CC

GUELPH-MS 94-139

SW 34 19

March 1994

FINITE TEMPERATURE CALCULATIONS IN THE TEMPORAL GAUGE

GEORGE LEIBBRANDT¹

*Department of Mathematics and Statistics, University of Guelph
and Guelph-Waterloo Program for Graduate Work in Physics - Guelph Campus
Guelph, Ontario, N1G 2W1, Canada*

and

MARK STALEY²

*Department of Physics, University of Guelph
and Guelph-Waterloo Program for Graduate Work in Physics - Guelph Campus
Guelph, Ontario, N1G 2W1, Canada*

Mathematical Series 1994-139

ABSTRACT

Working in the temporal gauge $A_0^a(x) = 0$, we develop a general technique for performing perturbative calculations at both zero and finite temperature. The new technique is based on a special version of the unified n_μ^* -prescription for axial-type gauges and on the method of ζ -functions. We apply our technique to pure, one-loop Yang-Mills theory, deriving the complete high-temperature expansion for the self-energy $\Pi_{00}^{ab}(k_0=0, \mathbf{k})$, and the Debye screening length m_{el} . It is shown that the temporal gauge works admirably in all temperature regimes.

PACS: 11.10.Wx, 12.38.Mh

CERN LIBRARIES, GENEVA

P00022957

¹E-mail address: gleibbra@msnet.mathstat.uoguelph.ca²E-mail address: staley@physics.uoguelph.ca

1. Introduction

It seems that the temporal gauge was the first noncovariant gauge to find application in field theory, having been used in the quantization of the Maxwell-Dirac field initially by Weyl in 1929 [1] and a year later by Heisenberg and Pauli [2]. In modern times, the temporal gauge has found numerous other applications [3], notably in the analysis of vacuum tunnelling by instantons [4], in connection with Nicolai maps [5,6] and one-loop thermodynamic potentials [7], as well as in quark-gluon plasma studies [8].

The temporal gauge is a ghost-free axial-type gauge defined by $n \cdot A^a(x) = 0$, where $A_\mu^a(x)$, $\mu = 0, 1, 2, 3$, is a Yang-Mills field, $a = 1, 2, \dots, N^2 - 1$, for $SU(N)$, and n_μ labels a fixed axis in space-time such that $n^2 > 0$, $n^2 = n_0^2 - \mathbf{n}^2$. The class of axial gauges also includes the light-cone gauge ($n^2 = 0$), the pure or homogeneous axial gauge ($n^2 < 0$), and the planar gauge ($n^2 < 0$). Despite the ghost-free nature of these noncovariant gauges, calculations of the corresponding Feynman integrals are known to become tricky at times, especially in the temporal gauge. The main difficulties are caused, of course, by the spurious poles of $(q \cdot n)^{-\lambda}$, $\lambda = 1, 2, \dots$, in the gauge boson propagator, which have to be treated with a meaningful prescription. But decent prescriptions, coupled with reliable techniques, are hard to come by! This fact may explain why progress in the temporal gauge has been modest and why detailed computations at zero and finite temperature remain the exception rather than the rule.

The purpose of this article is to develop a general technique for doing perturbative calculations in the temporal gauge at both zero and finite temperature T . Before going into details, however, we thought it might be instructive to give the reader a rough idea of just how our computation in the temporal gauge fits into the overall scheme of things at finite temperature. To this effect we shall review some earlier papers which are related to the present program and in which the computations are carried out either in the temporal gauge or in the covariant Feynman gauge.

In 1982, Caracciolo, Curci and Menotti employed the principal-value (PV) prescription to evaluate the Wilson loop in the temporal gauge [9]. They discovered that the PV-prescription was ill-suited for the temporal gauge as it did not lead to exponentiation of the time dependence of the Wilson loop operator at zero temperature ($T = 0$). A few

years later, Gaigg and Kreuzer [10] and Gaigg, Kreuzer, Schweda and Piguet [11] also studied Yang-Mills theory in the temporal gauge by using a variant of the so called n_μ^* -prescription (commonly referred to as the Mandelstam-Leibbrandt prescription) [12,13]. They concluded that computations were unfeasible at zero temperature if n_μ was given by $n_\mu = (1, 0, 0, 0)$. We shall show later in this article that their claim was unfounded.

In 1985, Kajantie and Kapusta [8] used the temporal gauge in the imaginary-time formalism to perform finite temperature calculations in QCD. They used the well-known method of introducing the Bose-Einstein distribution function, replacing the Matsubara frequency sum with a contour integral in complex p_0 space. They succeeded in deriving for the two-point function (static limit) the leading terms of the $1/T$ -expansion. However, they did not address the problem of the spurious singularities arising from the $n = 0$ terms in the Matsubara sums ($p_0 \equiv i2\pi nT$, p_0 being an internal momentum and $n = 0, 1, 2, \dots$). The finite temperature case was also studied by Landshoff and James [14] but from a different perspective. As for calculations in the Feynman gauge, we should stress once again the importance of the contour method for finite temperature calculations which played such a vital part in the development of the subject [15]. More recently, Brandt, Frenkel and Taylor employed another technique, known as the ζ -function method [16], to obtain (also in the Feynman gauge) the complete $1/T$ -expansion for the one-loop self-energy in Yang-Mills theory.

