ON THE ANYON TWO-POINT CORRELATION FUNCTION IN THE ABELIAN HIGGS MODEL WITH CHERN-SIMONS TERM

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Abstract:

In the non-compact Abelian Higgs model with Chern-Simons term, formulated on a 2+1 dimensional space-(Euclidean) time lattice, a Euclidean functional integral representation of a gauge invariant four-point correlation function with insertions is given. From this representation the two-point correlation function of the non-local anyon fields has to be generated by shifting two matter fields, which act as countercharges, to infinity after the infinite volume limit has been performed. Devising an expansion in a Coulomb-like phase of the model these limiting procedures are analyzed. The removal of the countercharges only leads to a finite limit after an appropriate renormalization which is worked out in some detail.

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1. INTRODUCTION

In the past years quantum fields in two space dimensions showing braid group statistics have attracted much attention. This structural property has been investigated mainly within an algebraic setting: in particular models of quantum field theory using a Hamiltonian approach, see [1–5] and references cited therein, and more generally in the model-independent framework of algebraic quantum field theory, see e.g. [6,7]. Despite these achievements there remain important dynamical issues still to be clarified, especially the creation of particle-like excitations by such fields, which have been termed anyons [8].

The goal of this article is to present and analyze a Euclidean functional integral representation for an anyon–antianyon correlation function in the non-compact Abelian Higgs model augmented by the Chern–Simons term [9]. The model is treated on a space–time lattice, thus being well-defined. One should notice, however, that in this model two types of anyons can be constructed: i) electrically charged vortices, and ii) transmuted matter fields. The former ones have been introduced in [10] by establishing Euclidean region functional integrals for the Euclidean Green functions of these objects. Correlation functions of transmuted matter fields in terms of Euclidean functional integrals have been set up by the author [11]. In these representations a transmuted matter field, being a locally gauge invariant charged field, manifests itself as a basic matter field to which a Dirac–Mandelstam string together with a corresponding disorder insertion are adjoint. The infinite extension of both these insertions has to be generated by a limit procedure, which is the main object of this article. Aiming at an anyon–antianyon correlation function we start from a particular four–point function of the matter field with insertions, where two of the fields act as countercharges to be eventually removed to spatial infinity. Since by this procedure the supports of the insertions become unbounded, an appropriate renormalization has to be performed, as will turn out.

To simplify the analysis the model is treated in a version where the matter field has "frozen length". One expects that charged states can only occur in a Coulomb-like phase of the model. To deal with such a phase we first integrate over the matter field in the functional integral in a region of sufficiently small Higgs coupling $\beta_H R^2$, (2.4). This is done starting from a finite volume formulation of the model where the correlation function considered is the quotient of a modified partition function divided by the standard one. Using Fourier expansion the integration over the matter field can be performed in both partition functions rigorously and, by exponentiation, an effective interaction of the gauge field in the form of a convergent Mayer series can be obtained. However, since its Gaussian part, given by the sum of the Maxwell and the Chern–Simons term, is complex, probabilistic methods are not applicable for further analysis. Therefore, in place of the exponentiation we form the remaining (Gaussian) gauge field expectations of the power series which emerged from integration over the matter field in the numerator and the denominator, respectively. The

quotient of the resulting series, obtained in the sense of formal power series in the (small) activities, yields a series representation of the four–point correlation function considered. Given this series the infinite volume limit has to be performed followed by removing the countercharges to infinity. These limit procedures are worked out explicitly in first and second order of the activities. The long range of the gauge field preserved in the imaginary part of its propagator in spite of the topological mass occuring, demands particular concern.

This article is organized as follows. In sect. 2 the non–compact Abelian Higgs model with Chern–Simons term and the matter field having "frozen length" is defined on a finite subcomplex $\Lambda \subset \mathbb{Z}^3$ which acts as a 2+1 dimensional space–(Euclidean) time lattice. Moreover, the Euclidean functional integral representation of the correlation function is given wherefrom the desired anyon–antianyon correlation function has to be obtained by limit procedures. In sect. 3 the above Euclidean functional integral is expanded in several steps. Combining cluster expansion techniques with a formal division of power series we finally obtain the series representation (3.19) involving sums of products of activities in rising order. The infinite volume limit of and the removal of the countercharges in the sum of terms linear in the activities is analyzed in sect. 4. In sect. 5 we first perform the corresponding analysis of the sum of terms bilinear in the activities. To remove the countercharges requires a renormalization which is worked out explicitly. This renormalization suggests a renormalization factor of exponential type. In the sum of terms trilinear in the activities, having settled their infinite volume limit, we observe the propagation of the renormalization as required by the factor suggested. Sect. 6 contains concluding remarks. In an Appendix some technical details relevant for sections 4 and 5 are listed.

