VACUUM VALUES FOR AUXILIARY STRING FIELDS.

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ABSTRACT

Auxiliary string fields are introduced in light-cone gauge string field theory in order to express contact interactions as contractions of cubic vertices. The auxiliary field in the purely closed-string bosonic theory may be given a non-zero expectation value, giving rise to a phase in which world-sheets have boundaries.
The sum over Riemann surfaces that represents the Feynman diagrams of perturbative string theory can be obtained from light-cone gauge string field theory \([1-3]\). In the first part of this paper the stringy Feynman rules will be reexpressed by the inclusion of auxiliary string fields which clarify the occurrence of contact interactions in both open and closed string theory. Furthermore, it will be argued that in the bosonic closed-string theory the auxiliary field may be given an expectation value consistent with Poincaré symmetry, thereby significantly altering the string vacuum state. The resulting theory is one that is described by a sum over world-sheets with boundaries on which the embedding coordinates are constant and with no net momentum flowing through any given boundary (\([4,5]\) and references therein describe some properties of the resulting theory). Such boundary insertions can also be formulated covariantly but it is only in the light-cone parametrization that they are described by an instantaneous interaction term in the hamiltonian. A related scheme might apply to open-string theory but whether similar considerations can be applied to superstring theories is not obvious.

In light-cone gauge string field theory the functional string field depends on the transverse components of the string coordinates, \(X^i(\sigma)\), \(i = 1, \ldots, D-2\), as well as \(p^+\) and \(\tau = X^+\) (the light-cone 'time'). The parameter \(\sigma\) has range \(\pi \alpha = 2 \pi p^+\). The string field may be expressed (at \(X^+ = 0\)) in terms of the Fock space states by

\[
\Phi[X] \equiv \Phi[X^i(\sigma), \alpha] = \langle X | \Phi \rangle. \tag{1}
\]

The free string hamiltonian may be written in this notation as

\[
H_2 = \int DX^i d\alpha \Phi[X^i, -\alpha] \frac{\pi \alpha}{\pi} d\sigma \left( -\frac{\delta^2}{\delta X^i} + \frac{1}{\pi^2} X'^2 \right) \Phi[X, \alpha] \tag{2}
\]

which defines the string propagator. In the light-cone parametrization Riemann surfaces are constructed from flat sectors of world-sheet by joining vertices with propagators, all the curvature being located at discrete 'interaction points'. A similar parametrization arises in the 'light-cone-like' version of covariant string field theory \([6–8]\), in which \(\alpha\) is an arbitrary parameter. Many of the issues discussed below have covariant generalizations within that framework.

The usual cubic interaction vertex is defined as the overlap between two incoming strings and the outgoing string (or the time-reversed process),

\[
H_3 = \int \left( \prod_{r=1}^3 DX^i_r d\alpha_r \right) \Phi[X_1, \alpha_1] \Phi[X_2, \alpha_2] \Phi[X_3, \alpha_3] \delta[X_1, X_2, X_3] \delta(\sum_{r=1}^3 \alpha_r) \tag{3}
\]

where \(\delta[X_1, X_2, X_3]\) is the functional delta function that imposes the condition that the transverse coordinates are continuous at the vertex (and the notation implies integration
over the coordinates of the complete sets of intermediate states). This defines the vertex, $\langle V_3 |$, as a state in the tensor product of the Fock spaces of the three interacting strings. This structure applies equally to open and closed strings and also to string theories with a richer world-sheet structure such as superstring theories (generally the vertex also includes an operator insertion at the interaction point that does not concern us here).

One way of obtaining the vertex is to consider the functional integral for the process in which three strings are in arbitrary asymptotic Fock space states and propagate from $\tau = \pm \infty$. The external legs are then lopped off to give the vertex, in the usual manner. However, this is not the whole story since some regions of the moduli space of multi-particle tree amplitudes are not reproduced by sewing such cubic vertices together. Although these regions are of zero measure in the bosonic closed-string case there are circumstances in which their presence can be important.