The purpose of the present article is, therefore, three-fold:

- (1) To develop, in the temporal gauge $A_0^a(x) = 0$, a general ambiguity-free technique for performing perturbative calculations at both zero and finite temperature T .
- (2) To test the technique in Yang-Mills theory by deriving, in the framework of the imaginary time formalism, the complete $1/T$ -expansion for the self-energy $\Pi_{00}^{ab}(k_0=0, \mathbf{k})$, along with the Debye chromo-electric screening length m_{el} .
- (3) To ascertain the validity of a special version of the unified n_μ^* -prescription designed to handle the double poles of $1/p_0^2$.

The paper is organized as follows. In Section 2, we first review the finite temperature Feynman rules and then derive a special version of the unified-gauge prescription[17] to

handle the spurious double poles of $(p_0)^{-2}$ in the gauge boson propagator. The first part of Section 3 contains a brief discussion of the contour method for evaluating Matsubara sums; in the second part we use the ζ -function method to illustrate a specific finite temperature calculation. Section 4 is devoted to a high-temperature expansion and to deriving the Debye screening length. We summarize our results in Section 5.

2. Feynman Rules and n_μ^* -Prescription

In the imaginary time formalism, the Feynman rules for Yang-Mills theory in the temporal gauge at finite temperature are the same as those at zero temperature, with the following expression for the gauge propagator:

$$G_{\mu\nu}^{ab}(p) = \frac{i\delta^{ab}}{(2\pi)^{2\omega}} G_{\mu\nu}(p), \quad \mu, \nu = 0, 1, 2, 3,$$

$$G_{\mu\nu}(p) = \frac{-1}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{p \cdot n} + \frac{n^2 p_\mu p_\nu}{(p \cdot n)^2} \right], \quad n^2 > 0, \quad \epsilon > 0. \quad (2.1)$$

For $n_\mu = (1, 0, 0, 0)$, the propagator simplifies to

$$G_{ij}(p) = \frac{-1}{p^2 + i\epsilon} \left(-\delta_{ij} - \frac{p_i p_j}{(p_0)^2} \right), \quad i, j = 1, 2, 3,$$

$$G_{00}(p) = G_{0i}(p) = G_{i0}(p) = 0, \quad (2.2)$$

but still contains a spurious double pole at $p_0 = 0$. The presence of this pole is due to the fact that the constraint $A_0^a(x) = 0$ does not fix the gauge entirely: the action is still invariant under time-independent gauge transformations. The finite-temperature Feynman rules also involve the following substitution for loop integrations:

$$\int \frac{d^4 p}{(2\pi)^4} \longrightarrow \frac{i}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 \mathbf{p}}{(2\pi)^3}, \quad (2.3)$$

where $p_0 \equiv i\omega_n$, $\omega_n = 2\pi nT$, with $\beta = 1/T$, $T \equiv$ temperature. Referring to the propagator in Eq.(2.2), we observe that the unphysical double pole at $p_0 = 0$ manifests itself as a singularity in the summation at $n = 0$. The proper treatment of this singularity is one of the objectives of this paper.

As already alluded to in the Introduction, we shall employ a special version of the unified-gauge prescription, devised by one of the authors for the light-cone, temporal and

axial gauge[17]. The unified prescription is an extension of the n_μ^* -prescription[13] which had been developed specifically for the light-cone gauge ($n \cdot A^a(x) = 0$, $n^2 = 0$):

$$\left. \frac{1}{p \cdot n} \right|^{lc} = \lim_{\epsilon \rightarrow 0} \frac{p \cdot n^*}{p \cdot n p \cdot n^* + i\epsilon}, \quad \epsilon > 0, \quad (2.4)$$

where $n_\mu = (n_0, +\mathbf{n})$, $n_\mu^* = (n_0, -\mathbf{n})$, and $n_0 = |\mathbf{n}|$. In the unified-gauge formalism, the form of the prescription for $(p \cdot n)^{-1}$ is identical to the form of Eq. (2.4). Thus,

$$\left. \frac{1}{p \cdot n} \right|_{\text{axial temporal}} = \lim_{\epsilon \rightarrow 0} \frac{p \cdot n^*}{p \cdot n p \cdot n^* + i\epsilon}, \quad \epsilon > 0, \quad (2.5)$$

n_μ and n_μ^* being defined as follows:[17]

$$\begin{aligned} \text{in the pure axial gauge } (n^2 < 0): \quad n_\mu &= (|\mathbf{n}_\perp|, \mathbf{n}_\perp, n_3); \quad n_3 \neq 0; \\ n_\mu^* &= (|\mathbf{n}_\perp|, -\mathbf{n}_\perp, -n_3); \quad n_3 \neq 0; \end{aligned} \quad (2.6a)$$

$$\begin{aligned} \text{in the temporal gauge } (n^2 > 0): \quad n_\mu &= (n_0, \mathbf{n}_\perp, -i|\mathbf{n}_\perp|); \quad n_0 \neq 0; \\ n_\mu^* &= (n_0, \mathbf{n}_\perp, i|\mathbf{n}_\perp|); \quad n_0 \neq 0. \end{aligned} \quad (2.6b)$$