2. THE MODEL

The lattice considered is a finite open subcomplex Λ of $\mathbb{Z}^3$, i.e. Λ does not contain the sites, bonds and plaquettes of its boundary [13]. Moreover, Λ is chosen symmetric under reflection in the time–zero lattice plane. Basic fields are the gauge field A which is a real–valued 1–form, and the scalar field ϕ which is a complex–valued 0–form, supported on the bonds and sites of Λ , respectively. The non–compact Abelian Higgs model with Chern–Simons term is considered in the limit of the scalar field having fixed absolute value $|\phi_x| = R$, hence $\phi_x = R \exp i\varphi_x$ with $\varphi_x \in [-\pi, \pi)$. Denoting by d and δ the lattice exterior derivative and co–derivative, respectively, (for details of the notation see the Appendix of [11]) the total action on Λ is given by the sum

$$S_\Lambda = S_\Lambda^{(0)} + S_\Lambda^{\text{g.f.}} + S_\Lambda^{(1)} + S_\Lambda^{\text{CS}} \quad (2.1)$$

formed of the Maxwell term, a gauge fixing, the minimal coupling and the Chern–Simons term

$$S_{\Lambda}^{(0)} = \frac{1}{2} \beta_G (dA, dA) \quad (2.2)$$

$$S_{\Lambda}^{g.f.} = \frac{1}{2} \beta_G \kappa (\delta A, \delta A) \quad (2.3)$$

$$S_{\Lambda}^{(1)} = \beta_H R^2 \sum_b [1 - \cos(A + d\varphi)(b)] \quad (2.4)$$

$$\begin{aligned} S_{\Lambda}^{CS} = i \frac{\alpha}{2} \sum_y \epsilon_{\lambda\mu\nu} [A_{\lambda}(y - \hat{\lambda}) + A_{\lambda}(y - \hat{\lambda} + \hat{0}) \\ + A_{\lambda}(y + \hat{1} + \hat{2} - \hat{\lambda}) + A_{\lambda}(y + \hat{1} + \hat{2} - \hat{\lambda} + \hat{0})] (dA)_{\mu\nu}(y) \end{aligned} \quad (2.5)$$

We used the notations $(.,.)$ for the ℓ^2 –inner products of p–forms and $\hat{\mu}$ for the lattice unit vectors in μ –direction, $\mu = 0, 1, 2$.

Although (2.5) is purely imaginary, it is invariant under Osterwalder–Schrader involution at the time–zero lattice plane. (We observe that the first pair of terms in (2.5), as well as the second one, already have this property).

The action (2.1) implies an Osterwalder–Schrader positive measure [10]. Moreover, disregarding the gauge–fixing, it is invariant under local gauge transformations $A \rightarrow A + df$, $\varphi \rightarrow \varphi - f$ where f is a real–valued 0–form on Λ . This latter property of the action is a consequence of the boundary conditions chosen. The action (2.1), together with the product Lebesgue measures $\mathcal{D}A$ and $\mathcal{D}\varphi$ extending over the positively oriented bonds and sites of Λ , respectively, defines the complex normalized measure

$$d\nu_{\Lambda} = \frac{1}{Z_{\Lambda}} \mathcal{D}A \mathcal{D}\varphi e^{-S_{\Lambda}} \quad (2.6)$$

where the normalization factor Z_{Λ} is determined by $\int d\nu_{\Lambda} = 1$.

An anyon–antianyon correlation function would emerge as a limit – provided it exists! – from the expectation

$$\frac{Z_{\Lambda}(\omega, j)}{Z_{\Lambda}} \equiv \int d\nu_{\Lambda} \phi_x \overline{\phi}_y e^{-i(A,j)} \overline{\phi}_{x^c} \phi_{y^c} \cdot \exp \left\{ -S_{\Lambda}^{(0)}(dA + \frac{1}{8\alpha} \omega) + S_{\Lambda}^{(0)}(dA) \right\} \quad (2.7)$$

To the matter fields, in order to be transmuted into anyons, both a disorder insertion ω , which is a singular external gauge field, and a related Dirac–Mandelstam string j have to be attached [11,12]. In addition, the matter fields $\overline{\phi}_{x^c}, \phi_{y^c}$, acting as countercharges, have to be removed to infinity after the infinite volume limit $\Lambda \nearrow \mathbb{Z}^3$ has been performed. The dual of ω is a pair of two straight line segments formed of bonds of the dual lattice in 1–direction. Denoting by $\delta(z,x)$ the Kronecker symbol on the lattice, i.e. $\delta(z,x) = 1$ if $z = x$ and vanishing otherwise, we have

$$\omega_{\mu\nu}(z) = (\delta_{\mu 2} \delta_{\nu 0} - \delta_{\mu 0} \delta_{\nu 2}) \sum_{n=1}^{L+1} \{ \delta(z, x - \hat{0} + n\hat{1}) - \delta(z, y + n\hat{1}) \} \quad (2.8)$$

satisfying

$$(d\omega)_{012}(z) = \delta(z, x - \hat{0}) - \delta(z, x^c) - \delta(z, y) + \delta(z, y^c - \hat{0}) \quad (2.9)$$

where $x^c \equiv x + (L+1)\hat{1} - \hat{0}$ and $y^c \equiv y + (L+1)\hat{1} + \hat{0}$. Moreover, we choose $y = x - T\hat{0}$ with $T \in 2\mathbb{N}$. The (external) current j corresponding to ω , which enters (2.7), reads

