(t)$. The result is particularly simple, and reads as [9]

$$\tilde{h}(\omega) = \tilde{h}_0(\omega) \left\{ 1 + \frac{2GM}{c^3} \left[\frac{\pi}{2} |\omega| + i\omega \ln(2|\omega|b') \right] \right\}. \quad (2)$$

Note that (2) is valid only for *low* frequencies such that $GM\omega/c^3$ is of small post-Newtonian order c^{-3} . The constant b' is related to b and to Euler's constant $C = 0.577\dots$ by $b' = b \exp(C - 11/12)$. In this Letter, we shall choose the value $b' = 1/2\omega_s$, where ω_s is the “seismic cutoff” frequency of a laser interferometric detector, below which the seismic noise prevents detection.

The brackets in (2) show the relative corrections which are induced, in the Fourier domain, by the wave tail. Since they involve both a real and a imaginary part, these corrections imply a modification of both the *amplitude* and the *phase* of the wave front $\tilde{h}_0(\omega)$. They also imply, in the case of an inspiralling binary, corresponding modifications in the time domain of the instantaneous amplitude and phase of the binary wave front $h_0(t)$. As the frequency $\Omega(t)$ of $h_0(t)$ increases, these tail-induced modifications of the amplitude and the phase grow like $\Omega(t)$ and $\Omega(t) \ln \Omega(t)$, respectively. Thus, it appears to be important to include the tail effect in the construction of the matched filters of inspiralling binaries, so as to ensure the most accurate possible detection [9].

Matched filtering is the appropriate technique for extracting the binary signal out of the detector noise. It consists in correlating the output of the detector, containing the signal $h(t)$ superposed with some Gaussian noise, with a filter $q(t)$ whose Fourier transform is

$$\tilde{q}(\omega) = k \tilde{h}(\omega) / S_h(\omega) . \quad (3)$$

In (3), $\tilde{h}(\omega)$ is the Fourier transform of the signal, $S_h(\omega)$ is the power spectral density of the noise, and k is an arbitrary real constant (see e.g. [4] for a review). The SNR obtained after matched filtering is the best achievable with a linear filter. It reads as

$$\rho = \left(\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{|\tilde{h}(\omega)|^2}{S_h(\omega)} \right)^{1/2} . \quad (4)$$

This SNR has contributions only from frequencies belonging to the bandwidth $[\omega_s, \omega_u]$ in which the power spectral density of the noise is small.

As we see, the matched filtering technique necessitates beforehand the knowledge of the signal. In practice, however, only the *form* of the signal is known (with some precision), and some unknown parameters, such as the two masses of the binary, are to be measured. This is accomplished by maximizing the correlation with a whole family of filters (3), corresponding to different values of the parameters. The parameters of the filter maximizing the correlation are the “measured” parameters attributed by the experimenters to the real signal. These parameters do not exactly agree with the real signal parameters, since they depend on a particular realization of noise in the detector. However, their statistical distribution over a large number of realizations of noise in a large ensemble of detectors is Gaussian (for Gaussian noise and for high enough

SNR), and centred on the signal parameters, with computable variances and correlation coefficients.

This consideration assumes, of course, that the filters are accurately matched, by (3), on the signal. If this is not the case, the expectation values of the measured parameters will disagree with the real signal parameters [12]. Thus, the inclusion of post-Newtonian correction terms in the filters, such as the tail-induced terms in (2), permits a more accurate determination of the signal parameters (and permits also the determination of new parameters). Note that the inclusion of a small correction term improves the value of the maximum SNR by a quantity of the order of the *square* of this term, which is in general negligible (see e.g. the appendix of [8]). Thus, if one wants a high SNR but accepts a poor determination of the parameters, one can use a simpler filter associated with the lowest-order radiation. This is what will be done in a preliminary on-line data analysis aiming at *searching* for the signal.

The wave front $h_0(t)$ or $\tilde{h}_0(\omega)$, generated by a binary, depends on the two masses m_1 and m_2 of the binary in a somewhat involved way. At the quadrupolar level, $\tilde{h}_0(\omega)$ depends on m_1 and m_2 through the so-called *chirp mass*, $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ [4]. At higher multipolar levels, corresponding to higher post-Newtonian orders, $\tilde{h}_0(\omega)$ depends on other combinations of the masses. However, inspection of the expression of $\tilde{h}_0(\omega)$ at the post-Newtonian orders c^{-1} , c^{-2} and c^{-3} [13] suggests to write these combinations as combinations only of the above chirp mass $\mathcal{M}$, of the reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$, and of the difference $\delta m = m_1 - m_2$ between the two masses of the binary. Thus, we can view the wave front $\tilde{h}_0(\omega)$ as parametrized by the three parameters $\mathcal{M}$, μ and δm , which of course are more than sufficient to determine the two separate masses m_1 and m_2 .