**Auxiliary string fields and contact terms**

To see that there are other contributions to the string interaction, consider the process in which legs 1 and 2 of the vertex couple to definite asymptotic on-shell Fock states, $| n_1 \rangle$ and $| n_2 \rangle$ (where the labels summarize the occupation numbers of all the oscillator states), with widths $\pi \alpha_1$ and $\pi \alpha_2$, while the third (of width $\pi \alpha_3 = -\pi \alpha_1 - \pi \alpha_2$) is attached to a propagator that propagates a finite time to the state $| X_3 \rangle$ at $\tau = T$.

The amplitude defined by this process is given by (to be specific the following argument will refer to a purely closed-string theory)

$$I(1, 2, X_3; T) = \langle V_3 | n_1 \rangle | n_2 \rangle | X_3 \rangle \Delta(X_3', X_3; T),$$

with

$$\Delta(X_3', X_3; T) = \frac{1}{\alpha_3} \langle X_3' | e^{P^- T} | X_3 \rangle,$$

where $P^-$ is the Hamiltonian in the light-cone frame and is given in terms of modes (for closed strings) by $\alpha P^- = p^2 + 2N + 2\tilde{N} - 4$, where $N$ and $\tilde{N}$ are the level numbers of the right-moving and left-moving modes, respectively.

The process described by $I$ defines a world-sheet with a boundary which can conveniently be mapped to the upper-half $z(= x + iy)$-plane, with the $x$ axis being the image of the string $X_3(\sigma)$. Strings 1 and 2 are mapped to two infinitesimal holes (i.e., punctures) that may be located at $z = i$ and $z = iq$ (where $q$ is the real modulus and $0 \leq q \leq 1$) with no loss of generality. The mapping from the string diagram to the upper-half plane is given by

$$\rho = \frac{\alpha_1}{2} \ln \left( \frac{z - i}{z + i} \right) + \frac{\alpha_2}{2} \ln \left( \frac{z - iq}{z + iq} \right).$$

Lines of constant $\tau = X^+$ are equipotentials (where the electrostatic potential is $\phi = \frac{1}{2}(\rho + \bar{\rho})$). Close to the points $z = i$ and $z = iq$ the equipotentials are circles in the $z$ plane, while near the real axis the equipotentials are horizontal lines. The electric field $\partial_\rho \phi$ is normal to any boundary (including the punctures) and its integrated value around the $i$th boundary is the enclosed charge $\alpha_i = \oint_i \partial_\nu \phi d\sigma_t$ (where $\nu$ denotes the
normal to the boundary. The $\alpha_i$'s are arbitrary real constants subject only to the condition $\sum_{i=1}^3 \alpha_i = 0$ (so that $\alpha_3 = -\alpha_1 - \alpha_2$). On a surface of genus zero with $B$ boundaries there are generically $B - 2$ zeroes of the electric field (where a zero on a boundary counts as half a zero). In the present case there is a single zero corresponding to a turning point, $z_0$, of the map given by the solution of

$$\frac{\partial \rho}{\partial z} = 0 = \frac{i\alpha_1}{z^2 + 1} + \frac{iq\alpha_2}{z^2 + q^2}. \quad (7)$$

The solution is
\[ z_0^2 = -q \left( \frac{q + \eta}{1 + \eta q} \right), \] (8)

where \( \eta \equiv \alpha_2/\alpha_1 \). The value of the modulus \( q \) is determined by the value of \( T \).

We shall choose \(|\alpha_1| > |\alpha_2|\) (so that \(|\eta| < 1\)) with no loss of generality but the properties of the map depend sensitively on the sign of \( \eta \).

(a) \( \eta > 0 \). In this case \( z_0^2 \) is negative so that the zero lies on the imaginary \( z \) axis and \( q \leq |z_0| \leq 1 \). The \( \rho \) plane has the form of two incoming (or outgoing) closed strings entering at \( \tau = \infty \) (or \( \tau = -\infty \)) and joining at the turning point \( \rho_0 \equiv -T + i\sigma = \rho(z_0) \) (which is real) to form a closed string of width \( -\pi(\alpha_1 + \alpha_2) \) which evolves to the state \( \langle X_3 \rangle \) at \( \rho = 0 \) (fig. 1(a)). The amplitude for this process is given by (4) (with legs 1 and 2 incoming) and as \( q \) spans its complete range, \( 0 \leq q \leq 1 \), the interaction point spans its complete range, \( 0 \leq T \leq \infty \).