$$j_{\mu} = r_{\mu} + j_{\mu}^{(L)} \quad (2.10)$$

$$\begin{aligned} j_{\mu}^{(L)}(z) = & \delta_{\mu 1} \frac{1}{4} \sum_{n=1}^L \left\{ \delta(z, x + n\hat{1}) + \delta(z, x + \hat{2} + n\hat{1}) \right. \\ & + \delta(z, x - \hat{0} + n\hat{1}) + \delta(z, x + \hat{2} - \hat{0} + n\hat{1}) \\ & - \delta(z, y + n\hat{1}) - \delta(z, y + \hat{2} + n\hat{1}) \\ & \left. - \delta(z, y + \hat{0} + n\hat{1}) - \delta(z, y + \hat{2} + \hat{0} + n\hat{1}) \right\} \end{aligned} \quad (2.11)$$

$$\begin{aligned}
4 \mathbf{r}_\mu(z) = & \delta_{\mu 0} \left\{ -\delta(z, x - \hat{0}) - \delta(z, x + \hat{1} + \hat{2} - \hat{0}) - \delta(z, x^c + \hat{2}) - \delta(z, x^c) \right. \\
& \left. - \delta(z, y) - \delta(z, y + \hat{1} + \hat{2}) - \delta(z, y^c - \hat{0}) - \delta(z, y^c + \hat{2} - \hat{0}) \right\} \\
& + \delta_{\mu 1} \left\{ 2\delta(z, x) + \delta(z, x + \hat{2}) + \delta(z, x - \hat{0}) \right. \\
& \left. - 2\delta(z, y) - \delta(z, y + \hat{2}) - \delta(z, y + \hat{0}) \right\} \\
& + \delta_{\mu 2} \left\{ \delta(z, x) + \delta(z, x + \hat{1}) - 2\delta(z, x^c) \right. \\
& \left. - \delta(z, y) - \delta(z, y + \hat{1}) + 2\delta(z, y^c) \right\}
\end{aligned} \tag{2.12}$$

(See fig. 1). The cumbersome shape of this current on the lattice is dictated by the desired commutation properties of anyon operators in a fixed-time plane [5,11]. Of course, there is some freedom in installing the countercharges, which are finally shifted to infinity. The current (2.10) satisfies

$$(\delta j)(z) = \delta(z, x) - \delta(z, x^c) - \delta(z, y) + \delta(z, y^c) \tag{2.13}$$

Hence, the integrand in (2.7) is gauge-invariant. Moreover, for positions x with $x_0 = \frac{1}{2}$ T this integrand has the form $F(\Theta F)$, where Θ denotes Osterwalder–Schrader involution at the $x_0 = 0$ lattice plane and F is supported in the sublattice $x_0 > 0$. The Osterwalder–Schrader positivity of $d\nu_\Lambda$ therefore implies that (2.7) is real and positive.

3. COMBINED EXPANSION OF THE CORRELATION FUNCTION.

In view of the known phase diagram of the non-compact Abelian Higgs model, see e.g. [14], one expects charged states to occur in this model when modified by including the Chern–Simons term only in a Coulomb-like phase, i.e. where the propagator of the electromagnetic potential is not summable. We assume $\beta_H R^2$ sufficiently small.

In order to evaluate the correlation function (2.7) we first integrate over the matter field. This is achieved by employing the Fourier expansion,

$$\exp \left\{ -S_\Lambda^{(1)} \right\} = \text{const} \sum_H \left[\prod_b c_{H_b}(\tau) \right] e^{-i(A + d\varphi, H)} \tag{3.1}$$

which involves summation over an integer-valued 1-form H , and $\tau \equiv \beta_H R^2$.

The expansion coefficients are formed by products of factors

$$c_n(\tau) = [I_0(\tau)]^{-1} I_n(\tau), \quad n \in \mathbb{Z} \quad (3.2)$$

emerging from each positively oriented bond b of Λ ; I_n is the standard modified Bessel function. Introducing the normalized complex Gaussian measure

$$d\mu_\Lambda(A) = \text{const } \mathcal{D}A \exp \left\{ -S_\Lambda^{(0)} - S_\Lambda^{\text{g.f.}} - S_\Lambda^{\text{CS}} \right\} \quad (3.3)$$

we obtain for the denominator of (2.7) – ignoring the constant factors in (3.1) and (3.3) which do not affect the quotient

$$Z_\Lambda = \int d\mu_\Lambda(A) \sum_{\substack{H \\ \delta H=0}} \left[\prod_b c_{H_b}(\tau) \right] e^{-i(A,H)} \quad (3.4)$$

The integer-valued co-closed 1-form H , $\delta H = 0$, can be decomposed into disjoint connected components, $H = \{h^{(1)}, \dots, h^{(r)}\}$. Hence, the $h^{(i)}$ are connected integer-valued co-closed 1-forms, the supports of which do not touch mutually. To each component $h^{(i)}$ an activity

$$z(h^{(i)}) = \prod_{b \subset \text{supp } h^{(i)}} c_{H_b}(\tau) \quad (3.5)$$

is attributed. Then (3.4) reads

$$Z_\Lambda = \int d\mu_\Lambda(A) \left\{ 1 + \sum_{\{h^{(1)}, \dots, h^{(r)}\}} \prod_{p=1}^r z(h^{(p)}) e^{-i(A, h^{(p)})} \right\} \quad (3.6)$$

the summation extending over sets of components $\{h^{(1)}, \dots, h^{(r)}\}$, $r \in \mathbb{N}$, with the property, that in each set its members are mutually disjoint.