Taking the limit $|\mathbf{n}_\perp| \rightarrow 0$ in the temporal gauge, Eq.(2.6b), we easily deduce from Eqs.(2.5) and (2.6b) the causal prescription

$$\left. \frac{1}{p_0} \right|_{\text{temp}} = \lim_{\epsilon \rightarrow 0} \frac{p_0}{p_0^2 + i\epsilon}, \quad \epsilon > 0, \quad (2.7a)$$

and

$$\left. \frac{1}{p_0^2} \right|_{\text{temp}} = \lim_{\epsilon \rightarrow 0} \frac{p_0^2}{(p_0^2 + i\epsilon)^2}, \quad \epsilon > 0, \quad (2.7b)$$

which is seen to be well defined for $p_0 = i2\pi nT = 0$, i.e. at $n = 0$. It is interesting to note, for comparison, that the principal-value prescription,

$$\left. \frac{1}{p_0^2} \right|_{PV}^{\text{temp}} = \frac{1}{2} \lim_{\epsilon \rightarrow 0} \left[\frac{1}{(p_0 + i\epsilon)^2} + \frac{1}{(p_0 - i\epsilon)^2} \right], \quad \epsilon > 0, \quad (2.8)$$

is not well defined at $p_0 = 0$ since it is singular in $1/\epsilon^2$. Also, in our prescription (2.7), the two poles are located in the second and fourth quadrants of the complex p_0 plane, so that it is possible to perform a Wick rotation to Euclidean space ($p_0 = ip_4$):

$$\left. \frac{1}{p_4} \right|_{\text{temp}} = \lim_{\mu \rightarrow 0} \frac{p_4}{p_4^2 + \mu^2}, \quad \mu > 0, \quad (2.9a)$$

$$\left. \frac{1}{p_4^2} \right|_{\text{temp}} = \lim_{\mu \rightarrow 0} \frac{p_4^2}{(p_4^2 + \mu^2)^2}, \quad \mu > 0. \quad (2.9b)$$

To see how prescription (2.9) works in practice, consider the following zero temperature integral (Euclidean space):

$$I = \lim_{\mu \rightarrow 0} \int \frac{d^{2\omega} p}{(2\pi)^{2\omega}} \frac{1}{(p+k)^2} \frac{p_4^2}{(p_4^2 + \mu^2)^2}. \quad (2.10)$$

The general procedure is to apply exponential parameterization to the denominators, integrate over momentum space and finally integrate over the parameters. The result is:

$$I = \frac{\pi^\omega}{2(2\pi)^{2\omega}} \Gamma(2-\omega) \int_0^1 d\sigma \sigma(1-\sigma)^{1/2-\omega} [\sigma(1-\sigma)k_4^2 + \sigma\mu^2]^{\omega-2} + (\text{finite part}). \quad (2.11)$$

In order to evaluate the remaining integral we may set $\mu = 0$, but keep ω general. The final integral may then be expressed in terms of a beta function which is well defined in the limit $\omega = 2$:

$$I = \frac{1}{8\pi^2} \frac{1}{\omega-2} + (\text{finite part}). \quad (2.12)$$

The integral in Eq.(2.10) has also been investigated by Lee and Nyeo [18] who obtain the same result, but using a different method of integration. Both results disagree with Gaigg and Kreuzer [10] (see also [11]) who argue that the σ -integral in Eq. (2.11) is singular. Their conclusion is based on the corresponding integral in Minkowski space:

$$I = \int \frac{d^{2\omega} p}{(2\pi)^{2\omega}} \frac{1}{(p+k)^2} \left(\frac{p \cdot n^*}{p \cdot n p \cdot n^* + i\epsilon} \right), \quad (2.13)$$

in which n_μ and n_μ^* are analytically continued from their light-cone gauge values. In order to obtain a general expression in terms of n_μ, n_μ^* they are forced to set $\omega = 2$ before computing the parameter integrals, with the result that their integrals are no longer valid for the case $n_\mu = n_\mu^* = (1, 0, 0, 0)$. Inspection of Eq. (2.11) shows very clearly that the difficulty stems from setting $\omega = 2$ before carrying out the final integration.

3. Calculations at Finite Temperature

a. Contour method and Matsubara sums

One of the goals of this paper is to develop a general technique for performing perturbative calculations at finite temperature in the temporal gauge. We shall work exclusively in the imaginary time formalism. A discussion of the real time formalism may be found in James [19]. Since the n_μ^* -prescription was originally devised for zero temperature calculations, care must be taken when applying it to the finite temperature regime. The main

problem at finite temperature is how to handle the spurious poles of the prescription when computing the Matsubara frequency sums.

The usual method of computing the Matsubara sums is to replace them with a contour integral, using the Bose-Einstein distribution function [15]:

$$T \sum_{n=-\infty}^{+\infty} \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} f(p_0=2\pi nTi, \mathbf{p}) = \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{2\pi i} \oint dp_0 f(p_0, \mathbf{p}) \frac{1}{2} \coth\left(\frac{p_0}{2T}\right), \quad (3.1)$$

where

$$\oint dp_0 = \int_{i\infty-\delta}^{-i\infty-\delta} dp_0 + \int_{-i\infty+\delta}^{i\infty+\delta} dp_0. \quad (3.2)$$

But this technique is only valid if $f(p_0, \mathbf{p})$ is regular along the imaginary p_0 axis and contains no poles between the two vertical contours. Thus in the case of the temporal gauge, we have to make sure that all the poles of $f(p_0, \mathbf{p})$, including those arising from the n_μ^* -prescription, are located outside this contour. We may achieve our goal by choosing an appropriate value for ϵ in the n_μ^* -prescription (see Eqs. 2.7a,b). The next step is to close the contours in the right and left halves of the complex p_0 plane. As an example, consider Figs. (1a,b) which show the pole locations for the function

$$f(p_0, \mathbf{p}) = \frac{1}{(p+k)^2} \frac{p_0^2}{(p_0^2 + i\epsilon)^2}, \quad (3.3)$$

which is just the finite temperature extension of the example given in Section 2. The final step consists of computing the residues of the poles inside these new contours. Details for the above example are given in the Appendix.