The tail-induced modifications of the wave front $\tilde{h}_0(\omega)$ in (2) now bring in the total mass $M = m_1 + m_2$ of the binary as a new parameter, which evidently is not independent of the parameters of the wave front since it is equal, for instance, to $\mathcal{M}^{5/2} \mu^{-3/2}$. In order to detect the tail effect, we propose, first, to correlate the output of the detector with the family of filters (3) in which $\tilde{h}(\omega)$ includes the tail-induced modifications given by (2), and, second, to maximize the correlation by varying the mass M in factor of the tail *independently* of the parameters of the wave front, i.e. for instance $\mathcal{M}$, μ and δm . (Thus, we assume that, in the filters, the values of M and the combination $\mathcal{M}^{5/2} \mu^{-3/2}$ do not a priori agree.) In this way, the measurement of M will permit a *test* of the existence of the tail effect. Indeed, if the tail effect exists, the best filter of the family, corresponding to the maximum correlation, will find a value of M which is compatible with the measured values of the other parameters (i.e., which is approximatively equal to $\mathcal{M}^{5/2} \mu^{-3/2}$). On the contrary, if the tail effect does *not* exist, the best filter of the

family will find a value of M which is compatible with zero, together with some realistic values of the other parameters.

The test could be best represented in the m_1, m_2 plane of the two masses of the binary, in a way somewhat similar to the test of the existence of gravitational radiation in the binary pulsar PSR 1913+16, where the change in the orbital period $\dot{P}$ of the pulsar, the relativistic periastron shift $\dot{\omega}$ of the orbit, and the red-shift-Doppler parameter γ are plotted in the m_p, m_c plane of the pulsar and its companion [3]. The test would consist in the intersection at *one* single point in the plane m_1, m_2 of four curves corresponding to the measurements of the total mass M which is in factor of the wave tail, and of the three parameters of the wave front $\mathcal{M}$, μ and δm (all the four parameters M , $\mathcal{M}$, μ and δm being independently varied and measured in the filtering process). The curves would be surrounded by $1 - \sigma$ error bars reflecting the uncertainties in the measurement. The intersecting point would give, within these uncertainties, the values of the two separate masses m_1 and m_2 of the binary, as determined by general relativity.

More complicated (or different) tests could also be done. For instance, one could verify that both the real and imaginary parts of the tail-induced corrections in (2) are present in the signal. To do so, it would suffice to introduce in the filters, instead of M , two parameters M_r and M_i in factor of the real and imaginary parts respectively, and to measure M_r and M_i independently.

We shall now determine the level at which the above process can be implemented, i.e. the level at which it is possible to detect the tail effect. For this purpose, it is sufficient to determine the level at which the parameter M in factor of the tail can be attributed, with some confidence, a *non-zero* value. Thus, we need to compute the $1 - \sigma$ error bar, say σ_M , in the measurement of M , and to compare σ_M with the value of M itself. The $1 - \sigma$ confidence level at which the tail effect can be detected is simply the level at which σ_M gets *smaller* than M . To compute σ_M , it is not necessary to include in the wave front $\tilde{h}_0(\omega)$ all the post-Newtonian corrections c^{-1} , c^{-2} and c^{-3} . We thus retain in $\tilde{h}_0(\omega)$ only the usual quadrupole radiation, depending on the chirp mass $\mathcal{M}$. In this case, the wave front $\tilde{h}_0(\omega)$ is given by [14, 15]

$$\tilde{h}_0(\omega) = \mathcal{H} \eta^{-1/2} \omega^{-7/6} e^{i[\omega T + \eta \omega^{-5/3} - \varphi - \pi/4]} \quad (5)$$

(with $\omega > 0$), where the amplitude $\mathcal{H}$ is inversely proportional to the distance of the source, where η is related to $\mathcal{M}$ by $\eta = \frac{3}{4}(4GM/c^3)^{-5/3}$, where T is the instant of coalescence of the binary, and where φ is some constant phase. With (2) and (5), we have four parameters relevant to the construction of the filters (3): three “Newtonian” parameters η , T and φ , and the parameter M in factor of the tail.