The support of a component $h^{(i)}$, $J = \text{supp } h^{(i)}$, is formed by a connected, generally branched lattice loop. These supports $\{J^{(\ell)}\}$ define our normal polymers. To the polymer $J^{(\ell)}$ we attribute the activity

$$\zeta(J^{(\ell)}; A) = \sum_{\substack{p \\ \text{supp } h(p) = J^{(\ell)}}} z(h(p)) e^{-i(A, h(p))} \quad (3.7)$$

which is obviously real, due to summation on complex conjugate contributions. Using the identity $\sum I_n(x) = \exp x$, with summation on n extending over $\mathbb{Z}$, the bound, uniform in A ,

$$|\zeta(J^{(\ell)}; A)| < \left[\frac{e^\tau}{I_0(\tau)} - 1 \right]^{|J^{(\ell)}|} \quad (3.8)$$

follows easily, where $|J^{(\ell)}|$ denotes the number of bonds composing $J^{(\ell)}$. In terms of the activities (3.7), we write (3.6) in the form

$$Z_\Lambda = \int d\mu_\Lambda(A) \left\{ 1 + \sum_{\{J^{(1)}, \dots, J^{(r)}\}} \prod_{\ell=1}^r \zeta(J^{(\ell)}; A) \right\} \quad (3.9)$$

where the summation extends over admissible sets of polymers, i.e. in each such set its members do not touch mutually. The curly bracket in (3.9) can be viewed as the polymer representation of a partition function (of a gas of loops). It can be exponentiated, due to the bound (3.8), for sufficiently small τ , yielding

$$Z_\Lambda = \int d\mu_\Lambda(A) \exp \sum_{r=1}^{\infty} \frac{1}{r!} \sum_{J^{(1)}, \dots, J^{(r)}} a(J^{(1)}, \dots, J^{(r)}) \prod_{p=1}^r \zeta(J^{(p)}; A) \quad (3.10)$$

where $a(J^{(1)}, \dots, J^{(r)})$ is the standard combinatorial coefficient entering the Mayer series [15].

In the case of the numerator in (2.7) the integration over the matter field requires the integer-valued 1-form H of (3.1) to satisfy

$$\delta H = -\rho, \quad \rho = \delta j \quad (3.11)$$

Therefore the decomposition into disjoint components is of the form $H = \{h_\rho, h^{(1)}, \dots, h^{(r)}\}$ with the $h^{(i)}$ already introduced before but h_ρ satisfying $\delta h_\rho = -\rho$. Moreover, $\text{supp } h_\rho$ may be connected or consist of two disjoint connected pieces. This latter case appears in two versions: either one piece joins x with y and the other x^c with y^c or x, x^c are joined by one and y, y^c by the other piece. We obtain

$$\begin{aligned}
 Z_{\Lambda}(\omega, j) = & R^4 e^{-\frac{1}{2} \beta_G \left[\frac{1}{8\alpha} \right]^2 (\omega, \omega)} \int d\mu_{\Lambda}(A) e^{-i(A, j) + \frac{\beta_G}{8\alpha} (A, \delta\omega)} \\
 & \cdot \sum_{\mathbf{h}_{\rho}} z(\mathbf{h}_{\rho}) e^{-i(A, \mathbf{h}_{\rho})} \left\{ 1 + \sum_{\{h_1^{(1)}, \dots, h^{(r)}\} \in \Sigma'} \prod_{p=1}^r z(\mathbf{h}^{(p)}) e^{-i(A, \mathbf{h}^{(p)})} \right\} \quad (3.12)
 \end{aligned}$$

where the prime restricts the summation on those sets $\{h_1^{(1)}, \dots, h^{(r)}\}$, having only members that are mutually disjoint and are disjoint from $\mathbf{h}_{\rho}$, too. Grouping furthermore the components $h^{(i)}$ to normal polymers as before, we get

$$\begin{aligned}
 Z_{\Lambda}(\omega, j) = & R^4 e^{-\frac{1}{2} \beta_G \left[\frac{1}{8\alpha} \right]^2 (\omega, \omega)} \int d\mu_{\Lambda}(A) e^{-i(A, j) + \frac{\beta_G}{8\alpha} (A, \delta\omega)} \\
 & \cdot \sum_{\mathbf{h}_{\rho}} z(\mathbf{h}_{\rho}) e^{-i(\mathbf{h}_{\rho}, A)} \left\{ 1 + \sum_{\{J^{(1)}, \dots, J^{(r)}\} \in \Sigma'} \prod_{\ell=1}^r \zeta(J^{(\ell)}; A) \right\} \quad (3.13)
 \end{aligned}$$