b. Zeta-function method in the temporal gauge

Although it is possible to use the above method for perturbative calculations, it is not very practical in general. The residues of the prescription poles are tricky to calculate and usually require a subtle cancellation of the $1/\epsilon$ singularities. This situation has prompted us to consider the alternative method devised by Brandt, Frenkel, and Taylor who use zeta functions to compute the Matsubara sums [16]. In this technique, one-loop diagrams are computed by first integrating over the internal momentum and *then* performing the summations (see Eq.(2.3)). Note that the zeta function, given by

$$\zeta(\alpha) \equiv \sum_{n=1}^{\infty} n^{-\alpha}, \quad (3.4)$$

is defined uniquely for *all* $\alpha \neq 1$, including $\alpha \leq 0$ by analytic continuation. Dimensional regularization is employed to handle the ultra-violet divergences, which manifest themselves as poles of the ζ -function:

$$\zeta(1 - 2(\omega - 2)) \simeq -\frac{1}{2(\omega - 2)} + \gamma + O(\omega - 2). \quad (3.5)$$

To illustrate the zeta-function method, we shall consider the integral in Eq.(2.10) again, but generalized to the finite temperature case (Minkowski space):

$$I = \sum_{n=-\infty}^{+\infty} \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{(p+k)^2} \frac{p_0^2}{(p_0^2 + i\epsilon)^2}. \quad (3.6)$$

Substituting $p_0 = ip_4$ and integrating over $\mathbf{p}$, we obtain:

$$\begin{aligned} I &= T \sum_{n=-\infty}^{+\infty} \frac{p_4^2}{(p_4^2 + \mu^2)^2} \frac{\pi^{\omega-1/2}}{(2\pi)^{2\omega-1}} \Gamma(3/2 - \omega) [(k_4 + p_4)^2]^{\omega-3/2}, \quad p_4 = 2\pi nT, \\ &= T \sum_{n=1}^{\infty} \frac{p_4^2}{(p_4^2 + \mu^2)^2} \frac{\pi^{\omega-1/2}}{(2\pi)^{2\omega-1}} \Gamma(3/2 - \omega) \left\{ [(k_4 + p_4)^2]^{\omega-3/2} + [(k_4 - p_4)^2]^{\omega-3/2} \right\}, \end{aligned} \quad (3.7)$$

where the $n = 0$ term in the last line has been dropped. Setting $\omega = 2$ we may show that:

$$I_{(\omega=2)} = -\frac{T}{4\pi} \sum_{n=-\infty}^{+\infty} \frac{p_4^2}{(p_4^2 + \mu^2)^2} |k_4 + p_4|, \quad p_4 = 2\pi nT. \quad (3.8)$$

By considering the two cases $k_4 > 0$, $k_4 < 0$, we can simplify Eq. (3.8) to the form:

$$I_{(\omega=2)} = -\frac{T}{2\pi} \sum_{n=1}^{\infty} \left\{ \frac{p_4^2}{(p_4^2 + \mu^2)^2} |k_4| + \frac{(p_4 + |k_4|)^2}{[(p_4 + |k_4|)^2 + \mu^2]^2} p_4 \right\}. \quad (3.9)$$

In the high-temperature limit, we first expand I in $1/p_4$, and then take the limit $\mu \rightarrow 0$ to get:

$$I_{(\omega=2)} = -\frac{T}{2\pi} \sum_{n=1}^{\infty} \left\{ \frac{1}{p_4} - \frac{|k_4|}{p_4^2} + 3\frac{k_4^2}{p_4^3} - 4\frac{|k_4|^3}{p_4^4} + \dots \right\}, \quad T \gg k_4, \quad P_4 = 2\pi nT. \quad (3.10)$$

Notice that the sum over $1/p_4$ in Eq. (3.10) is divergent due to our setting $\omega = 2$. The ultra-violet part of I may be found by referring back to Eq. (3.7) and expanding in $1/p_4$ and μ :

$$I = \frac{\pi^{\omega-1/2}}{(2\pi)^{2\omega-1}} \Gamma(3/2 - \omega) T \sum_{n=1}^{\infty} \frac{1}{(p_4)^{5-2\omega}} \left(2 + (\text{higher order in } 1/p_4, \mu) \right). \quad (3.11)$$

The ultra-violet part of I reads:

$$I^{\text{div}} = -\frac{1}{4\pi^2} \zeta(1 - 2(\omega - 2)) \simeq \frac{1}{8\pi^2} \frac{1}{(\omega - 2)}. \quad (3.12)$$

Combining Eqs. (3.10) and (3.12) we obtain:

$$I = I^{\text{div}} + I^{\text{finite}}, \quad (3.13)$$

with I^{div} given in Eq. (3.12) and

$$I^{\text{finite}} = -\frac{T}{2\pi} \sum_{n=1}^{\infty} \left\{ -\frac{|k_4|}{p_4^2} + 3\frac{k_4^2}{p_4^3} - 4\frac{|k_4|^3}{p_4^4} + \dots \right\}. \quad (3.14)$$

Using the definition of the zeta function to evaluate each term in the infinite series we eventually get for the integral in Eq. (3.6):

$$I = \frac{1}{8\pi^2} \frac{1}{\omega - 2} + \frac{\zeta(2)}{8\pi^3} \frac{|k_4|}{T} - \frac{3\zeta(3)}{16\pi^4} \frac{k_4^2}{T^2} + (\text{higher orders in } \frac{|k_4|}{T}). \quad (3.15)$$

The above detailed example should convince the reader of the practicality of our new method. We shall now proceed to analyze a more challenging problem.