Some simple models of noise in the detector are then used. We assume first that the spectral density $S_h(\omega)$ of the noise is infinite outside the bandwidth $[\omega_s, \omega_u]$, where ω_s ,

is the “seismic” cutoff frequency to which b' in (2) has been related above. Inside the bandwidth, we assume that the noise is either white,

$$S_h(\omega) = \text{const} , \quad (6)$$

or colored in the sense appropriate for shot noise in the standard recycling configuration of a laser interferometric detector (see e.g. [4]),

$$S_h(\omega) = \text{const } \omega_k [1 + (\omega/\omega_k)^2] . \quad (7)$$

The frequency ω_k in (7) is the so-called “knee” frequency. We adopt here the value $\omega_k = 1.44 \omega_s$, which maximizes, all other parameters being fixed, the SNR (4).

The optimal filtering process is now simulated. A signal $\tilde{h}(\omega)$ given by (2) with (5), and depending on a known set of signal parameters, is added to a particular realization of simulated Gaussian noise with spectral density (6) or (7). The resulting noisy data are correlated with a lattice of filters (3) which are matched on the signal (2) with (5), and constructed using the method of [15,16]. By maximizing the correlation, we determine a first measured value of the parameter M , and by repeating the process for a large number of realizations of noise (~ 50), we obtain the whole statistical distribution of the measured values of M . The standard deviation σ_M of this distribution is then easily deduced. Finally, the computation is redone with other signal parameters and the variations of σ_M in terms of these parameters are obtained. It is convenient to express σ_M , for a given type of noise, as a function of the optimal SNR ρ of the signal, given by (4), and of the total mass M itself (no other parameters are needed). The result of the computation, for both the white and colored noises (6) and (7) (where $\omega_s/2\pi = 100$ Hz and $\omega_u/2\pi = 1000$ Hz are used), is presented in figure 1.

As figure 1 shows, the precision σ_M in the measurement of M is a decreasing function of the SNR. This is to be expected since the more signal we have, the more accurate is the determination of M . Note also that for a given type of noise, σ_M is fairly insensitive on the value of M . This means that a lower SNR is needed to detect the tail from a comparatively higher total mass binary. Finally, we see that, for a given SNR, σ_M is larger in the colored noise case than in the white noise case. This results from the fact that the colored noise (7) is relatively narrowband as compared to the white noise (6) (i.e., most of the signal power is extracted in a smaller range of frequencies). Thus, it is more difficult to detect the tail in the colored noise case than in the white noise case.

The minimal SNR required to detect the tail effect from a coalescing binary with total mass M is directly read from figure 1. All signals whose optimal SNR is such that σ_M in figure 1 is smaller than M have sufficient strength to detect the tail. Note that, in practice, we shall compare the (anticipated) value of σ_M in figure 1 not with M , but with some *measured* value of M . However, this does not make any difference if the SNR

is high enough. For a neutron-star binary with total mass $M = 2.8M_{\odot}$, we obtain a minimal SNR ~ 40 -50 in the colored noise case, and ~ 25 -30 in the white noise case. For a black-hole binary with $M = 10M_{\odot}$, we obtain in both cases a minimal SNR which is less than ~ 15 . These results are in good agreement with the results of an analytical computation, based on the theory of the covariance matrix, which is reported, together with the details of the numerical simulation, in [8].

Our conclusion is that the detection of the tail effect is likely to be achievable, at a reasonable rate of occurrence, in future laser interferometric detectors. This conclusion of course relies very much on published estimates of the number of coalescence events [5,6], and on the anticipated sensitivity of future detectors [7,14]. If, following [5], three neutron-star coalescences occur each year out to a distance of 200 Mpc, and if LIGO and VIRGO can detect these events with a SNR ~ 10 [7], then about one event each twenty years will have the required SNR ~ 40 . This rate is too small, and we shall probably have to wait for a second generation of detectors, next to LIGO and VIRGO. For instance, the advanced LIGO detector envisaged in [14] could reach a SNR ~ 40 once each two years. Finally, we notice that even LIGO and VIRGO with their present design could detect the tail effect in the case of the radiation generated by a massive black-hole binary.

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FIG. 1. The standard deviation σ_M in the measurement of the mass M is plotted against the SNR ρ .