The prime restricts summation to such collections of normal polymers where the members of each collection neither touch mutually nor touch $\text{supp } \mathbf{h}_{\rho}$. The curly bracket in (3.13) can again be exponentiated and the exponent arising can be written as a difference,

$$\begin{aligned}
 Z_{\Lambda}(\omega, j) = & R^4 e^{-\frac{1}{2} \beta_G \left[\frac{1}{8\alpha} \right]^2 (\omega, \omega)} \\
 & \cdot \int d\mu_{\Lambda}(A) \exp \left\{ \sum_{r=1}^{\infty} \frac{1}{r!} \sum_{\{J^{(1)}, \dots, J^{(r)}\} \in \Sigma} a(J^{(1)}, \dots, J^{(r)}) \prod_{\ell=1}^r \zeta(J^{(\ell)}; A) \right\} \\
 & \cdot e^{-i(A, j) + \frac{\beta_G}{8\alpha} (A, \delta\omega)} \sum_{\mathbf{h}_{\rho}} z(\mathbf{h}_{\rho}) e^{-i(A, \mathbf{h}_{\rho})} \\
 & \cdot \exp \left\{ - \sum_{r=1}^{\infty} \frac{1}{r!} \sum_{\substack{\{J^{(1)}, \dots, J^{(r)}\} \\ (\cup_{\ell} J^{(\ell)}) \cap \text{supp } \mathbf{h}_{\rho} \neq \emptyset}} a(J^{(1)}, \dots, J^{(r)}) \prod_{\ell=1}^r \zeta(J^{(\ell)}; A) \right\} \quad (3.14)
 \end{aligned}$$

For sufficiently small τ the additional action of the electromagnetic potential A appearing in (3.10) and (3.14) and modifying the Gaussian measure $d\mu_\Lambda(A)$ is well-behaved: its absolute value is bounded and the bound proportional to the volume $|\Lambda|$. However, in an analysis of the arising non-Gaussian measure probabilistic techniques, e.g. the method of complex translations [16], are not applicable, since the Gaussian measure $d\mu_\Lambda(A)$ is complex. Our treatment of the quotient (2.7) is to generate an expansion dividing (3.12) by (3.6) as formal power series in the number of activity factors. The external currents j , $\delta\omega$ and the matter currents h_ρ , $\{h^{(p)}\}$ interact via the Gaussian measure $d\mu_\Lambda(A)$. Hence, for (complex) currents g , g_1 , g_2 with supports contained in Λ we have

$$\int d\mu_\Lambda(A) e^{-i(A,g)} = \exp \left\{ -\frac{1}{2\beta_G} (g, C_\Lambda g) \right\} \quad (3.15)$$

and the covariance C_Λ is assumed to satisfy

$$(g_1, C_\Lambda g_2) = (g_2, C_\Lambda g_1) \quad (3.16)$$

Introducing the complex currents, called "mixed currents",

$$\mathcal{H}_\rho = h_\rho + j + i \frac{\beta_G}{8\alpha} \delta\omega \quad (3.17)$$

which are conserved, $\delta\mathcal{H}_\rho = 0$, due to (3.11), we obtain the expansion, with

$$\beta \equiv \beta_G, \quad (3.18)$$

$$\begin{aligned} \frac{Z_\Lambda(\omega, j)}{Z_\Lambda} &= R^4 \sum_{h_\rho} z(h_\rho) \exp \left[-\frac{\beta}{2} \left[\frac{1}{8\alpha} \right]^2 (\omega, \omega) - \frac{1}{2\beta} (\mathcal{H}_\rho, C_\Lambda \mathcal{H}_\rho) \right] \\ &\quad \cdot \left\{ 1 + \sum_{n=1}^{\infty} V_\Lambda^{(n)}(\mathcal{H}_\rho) \right\} \end{aligned} \quad (3.19)$$

$$V_\Lambda^{(1)}(\mathcal{H}_\rho) = \sum_h' \tilde{z}(h) \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] - \sum_h \tilde{z}(h) \quad (3.20)$$

$$\begin{aligned} V_\Lambda^{(2)}(\mathcal{H}_\rho) &= \sum_{\{h^{(1)}, h^{(2)}\}}' \tilde{z}(h^{(1)}) \tilde{z}(h^{(2)}) \exp \left[-\frac{1}{\beta} (h^{(1)}, C_\Lambda h^{(2)}) - \frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h^{(1)}) \right. \\ &\quad \left. - \frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h^{(2)}) \right] \\ &\quad - \sum_h' \tilde{z}(h) \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] \cdot \sum_h \tilde{z}(h) \\ &\quad - \sum_{\{h^{(1)}, h^{(2)}\}} \tilde{z}(h^{(1)}) \tilde{z}(h^{(2)}) \exp \left[-\frac{1}{\beta} (h^{(1)}, C_\Lambda h^{(2)}) \right] + \left[\sum_h \tilde{z}(h) \right]^2 \end{aligned} \quad (3.21)$$

In (3.20), (3.21) the renormalized activity

$$\tilde{z}(\mathbf{h}^{(i)}) \equiv z(\mathbf{h}^{(i)}) \exp \left[-\frac{1}{2\beta} (\mathbf{h}^{(i)}, C_\Lambda \mathbf{h}^{(i)}) \right] \quad (3.22)$$

of a current $\mathbf{h}^{(i)}$, compare (3.5), caused by "electromagnetic" self-energy, has been introduced. The expansion (3.19) can be seen in the following way: summation on $\mathbf{h}_\rho$ describes fluctuations of the internal current line(s), which connect the charges and emerge from integrating the matter field, whereas the sums $V_\Lambda^{(n)}(\mathcal{H}_\rho)$ represent the interaction of the mixed currents $\mathcal{H}_\rho$, (3.17), with vacuum polarization currents $\mathbf{h}^{(i)}$ resulting from the integration of the matter field, too.