4. Debye Screening Length in Yang-Mills Theory

To test prescription (2.9), we shall calculate the Debye screening length m_{el} in pure Yang-Mills theory to one loop. The screening length is given in terms of the gauge field self-energy [20] by

$$m_{el} = -\lim_{\mathbf{k} \rightarrow 0} \Pi_{00}^{ab}(k_4 = 0, \mathbf{k}). \quad (4.1)$$

Π_{00}^{ab} may be calculated from the diagrams in Fig. 1 and reads:

$$\Pi_{00}^{ab}(k_4, \mathbf{k}) = \delta^{ab} g^2 N_c [I_1(k_4, \mathbf{k}) + I_2(k_4, \mathbf{k}) + I_3(k_4, \mathbf{k}) + I_4(k_4, \mathbf{k})], \quad (4.2)$$

where

$$\begin{aligned} I_1(k_4, \mathbf{k}) &= \frac{1}{2} T \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{p^2(p+k)^2}, \\ I_2(k_4, \mathbf{k}) &= T \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{(p+k)^2 p_4^2}, \\ I_3(k_4, \mathbf{k}) &= \frac{1}{2} T \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{(\mathbf{p}^2 + \mathbf{k} \cdot \mathbf{p})^2}{p^2(p+k)^2(p_4 + k_4)^2 p_4^2}, \\ I_4(k_4, \mathbf{k}) &= -2 T \sum_{n=-\infty}^{+\infty} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{p^2}. \end{aligned} \quad (4.3)$$

here g is the gauge coupling, N_c the number of colours, $p_4 = 2\pi nT$, $p^2 = p_4^2 + \mathbf{p}^2$ and T is the temperature. The factors $1/p_4^2$ are defined by the prescription in Eq. (2.9b).

After performing the momentum integrations and setting $\omega = 2$, we obtain the following expressions for $I_1, \dots, I_4$:

$$\begin{aligned}
I_1(k_4, \mathbf{k}) &= \frac{T}{8\pi|\mathbf{k}|} \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 \text{sign}(p_4) \frac{i}{2} \left[\ln \left(\frac{|\mathbf{k}| - ik_4}{|\mathbf{k}| + ik_4} \right) + \ln \left(\frac{2p_4 + k_4 - i|\mathbf{k}|}{2p_4 + k_4 + i|\mathbf{k}|} \right) \right], \\
I_2(k_4, \mathbf{k}) &= -\frac{T}{4\pi} \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 |k_4 + p_4| \frac{p_4^2}{(p_4^2 + \mu^2)^2}, \\
I_3(k_4, \mathbf{k}) &= \frac{T}{64\pi} \sum_{n=-\infty}^{+\infty} (2p_4 + k_4)^2 \text{sign}(p_4) \frac{p_4^2}{(p_4^2 + \mu^2)^2} \frac{(p_4 + k_4)^2}{((p_4 + k_4)^2 + \mu^2)^2} \left\{ 4p_4(\mathbf{k}^2 + k_4^2 + 2k_4p_4 + 4p_4^2) \right. \\
&\quad \left. + \frac{2}{|\mathbf{k}|}(\mathbf{k}^2 + k_4^2 + 2k_4p_4 + 2p_4^2)^2 \frac{i}{2} \left[\ln \left(\frac{|\mathbf{k}| - ik_4}{|\mathbf{k}| + ik_4} \right) + \ln \left(\frac{2p_4 + k_4 - i|\mathbf{k}|}{2p_4 + k_4 + i|\mathbf{k}|} \right) \right] \right\}, \\
I_4(k_4, \mathbf{k}) &= \frac{T}{2\pi} \sum_{n=-\infty}^{+\infty} |p_4|. \tag{4.4}
\end{aligned}$$

In the limit $\mu \rightarrow 0$ and $k_4 \rightarrow 0$ the right hand sides of Eq. (4.4) simplify to:

$$\begin{aligned}
I_1(k_4 = 0, \mathbf{k}) &= \frac{iT}{2\pi|\mathbf{k}|} \sum_{n=1}^{\infty} p_4^2 \ln \left(\frac{2p_4 - i|\mathbf{k}|}{2p_4 + i|\mathbf{k}|} \right), \\
I_2(k_4 = 0, \mathbf{k}) &= -\frac{2T}{\pi} \sum_{n=1}^{\infty} p_4, \\
I_3(k_4 = 0, \mathbf{k}) &= \frac{|\mathbf{k}|T}{4} + \frac{T}{8\pi} \sum_{n=1}^{\infty} \frac{1}{p_4^2} \left\{ 4p_4(\mathbf{k}^2 + 4p_4^2) + \frac{2}{|\mathbf{k}|}(\mathbf{k}^2 + 2p_4^2)^2 \frac{i}{2} \ln \left(\frac{2p_4 - i|\mathbf{k}|}{2p_4 + i|\mathbf{k}|} \right) \right\}, \\
I_4(k_4 = 0, \mathbf{k}) &= \frac{T}{\pi} \sum_{n=1}^{\infty} p_4, \quad p_4 = 2\pi nT. \tag{4.5}
\end{aligned}$$