(b) \( \eta < 0 \). This corresponds to the case in which one string (say, the one of width \( \pi\alpha_1 \)) is incoming and the other is outgoing. Now \( z_0^2 \) is either positive or negative depending on the value of \( q \). For \( q > -\eta \), \( z_0^2 \) is again negative and \( 0 \leq |z_0| \leq q \). The string diagram is one describing an incoming string of width \( \pi\alpha_1 \) evolving from \( \tau = -\infty \) until it breaks at \( \tau = -T < 0 \) into a string of width \( \pi\alpha_2 \) which evolves to \( \tau = \infty \) and a string of width \( -\pi(\alpha_1 + \alpha_2) \) that evolves to the boundary at \( \tau = 0 \) (fig. 1(bi)). \( T \) decreases as \( q \) is reduced from \( q = 1 \) until a critical value \( q_c = -\eta \) is reached where \( z_0 = 0 \) so that the zero hits the real \( z \) axis and the interaction time is \( T = 0 \). In this range of \( q \) the process is again defined by (4) (now with leg 2 outgoing), but this covers only part of the complete moduli space.

When \( q \) is further reduced there are two real solutions to (8) so there are two zeroes of the electric field on the boundary – the boundary has two turning points. These points \( \rho^+ = \rho(z_0) = iR/2 \) and \( \rho^- = \rho(-z_0) = -iR/2 \) are on the imaginary \( \rho \) axis (\( \tau = 0 \)) and the string diagram is now one in which the boundary is represented by a vertical slit (fig. 1(bii)) (so the parametrization of string 3 is double-valued). This situation was discussed in detail in [4] in a theory with Neumann as well as a Dirichlet boundaries. The string diagram in this region of moduli space is described by a new vertex that couples fields on legs 1 and 2 to a field in the third Fock space that does not propagate in the usual manner. This process may be expressed in the form

\[ I'(1, 2, X_3; R) = \langle W(R)|n_1|n_2\rangle|X_3\rangle, \] (9)

where \( \langle W(R)\rangle \) is a new vertex describing this second term. The modulus, \( R \), changes from \( a = \pi\alpha_1 + \pi\alpha_2 \) to \( b = \pi\alpha_1 \) as \( q \) is reduced from \( q_c \) to 0. In other words, when \( \eta < 0 \) the string diagram of fig. 1(bi) does not cover the whole of the moduli space for the world-sheet with boundary and the contribution of fig. 1(bii) must be added.

Scattering amplitudes are constructed by sewing world-sheets together and integrating over \( X_3 \). One contribution to the four-string interaction is a \( t \)-channel 'pole term'
that is an integral over the two-dimensional moduli space,

\[
\int DX_5' DX_5 \int_0^\infty dT \int_0^{2\pi} d\theta \ I^I(1, 4, X_5'; 0) \ \Delta(X_5', X_5; T) \ I(3, 2, X_5; 0) e^{-p^- T + i(N - \tilde{N})\theta}
\]

\[
= \int_0^\infty dT \int_0^{2\pi} d\theta \ V_3|n_1\rangle|n_4\rangle|X_5\rangle \langle X_5|e^{(P^- - p^-) T + i(N - \tilde{N})\theta}|X_5'\rangle \langle X_5'|n_3\rangle\langle n_2|V_3),
\]

where the states 1 and 2 are incoming and states 3 and 4 are outgoing with \(\alpha_1 > -\alpha_4 > 0\) and \(-\alpha_3 > \alpha_2 > 0\) (and \(p^- = p_1^- + p_2^-\)). In addition to the other pole terms, the complete expression for the four-particle amplitude requires the less familiar contact term which arises by sewing two \(I'\) factors,

\[
\int DX_5 \int_a^b dR \ I'(1, 4, X_5; R) \ I'(3, 2, X_5; R)
\]

\[
= \int_a^b dR \ V(R)|n_1\rangle|n_4\rangle|X_5\rangle \langle X_5|\Phi[|X_1\rangle\langle X_2|X_3\rangle \Phi[X_1]\Phi[X_2]\Sigma[X_3],
\]

where \(\Sigma[X] \equiv \langle X|\Sigma\rangle\) (and the limits \(a\) and \(b\) again depend on the values of the suppressed
integration variables $\alpha_1$ and $\alpha_2$). The sum over intermediate states in the second line includes an integration over $\alpha_1$ and $\alpha_2$ (with $\alpha_3$ determined by momentum conservation).