In order to convert (3.19) into an anyon–antianyon correlation function two successive limits have still to be performed: i) the infinite volume limit $\Lambda \nearrow \mathbb{R}^3$, then ii) the removal of the countercharges, i.e. $L \nearrow \infty$ with T kept fixed.

4. FLUCTUATIONS OF THE INTERNAL CURRENT LINES

We first present the covariance C_Λ of the Gaussian measure (3.3), (3.15) in the infinite volume limit $\Lambda \nearrow \mathbb{R}^3$ and denote it by C . Recalling (3.18) and introducing the notation

$$M(\mathbf{k}) = \frac{2\alpha}{\beta} (1 + e^{-i\mathbf{k}_0}) (1 + e^{-i\mathbf{k}_1 - i\mathbf{k}_2}) \quad (4.1)$$

$$K_\mu = i(1 - e^{i\mathbf{k}_\mu}), \quad \lambda(\mathbf{k}) = \sum_{\mu=0}^2 |K_\mu|^2 \quad (4.2)$$

we find

$$C_{\nu\sigma}(z) = \left[\prod_{\mu=0}^2 \int_{-\pi}^{\pi} \frac{d\mathbf{k}_\mu}{2\pi} \right] \frac{e^{i\mathbf{k}z}}{\lambda(\mathbf{k}) + |M(\mathbf{k})|^2} \quad (4.3)$$

$$\cdot \left\{ \delta_{\nu\sigma} + \frac{M(\mathbf{k})}{\lambda(\mathbf{k})} e^{i\mathbf{k}_\nu} \epsilon_{\nu\rho\sigma} K_\rho - \left[1 - \frac{1}{\kappa} - \frac{|M(\mathbf{k})|^2}{\kappa\lambda(\mathbf{k})} \right] \frac{K_\nu K_\sigma}{\lambda(\mathbf{k})} \right\}$$

which we decompose according to the sum of the three terms occuring in the curly bracket

$$C_{\nu\sigma}(z) = C_{\nu\sigma}^{(1)}(z) + iC_{\nu\sigma}^{(2)}(z) + C_{\nu\sigma}^{(3)}(z) \quad (4.4)$$

We observe that $C^{(3)}$ vanishes upon contraction with a conserved current. Both $C^{(1)}$ and $C^{(2)}$ are real-valued and satisfy the relation (3.16) individually. Most important for the sequel: whereas $C^{(1)}$ decays exponentially, due to the "topological mass" $M(k)$, $iC^{(2)}$ decays only with the square of the inverse distance, because of the factor $K_\rho[\lambda(k)]^{-1}$.

Considering now the weight factor $\exp \left\{ -\frac{\beta}{2} \left[\frac{1}{8\alpha} \right]^2 \left[(\omega, \omega) + \left[\frac{8\alpha}{\beta} \right]^2 (\mathcal{H}_\rho, C_\Lambda \mathcal{H}_\rho) \right] \right\}$ in the expansion (3.19) which multiplies the activity $z(h_\rho)$ of a given h_ρ , it obviously has an infinite volume limit. Decomposition into real and imaginary part gives

$$\begin{aligned}
 & (\omega, \omega) + \left[\frac{8\alpha}{\beta} \right]^2 (\mathcal{H}_\rho, C \mathcal{H}_\rho) \\
 &= (\omega, \omega) - (\delta\omega, C^{(1)} \delta\omega) \\
 &+ \left[\frac{8\alpha}{\beta} \right]^2 (h_{\rho+j}, C^{(1)} [h_{\rho+j}]) - \frac{16\alpha}{\beta} (h_{\rho+j}, C^{(2)} \delta\omega) \\
 &+ i \left\{ \frac{16\alpha}{\beta} (h_{\rho+j}, C^{(1)} \delta\omega) - (\delta\omega, C^{(2)} \delta\omega) \right. \\
 &\quad \left. + \left[\frac{8\alpha}{\beta} \right]^2 (h_{\rho+j}, C^{(2)} [h_{\rho+j}]) \right\}
 \end{aligned} \tag{4.5}$$

Since the model is treated here in the domain where the parameter τ is (sufficiently) small, due to the dependence of the activity $z(h_\rho)$ on τ we only need to consider such h_ρ which consist of a pair of current lines (generally branched), one leading from x to y and the other from x^c to y^c , see text after (3.11). The minimal version of this type – the "static approximation" – reads

$$(h_\rho^{\min})_\mu(z) = \delta_{\mu 0} \left\{ \sum_{n=0}^{T-1} \delta(z, y+n\hat{0}) - \sum_{n=0}^{T-3} \delta(z, y^c+n\hat{0}) \right\} \tag{4.6}$$