The term $|\mathbf{k}|T/4$ in I_3 is obtained from the $n = 0$ part of the Matsubara sum in the limit $\mu \rightarrow 0$. I_2 and I_4 are easily computed using $\zeta(-1) = -1/12$. For I_1 and I_3 we must use a high-temperature expansion:

$$\frac{i}{2} \ln \left(\frac{2p_4 - i|\mathbf{k}|}{2p_4 + i|\mathbf{k}|} \right) = \sum_{j=0}^{\infty} \frac{(-1)^j}{2j+1} \left(\frac{|\mathbf{k}|}{2p_4} \right)^{2j+1}, \quad |\mathbf{k}| \ll T. \tag{4.6}$$

Note that the $j = 0$ term gives a contribution proportional to $\zeta(1)$. If we go back to the original expressions in Eq.(4.3) and keep ω general, it is possible to extract the pieces proportional to $\zeta(1 - 2(\omega - 2))$ which have the same overall coefficient as the $\zeta(1)$ terms in Eq.(4.5). The final result for the 00-component of the gauge field self-energy reads:

$$\begin{aligned}
\Pi_{00}^{ab}(k_4 = 0, \mathbf{k}) &= \delta^{ab} g^2 N_c \left\{ -\frac{T^2}{3} + \frac{|\mathbf{k}|T}{4} + \frac{11\mathbf{k}^2}{24\pi^2} \zeta(1 - 2(\omega - 2)) + \frac{11\mathbf{k}^2}{24\pi^2} \ln \left(\frac{|\mathbf{k}|}{2\pi T} \right) \right. \\
&\quad \left. + \sum_{j=1}^{\infty} \left(\frac{8}{4j^2 - 1} + \frac{2}{2j + 3} \right) \frac{(-1)^{j+1}}{(4\pi)^{2j+2}} \frac{(\mathbf{k}^2)^{j+1}}{T^{2j}} \zeta(2j + 1) \right\}, \quad |\mathbf{k}| \ll T. \tag{4.7}
\end{aligned}$$

The first four terms of this expansion agree with the results of reference [8], while the screening length is given by (see Eq.(4.1)):

$$m_{el} = \frac{1}{3}g^2 N_c T^2. \quad (4.8)$$

5. Discussion

We have developed a general technique for perturbative calculations in the temporal gauge at both zero and finite temperature. It is based on a special version of the n_μ^* -prescription which is used to handle the spurious poles present in the temporal gauge. At zero temperature, one-loop integrals may be computed unambiguously using dimensional regularization and the technique of exponential parameterization, but at finite temperature, the method of zeta functions must be incorporated in order to facilitate the calculation of Matsubara frequency sums. The advantage of the zeta-function method over the contour method lies in the ease with which high-temperature expansions may be computed to any order in $1/T$.

Our technique has been tested in Yang-Mills theory by calculating in the imaginary time formalism the complete $1/T$ -expansion for the self-energy $\Pi_{00}^{ab}(k_4 = 0, \mathbf{k})$. The first four terms of Eq. (4.7) agree with those of Kajantie and Kapusta [8], while the infrared limit yields the correct expression for the Debye screening length. These results give us some confidence in the validity of our approach. In particular, it is re-assuring that the n_μ^* -prescription yields satisfactory results also at finite temperature, and that there is no ambiguity when the summation index n in the Matsubara sum is zero, i.e. when $n = 0$ ($p_0 = i2\pi nT$). In summary, all three objectives listed toward the end of the Introduction have been achieved.

As emphasized in Section 2, prescription (2.7), or equivalently (2.9), is necessary because the condition $A_0^a(x) = 0$ does not completely fix the gauge: the Yang-Mills action is still invariant under time-independent gauge transformations. For this reason it would be interesting to investigate the relationship between gauge fixing and the n_μ^* -prescription. One possibility, suggested by Andraši and Taylor [21], is to modify the gauge condition

so as to fix the gauge completely. For instance, one may try to replace $A_0^a(x) = 0$ by the constraint

$$A_0^a(x) + i\epsilon \frac{\partial \cdot A^a(x)}{\partial_0^2 \partial^2} = 0. \quad (5.1)$$

Addition of this gauge constraint to the Yang-Mills Lagrangian density leads to the following propagator:

$$G_{\mu\nu}(p) = \frac{-1}{p^2 + i\epsilon} \left(-\delta_{ij} - p_i p_j \frac{p_0^2}{(p_0^2 + i\epsilon)^2} \right) + \epsilon^2 \frac{p_\mu p_\nu}{(p^2 + i\epsilon)^2 (p_0^2 + i\epsilon)}, \quad (5.2)$$

where $\mu, \nu = 0, 1, 2, 3$ and $i, j = 1, 2, 3$. Expression (5.2) is just the propagator given in Eq. (2.2) with the poles treated by the prescription in Eq. (2.7b), plus an extra ϵ^2 -dependent term. There is also a ghost-gauge coupling proportional to ϵ^2 . If one could show that the integrals resulting from these extra terms vanished as $\epsilon \rightarrow 0$, one would have a much better handle proving at least the formal equivalence between this gauge fixing scheme and the prescription given in Eq. (2.7b).