\( \text{(c) The special case } \eta = -1. \) In this case the incoming and outgoing strings have $\alpha_1 = -\alpha_2$ so that the integral of the electric field along the boundary vanishes. Since the electric field, $\partial_n \phi$, is periodic this can only happen if it has an even number of zeroes (at least two). Indeed, since $z_0^2 = q$, there are precisely two zeroes on the real $z$ axis for all values of $q$. The process now described is a string of width $\pi \alpha_1$ evolving from $\tau = -\infty$ to $\tau = \infty$ with a vertical slit inserted at $\tau = 0$ (fig. 1(c)). In this case the boundary carries no net $P^+$ component of momentum. The turning points are given by substituting $z_0^2 = q$ into (6) so that the length of the slit is given by

$$R = \pi \alpha_1 - 4 \alpha_1 \tan^{-1}(\sqrt{q}).$$

(13)

As $q$ varies from $q = 1$ to $q = 0$ this modulus varies from $R = 0$ to $R = \pi \alpha_1$.

A parallel discussion can be given for open-string theories. Again, the usual three-string interaction does not cover the whole of moduli space and there is a vertex coupling to an auxiliary open-string field,

$$\int_a^b dR \langle w(R) | \psi_1 \rangle | \psi_2 \rangle | \rho \rangle,$$

(14)

where $| \psi \rangle$ represents an open-string light-cone field and $| \rho \rangle$ is the auxiliary open-string field. The four-string contact term that arises by sewing two of these vertices together on the third leg is an integral over $R$ as before but now this is of finite weight since the moduli space for the tree amplitude with four external open strings is one-dimensional. It is the familiar instantaneous interaction (occurring when two open strings touch at internal points) that arises in the light-cone description of the Veneziano model [2]. The open-string diagram in which $\alpha_2 = -\alpha_1$ (analogous to fig. 1(c)) is one in which the world-sheet is a strip of width $\pi \alpha_1$. The coordinates $X^i$ satisfy Neumann conditions on the horizontal boundaries of the strip and the new vertex is represented by a vertical section of the boundary of length $R \leq \pi \alpha_1$ at $\tau = 0$, which has one turning point and on which the coordinates take the values $X^i_3(\sigma)$.

In addition, in any open-string field theory there is a vertex coupling a closed string to two open strings. In order to cover the whole of moduli space in this case it is necessary to add a contribution from $\Sigma$, the auxiliary closed-string field, coupling to the interior of the world-sheet.

**Vacuum expectation values**

An interesting feature of the vertex shown in fig. 1(c) (and the corresponding open-string diagram) is that it makes a non-trivial contribution even when the momentum in leg 3 vanishes, in which case the total length of the boundary is $2R$. This raises the possibility of modifying the usual theory by assigning a non-zero expectation value to the zero-momentum auxiliary field so that fig. 1(c) would be a ‘tadpole’ diagram that
redefines the vacuum state. There are strong constraints on the form of this vacuum value due to the requirement that the theory remain invariant under Poincaré transformations generated by $J^{\mu\nu} = \int d\sigma P^{[\mu} X^{\nu]}$ and $P^\mu$.

The idea is to couple $\Sigma$ to a source in order to shift the auxiliary field in $H'_3$ to $\Sigma'$ defined by

$$\Sigma = \Sigma' + \lambda B$$

(15)

where $\lambda$ is an arbitrary constant weight and $B[X(\sigma)]$ is the string field expectation value. In the closed bosonic string theory there is an obvious candidate for this expectation value that maintains Poincaré symmetry, namely, the point-like field

$$B[X(\sigma)] \equiv \langle X|B \rangle = \delta^{D-2}[\partial_\sigma X^i(\sigma)],$$