Taking in (4.5) finally L to infinity seems at first sight not to lead to a finite limit, since (ω, ω) and all bilinears composed of j or $\delta\omega$ grow linearly with L , if they are different from zero. They are listed in the Appendix. Whereas those appearing in the imaginary part of (4.5) vanish, (A.12), it is very remarkable that in the sum of the terms contributing to the real part, (A.8–10), the behaviour proportional to L cancels and a finite limit results, (A.11). Close inspection of the remaining terms in (4.5) reveals that they remain finite, too, for $L \nearrow \infty$. Hence, the limit

$$\lim_{L \rightarrow \infty} \left\{ (\omega, \omega) + \left[\frac{8\alpha}{\beta} \right]^2 (\mathcal{H}_\rho, C \mathcal{H}_\rho) \right\} = a_\rho(T) \quad (4.7)$$

is a well-defined function of the distance T. The large T behaviour in (4.5) emerges from

$$\lim_{L \rightarrow \infty} \left[\frac{8\alpha}{\beta} \right]^2 (h_\rho, C^{(1)} h_\rho) \quad (4.8)$$

which yields in the case of the static approximation (4.6)

$$\begin{aligned} & \lim_{L \rightarrow \infty} \left[\frac{8\alpha}{\beta} \right]^2 (h_\rho^{\min}, C^{(1)} h_\rho^{\min}) \\ &= \left[\frac{8\alpha}{\beta} \right]^2 \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\lambda(k) + |M(k)|^2} \left\{ \frac{1 - \cos k_0 T}{1 - \cos k_0} + \frac{1 - \cos k_0 (T-2)}{1 - \cos k_0} \right\} \end{aligned} \quad (4.9)$$

Stating the main result of this section: due to (4.7), the weight factors multiplying the activities $z(h_\rho)$ in the expansion (3.19) have finite limits when L is taken to infinity.

5. CORRECTIONS BY VACUUM POLARIZATION

In this section the rôle of vacuum polarization currents in the expansion (3.19) is considered. Beginning with (3.20), we rewrite this sum in view of the limits to be investigated in the form

$$\begin{aligned} V_\Lambda^{(1)}(\mathcal{H}_\rho) &= \sum_h \tilde{z}(h) \left\{ \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] - 1 \right\} \\ &- \sum_{\substack{h \\ h \cap h_\rho \neq \emptyset}} \tilde{z}(h) \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] \end{aligned} \quad (5.1)$$

Because of the (renormalized) activities appearing in the sums there is a natural order to perform the summations: the set $\{h\}$ is partitioned into disjoint subsets, called "classes". The class $[\underline{h}]$ contains all those h which are lattice translates of $\underline{h}$. Then one sums first over the elements of a given class, and hereafter over different classes

$$\begin{aligned} & \sum_h \tilde{z}(h) \left\{ \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] - 1 \right\} \\ &= \sum_{[\underline{h}]} \tilde{z}(\underline{h}) \sum_{h \in [\underline{h}]} \left\{ \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C_\Lambda h) \right] - 1 \right\} \end{aligned} \quad (5.2)$$

In the infinite volume limit, the elements of a class are indexed by $u \in \mathbb{Z}^3$.

The limits to be performed in (5.1) are obvious in the second sum. Because of the constraint in each class only a finite number of elements contributes, not depending on Λ . Letting then $L \nearrow \infty$ in the infinite volume limits $(\mathcal{H}_\rho, C h)$ leads again to finite limits.

We claim that summation over the elements of a class in (5.2) has an infinite volume limit, since the contraction of C with two conserved localized currents improves the decay leading to the bound

$$|(\mathcal{H}_\rho, C h)| < \text{const} [\text{distance} < \mathcal{H}_\rho, h >]^{-4} \quad (5.3)$$

for large distances. However, this bound is not sufficient to deduce the limit $L \nearrow \infty$ still to be performed. To this end finer knowledge has to be gained. We define on $\mathbb{Z}^3$ the Fourier-transform of a 1-form J with bounded support by

$$\hat{J}_\mu(k) = \sum_{z \in \mathbb{Z}^3} e^{ikz} J_\mu(z) \quad (5.4)$$

and state the

Lemma: Let h be a co-closed 1-form (conserved current), $\delta h = 0$, with bounded connected support on the bonds of $\mathbb{Z}^3$, then its Fourier transform $\hat{h}$ can be given the representation

$$\hat{h}_\sigma(k) = e^{iku} \sum_{\mu=0}^2 (e^{ik_\mu-1}) p_{\sigma\mu}(e^{ik_0}, e^{ik_1}, e^{ik_2}) \quad (5.5a)$$

where $u \in \mathbb{Z}^3$ and the functions $p_{\sigma\mu}$ are polynomials in the variables $e^{ik_0}, e^{ik_1}, e^{ik_2}$ satisfying

$$p_{\mu\sigma}(e^{ik_0}, e^{ik_1}, e^{ik_2}) = -p_{\sigma\mu}(e^{ik_0}, e^{ik_1}, e^{ik_2}) \quad (5.5b)$$

The simple proof is omitted.

In (5.5) the point u characterizes the position of the support of h . Considering as an example a current of strength m supported on the boundary of the plaquette $(u; \mu, \nu)$ we have $p_{\nu\mu} = -p_{\mu\nu} = m$ and all other $p_{\alpha\beta}$ vanishing.