Future tests of our general technique might also include the computation of gauge invariant quantities to higher orders. The Wilson loop, for instance, would provide a particularly sensitive test at $T = 0$, while calculation of the thermodynamic equation of state to two loops in Yang-Mills theory would constitute an excellent test at finite temperature. Equally rewarding should be a careful study of the analytic continuation of Green functions to real time. Both the structure of the n_μ^* -prescription and of the zeta function seem to suggest the possibility of continuing the imaginary time formalism directly to the real time formalism.

Acknowledgement

This research was supported in part by a grant from the Natural Sciences and Engineering Research Council of Canada under grant No. A8063.

Appendix

The purpose of this appendix is to show how one may compute the contour integral (see Fig. 2a,b) in Eq.(3.1):

$$I = \lim_{\epsilon \rightarrow 0} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{2\pi i} \oint dp_0 f(p_0, \mathbf{p}) \frac{1}{2} \coth \left(\frac{p_0}{2T} \right), \quad (\text{A.1})$$

where

$$f(p_0, \mathbf{p}) = \frac{1}{(p+k)^2} \frac{p_0^2}{(p_0^2 + i\epsilon)^2}. \quad (\text{A.2})$$

First note that the two contours in Fig. 2b may be combined to give

$$\begin{aligned} I &= \lim_{\mu \rightarrow 0} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{2\pi i} \int_{-i\infty+\delta}^{i\infty+\delta} dp_0 \frac{1}{2} [f(p_0) + f(-p_0)] \coth \left(\frac{p_0}{2T} \right), \\ &= \frac{1}{2} \lim_{\mu \rightarrow 0} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{2\pi i} \int_{-i\infty+\delta}^{i\infty+\delta} dp_0 f(p_0) \coth \left(\frac{p_0}{2T} \right) + (k_0 \rightarrow -k_0). \end{aligned} \quad (\text{A.3})$$

Closing the contour in the right hand plane, we are left with two residues, namely

$$I = I_1 + I_2, \quad (\text{A.4})$$

where

$$I_1 = -\frac{1}{2} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \left\{ \lim_{\epsilon \rightarrow 0} \lim_{(p_0 \rightarrow -(p_0)_{pr})} \frac{d}{dp_0} \left[\frac{1}{(p_0 + k_0)^2 - (\mathbf{p} + \mathbf{k})^2} \frac{p_0^2}{(p_0 + (p_0)_{pr})^2} \coth \left(\frac{p_0}{2T} \right) \right] \right\} + (k_0 \rightarrow -k_0), \quad (\text{A.5})$$

and

$$I_2 = -\frac{1}{2} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \left\{ \lim_{\epsilon \rightarrow 0} \lim_{(p_0 \rightarrow -k_0 + |\mathbf{p} + \mathbf{k}|)} \left[\frac{1}{p_0 + k_0 + |\mathbf{p} + \mathbf{k}|} \frac{p_0^2}{(p_0^2 + i\mu)^2} \coth \left(\frac{p_0}{2T} \right) \right] \right\} + (k_0 \rightarrow -k_0), \quad (\text{A.6})$$

where $(p_0)_{pr} = \sqrt{\epsilon} e^{i3\pi/4}$ is the pole position associated with the prescription (2.7b) (see Eq.(A.2)). For I_1 we obtain

$$\begin{aligned} I_1 &= \frac{1}{8} \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \left\{ \lim_{\epsilon \rightarrow 0} \frac{1}{(-p_{0r} + k_0)^2 - (\mathbf{p} + \mathbf{k})^2} \right. \\ &\quad \left. \left[\left(\frac{2(-p_{0r} + k_0)}{(-p_{0r} + k_0)^2 - (\mathbf{p} + \mathbf{k})^2} + \frac{1}{p_{0r}} \right) + (k_0 \rightarrow -k_0) \right] \right\}, \\ &= \int \frac{d^{2\omega-1} \mathbf{p}}{(2\pi)^{2\omega-1}} \left\{ \frac{-4Tk_4^2}{(\mathbf{p}^2 + k_4^2)^3} + \frac{T}{(\mathbf{p}^2 + k_4^2)^2} + \frac{1}{12T(\mathbf{p}^2 + k_4^2)} \right\}. \end{aligned} \quad (\text{A.7})$$

The first two terms cancel in the limit $\omega \rightarrow 2$, so that we are left with

$$I_1 = -\frac{|k_4|}{48\pi T}. \quad (\text{A.8})$$

For I_2 we shift $\mathbf{p} \rightarrow \mathbf{p} + \mathbf{k}$ to get:

$$I_2 = -\frac{1}{4} \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{|\mathbf{p}|(-k_0 + |\mathbf{p}|)^2} \coth\left(\frac{-k_0 + |\mathbf{p}|}{2T}\right) + (k_0 \rightarrow -k_0). \quad (\text{A.9})$$