(16)

which has support only when $X^i(\sigma)$ is constant (which further implies that $X^-(\sigma)$ is constant). The state $|B\rangle$ is defined by $\partial_\sigma X^i(\sigma)|B\rangle = 0$ and $p^i|B\rangle = 0$ (where $p^i$ is the transverse momentum). It may be expressed in terms of modes as [5]

$$|B\rangle = \exp \left( \alpha_n^{i\dagger} \tilde{\alpha}_n^{i\dagger} \right) |0\rangle$$

(17)

(where $\alpha_n$ and $\tilde{\alpha}_n$ are the usual modes of the closed string and $|0\rangle$ is the zero-momentum ground state). Upon shifting $\Sigma$ in this way the auxiliary quantum field $\Sigma'$ couples in $H'_3$ but in addition there is a new quadratic vertex,

$$H'_2 = \lambda \int_0^{\pi \alpha_1} dR \langle W(R)|\Phi_1|\Phi_2|B\rangle$$

$$= \lambda \int DX^1_i DX^1_2 DX^1_3 d\alpha_1 \int_0^{\pi \alpha_1} dR$$

$$\langle W(R)|X_1|X_2|X_3\rangle \Phi[X_1, \alpha_1] \Phi[X_2, -\alpha_1] \delta^{D-2} \left[ \partial_\sigma X^3_3 \right],$$

(18)

This describes the insertion of a zero-momentum boundary (of length $2R$) in the worldsheet on which the $X^\mu$ coordinates satisfy Dirichlet boundary conditions. We need to check that $H_2 + H'_2$ is invariant under Poincaré transformations of the fields, at least to $O(\lambda)$. The Lorentz transformations in the $D - 2$ transverse directions, generated by $J^{ij}$, are obvious symmetries. Transforming the fields in $H'_2$ by the non-linearly realized Lorentz generator, $J^{i-}$, results in a term proportional to $J^{i-}|B\rangle = P^0_0 x^i|B\rangle$, where $P^0_0 = \oint P^- (\sigma)$ is the integral of $P^-$ along a contour enclosing the boundary in fig. 1(c). However, this contour may be distorted into the sum of contours on legs 1 and 2. The transformation of $H'_2$ is then seen to cancel with terms coming from a new $O(\lambda)$ variation of fields in the free-field term, $H_2$. The closure of this Noether procedure has not been checked at higher orders in $\lambda$. 

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Furthermore, possible anomalies of the kind that usually arise in the bosonic theory have been ignored. In particular, the singularities of $H'_2$ arising from the boundary of moduli space, $R = 0$, are due to the closed-string dilaton and tachyon states coupling to the vacuum. These are similar to the singularities of the usual theory with Neumann boundaries, except that the relative sign of the dilaton and tachyon singularities is reversed (in addition the trace of the graviton couples to the Neumann boundary). The singularity at $R = \pi \alpha_1$ is less familiar. It can be related to the presence of an open-string Lagrange multiplier state that leads to a constraint on the low-energy spectrum of the theory [11] that can only be properly interpreted after summing over all iterations of the vertex.

![Diagram](image)

**Fig. 2:** (a) The closed-string propagator with iterated vacuum insertions giving boundaries that are vertical slits at $\tau = \tau_i$. (b) The open-string propagator. Vertical segments of the boundary at $\omega_i = \omega_i$ represent interactions involving auxiliary open-string fields while closed strings couple to the slits on the interior. The boundary condition on all boundaries is $\partial_e X^i = 0$.

The kinetic terms of the bare theory are modified by the presence of $H'_2$ so that, in principle, the tree-level spectrum of the theory in this phase can be determined by re-diagonalizing the complete quadratic hamiltonian which is equivalent to summing over all boundary insertions. Diagrammatically, this amounts to the iteration of the vertex, leading to ‘Dirichlet’ string theory in which the usual closed-string propagator is modified by a condensate of such boundaries as in fig. 2(a). Even though summing over boundaries is a formidable task it can be argued that the resulting theory exhibits point-like substructure induced by the presence of the boundaries which introduce a non-locality of the mapping of the world-sheet into the target space [4] – which was the original motivation for studying it.
By contrast, the insertion of a Neumann boundary is not described by a local addition to the quadratic light-cone hamiltonian since it is not an insertion at fixed \( \tau = X^+ \). It might be viewed as a vacuum insertion in a covariant treatment (to be outlined below) in which \( \tau \) is not identified with the physical time coordinate.

The closed-string expectation value \( \lambda |B\rangle \) also affects the open-string theory since it modifies the usual coupling between a pair of open strings and a closed string. This gives a term quadratic in open-string fields, illustrated by the vertical slits in fig. 2(b) in the interior of the open-string strip. The theory also possesses the usual Neumann boundary insertions representing intermediate open-string states. Since \( \lambda \) is arbitrary there is the intriguing possibility of adjusting its value so that the dilaton singularity cancels (at least to leading order in string perturbation theory) in the sum of the Dirichlet and the Neumann boundaries.

The situation with open-string theory also involves some new features. The obvious candidate for an open-string vacuum value for the auxiliary field is \( \langle \rho[X] \rangle = b[X] = \langle X|b \rangle = \delta_{D-2}[\partial_\sigma X^i(\sigma)] \), so that

\[
\partial_\sigma X^i(\sigma)|b\rangle = 0.
\]

The open-string point-like state \( |b\rangle \) (considered in [12] in the context of the off-shell states of non-zero momentum of [13, 14]) is not a Lorentz scalar since the action of \( J^{\perp} \) on the state is anomalous (it is a mixed angular momentum state of zero helicity). If this could be remedied, the result of the vacuum modification to the open string would be the world-sheet in fig. 2(b), in which \( \partial_\sigma X^i = 0 \) along the boundary so that the vertical segments have Dirichlet conditions and the horizontal Neumann. The world-sheet may be mapped to the upper-half plane with the boundary as the real axis which is composed of segments on which the boundary conditions are alternately Dirichlet and Neumann. Summing over all possible insertions results in an open string with point-like energy densities at its end-points, whereas the expectation value for the closed-string auxiliary field generated point-like densities at interior points. Correspondingly, external (‘flavour’) currents can couple to the point-like densities at the ends of the string whereas off-shell closed-string currents can couple to interior point-like sections.

**Covariant string field theory**

The above treatment emphasized the light-cone gauge in which perturbative string field theory can be understood precisely. However, the general idea that open-string world-sheets can be obtained by attaching tadpoles with zero-momentum boundaries to closed world-sheets is a well-known feature of the conformal symmetry of string theory. It should therefore be possible to express the earlier results using a covariant formulation of string field theory such as that of [15] or the ‘light-cone-like’ formalism of [6–8]. In the latter approach many of the earlier manipulations remain valid but the expressions, that now include Faddeev–Popov ghost coordinates, are manifestly Poincaré-covariant—it is BRST invariance that needs to be checked. The arguments involving fig. 1 show that contact terms again arise for general values of \( \alpha \). However, since \( \alpha \) is no longer a component of the string momentum the off-shell pole contributions in fig. 1(a) and fig. 1(bi) can contribute even when the momentum in leg 3 is zero. The end-state of the cylinder at \( \tau = 0 \) may therefore couple to the vacuum. The covariant expectation
value of a string field, \( B[X] \), may now either correspond to a Dirichlet boundary, on which \( X^\mu(\sigma)|B|_D = 0 \) (where \( \mu = 0, 1, \ldots, D - 1 \)) or to a Neumann boundary, on which \( P^\mu(\sigma)|B|_N = 0 \). These boundary states are both BRST invariant (after incorporating the appropriate ghost coordinates) \([16, 17, 5]\). In other words, either Neumann or Dirichlet boundaries can arise as vacuum values of the closed-string field.

The constraints of supersymmetry further limit the possibility of assigning non-zero expectation values to the fields in superstring theories. The discussion in \([16, 17]\) suggests that the usual type 1 theory (in which the open-string sector has Neumann boundary conditions) may be expressed as a type 2b closed-string theory in which the closed-string field acquires an expectation value that preserves a linear combination of world-sheet supersymmetries and is equal to a very particular sum of a Neumann boundary state and a cross-cap state. However, it is not obvious that a Dirichlet version of superstring theory exists since the two types of point-like boundary states of the type 2b theory transform non-trivially (they are off-shell versions of the massless scalar and pseudo-scalar states that form the end-states of a string supermultiplet \([18]\)).

REFERENCES