Separating the temperature-independent vacuum piece I_2^{vac} from the matter piece I_2^{mat} by means of the relation

$$\coth(x) = 1 + \frac{2}{e^{2x} - 1}, \quad (\text{A.10})$$

we find that

$$I_2 = I_2^{\text{vac}} + I_2^{\text{mat}}, \quad (\text{A.11})$$

where

$$I_2^{\text{vac}} = -\frac{1}{4} \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{|\mathbf{p}|(-k_0 + |\mathbf{p}|)^2} + (k_0 \rightarrow -k_0). \quad (\text{A.12})$$

$$I_2^{\text{mat}} = -\frac{1}{4} \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} \frac{1}{|\mathbf{p}|(-k_0 + |\mathbf{p}|)^2} \frac{1}{\exp\{\beta(-k_0 + |\mathbf{p}|)\} - 1} + (k_0 \rightarrow -k_0), \quad (\text{A.13})$$

with $\beta = 1/T$. The vacuum term is easily calculated by utilizing the technique of exponential parameterization:

$$\begin{aligned} I_2^{\text{vac}} &= -\frac{1}{2} \int_0^\infty d\alpha d\gamma \frac{\alpha^{-1/2}}{\Gamma(1/2)} \gamma \exp(-\gamma k_4^2) \int \frac{d^{2\omega-1}\mathbf{p}}{(2\pi)^{2\omega-1}} (-k_4^2 + \mathbf{p}^2) \exp\{-(\alpha + \gamma)\mathbf{p}^2\}, \\ &= \frac{1}{8\pi^2} \left(\Gamma(\omega - 2) + 1 \right). \end{aligned} \quad (\text{A.14})$$

The divergent part of I_2^{vac} agrees with both the zero temperature answer in Eq.(2.12), and the zeta-function result, Eq.(3.12). It is not possible to compute I_2^{mat} exactly but we may simplify it somewhat by integrating over the spatial angles:

$$\begin{aligned} I_2^{\text{mat}} &= -\frac{1}{4\pi^2} \int_0^\infty dp \frac{p}{(-k_0 + p)^2} \frac{1}{\exp\{\beta(-k_0 + p)\} - 1} + (k_0 \rightarrow -k_0), \\ &= -\frac{1}{2\pi^2} \text{Re} \left\{ \int_0^\infty dp \frac{p}{(-k_0 + p)^2} \frac{1}{\exp\{\beta(-k_0 + p)\} - 1} \right\}, \quad p = |\mathbf{p}|. \end{aligned} \quad (\text{A.15})$$

The final result for $I = I_1 + I_2$, $\omega \rightarrow 2$ reads

$$I = \frac{1}{8\pi^2} \left(\frac{1}{(\omega - 2)} + 1 \right) - \frac{|k_4|}{48\pi T} - \frac{1}{2\pi^2} \text{Re} \left\{ \int_0^\infty dp \frac{p}{(-k_0 + p)^2} \frac{1}{\exp\{\beta(-k_0 + p)\} - 1} \right\}. \quad (\text{A.16})$$

This example demonstrates that the prescription in Eq.(2.7b) may indeed be used in the contour method to give well defined results.

References

1. H. Weyl, *Z. Phys.* **56** (1929) 330.
2. W. Heisenberg, and W. Pauli, *Z. Phys.* **59** (1930) 168.
3. G. Leibbrandt, *Rev. Mod. Phys.* **59** (1987) 1067.
4. G.C. Rossi, and M. Testa, *Phys. Rev.* **D29** (1984) 2997.
5. M. Claudson, and M.B. Halpern, *Phys. Lett.* **B151** (1985) 281.
6. Z. Bern, and H.S. Chan, *Nucl. Phys.* **B266** (1986) 509.
7. A. Actor in *New Perspectives in Quantum Field Theories*, edited by J. Abad, M. Asorey, and A. Cruz (World Scientific, Singapore, 1986), p. 345.
8. K. Kajantie and J.I. Kapusta, *Ann. Phys.* **160** (1985) 477.
9. S. Caracciolo, G. Curci, and P. Menotti, *Phys. Lett.* **B113** (1982) 311.
10. P. Gaigg, and M. Kreuzer, *Phys. Lett.* **B205** (1988) 530.
11. P. Gaigg, M. Kreuzer, M. Schweda, and O. Piguet, *J. Math. Phys.* **28** (1987) 2781.
12. S. Mandelstam, *Nucl. Phys.* **B213** (1983) 149.
13. G. Leibbrandt, *Phys. Rev.* **D29** (1984) 1699.
14. K.A. James and P.V. Landshoff, *Phys. Lett.* **B251** (1990) 167.
15. J.I. Kapusta, *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, 1989).
16. F.T. Brandt, J. Frenkel and J.C. Taylor, *Phys. Rev.* **D44** (1991) 1801.
17. G. Leibbrandt, *Nucl. Phys.* **B310** (1988) 405.
18. K. Lee, and S.-L. Nyeo; National Cheng Kung University Preprint NCKU-HEP/93-09, 1993.

19. K.A.James, *Z. Phys.* **C48** (1990) 169.
20. G. Leibbrandt, and M. Staley; to appear in *Proceedings of the Third Workshop on Thermal Field Theories, Banff, Alberta, Canada, Aug. 1993* (World Scientific, Singapore, 1994).
21. A. Andraši and J.C. Taylor, *Nucl. Phys.* **B310** (1988) 222.

Figure Captions

Fig. 1. Diagrams which contribute to the one loop self-energy in Yang-Mills theory in the temporal gauge.

Fig. 2(a). Contour plot showing prescription poles (circled), propogator poles (boxed), and poles arising from the Bose-Einstein factor (on imaginary axis).

Fig. 2(b). Same contour plot as in Fig. 1(a) except with the contours closed in the right and left-hand planes.

Fig. 1. Diagrams which contribute to the one loop self-energy in Yang-Mills theory in the temporal gauge.

Fig. 2(a). Contour plot showing prescription poles (circled), propogator poles (boxed), and poles arising from the Bose-Einstein factor (on imaginary axis).

Fig. 2(b). Same contour plot as in Fig. 1(a) except with the contours closed in the right and left-hand planes.