Turning to the dependence of $(\mathcal{H}_\rho, C h)$ on u and L , we separate into real and imaginary part

$$\frac{1}{\beta} (\mathcal{H}_\rho, C h) = X_\rho + i Y_\rho \quad (5.6)$$

$$X_\rho = \frac{1}{\beta} (h_{\rho+j}, C^{(1)} h) - \frac{1}{8\alpha} (\delta\omega, C^{(2)} h) \quad (5.7)$$

$$Y_\rho = \frac{1}{8\alpha} (\delta\omega, C^{(1)} h) + \frac{1}{\beta} (h_{\rho+j}, C^{(2)} h) \quad (5.8)$$

Using (2.10), (A.1–4) and in addition (5.5) to extract a particular L –dependence, X_ρ and Y_ρ can be represented after a somewhat lengthy calculation as follows

$$\begin{aligned} X_\rho = \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\lambda(k) + |M(k)|^2} \left\{ -\frac{1}{4} (1 + e^{ik_0}) [1 + e^{ik_2 + ik_1(L+1)}] \kappa \hat{h}_1(-k) \right. \\ \left. + \widehat{(h_{\rho+r})}_\nu(k) \hat{h}_\nu(-k) - \frac{\beta}{8\alpha} \frac{M(k)}{\lambda(k)} [1 - e^{ik_1(L+1)}] \kappa \sum_{\mu=0}^2 (e^{-ik} \mu_{-1}) \hat{h}_\mu(-k) \right\} \quad (5.9) \end{aligned}$$

$$Y_\rho = Y + \tilde{Y}_\rho \quad (5.10)$$

$$\begin{aligned} \tilde{Y}_\rho = \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\lambda(k) + |M(k)|^2} \frac{M(k)}{\lambda(k)} \\ \cdot \left\{ \widehat{(h_{\rho+r})}_\nu(k) e^{ik_\nu} \epsilon_{\nu\rho\sigma} (1 - e^{ik_\rho}) \hat{h}_\sigma(-k) \right. \\ \left. - \frac{1}{4} (1 + e^{ik_0}) (e^{ik_1} + e^{ik_2 + ik_1(L+2)}) \kappa \right. \\ \left. \cdot [(1 - e^{ik_2}) \hat{h}_0(-k) - (1 - e^{ik_0}) \hat{h}_2(-k)] \right\} \quad (5.11) \end{aligned}$$

$$\begin{aligned} + \frac{1}{8\alpha} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\lambda(k)} [1 - e^{ik_1(L+1)}] \kappa e^{-iku} \\ \cdot [(1 - e^{ik_0}) p_{12}(e^{-ik}) + (1 - e^{ik_1}) p_{20}(e^{-ik}) + (1 - e^{ik_2}) p_{01}(e^{-ik})] \\ Y = \frac{1}{8\alpha} \int \frac{d^3 k}{(2\pi)^3} \sum_{n=1}^{L+1} e^{ink_1} [e^{ik(x-\hat{0})} - e^{iky}] e^{-iku} p_{20}(e^{-ik}) \quad (5.12) \end{aligned}$$

with the concise notation $p_{\mu\sigma}(e^{-ik}) = p_{\sigma\mu}(e^{-ik_0}, e^{-ik_1}, e^{-ik_2})$. Inspection of these integrals reveals the following crucial properties: For all X_ρ, Y_ρ of a given class $[\underline{h}]$, i.e. given $\{p_{\sigma\mu}\}$, holds

$$i) \quad |X_\rho| \text{ and } |Y_\rho| \text{ are uniformly bounded in } (L, u). \quad (5.13)$$

$$ii) \quad X_\rho = E_1(u) + \dots + E_a(u) + E_{a+1}(u-L\hat{1}) + \dots + E_{a+b}(u-L\hat{1}) \\ + W(u) - W(u-(L+1)\hat{1}) \quad (5.14a)$$

with a fixed number of L -independent functions $E_1, \dots, E_{a+b}, W$ satisfying

$$|u| \rightarrow \infty: \begin{cases} |E_j(u)| \leq \text{const} \exp(-m|u|), j=1, \dots, a+b \\ |W(u)| \leq \text{const} |u|^{-4} \end{cases} \quad (5.14b)$$

$$iii) \quad \tilde{Y}_\rho = V_1(u) + \dots + V_c(u) + V_{c+1}(u-L\hat{1}) + \dots + V_{c+d}(u-L\hat{1}) \\ + W_0(u) + W_0(u-(L+1)\hat{1}-\hat{2}) \quad (5.15a)$$

with a fixed number of L -independent functions $V_1, \dots, V_{c+d}, W_0$ satisfying

$$|u| \rightarrow \infty: \begin{cases} |V_j(u)| \leq \text{const} |u|^{-3}, j=1, \dots, c+d \\ |W_0(u)| \leq \text{const} |u|^{-4} \end{cases} \quad (5.15b)$$

The claim i) is immediate in (5.9) and (5.11) and follows in (5.12) from orthogonality. The particular power law behaviour in (5.14b) and (5.15b) results from the inverse Laplacian $[\lambda(k)]^{-1}$ being multiplied by factors which have L -independent zeros.

After these preparations we come back to the sum (5.2). Due to the relation $\tilde{z}(-h) = \tilde{z}(h)$ it can be rewritten, the infinite volume limit granted,

$$\sum_h \tilde{z}(h) \{ \exp \left[-\frac{1}{\beta} (\mathcal{H}_\rho, C h) \right] - 1 \} \\ = 2 \sum_{[\underline{h}]_+} \tilde{z}(\underline{h}) \sum_{h \in [\underline{h}]_+} \left\{ \cosh \frac{1}{\beta} (\mathcal{H}_\rho, C h) - 1 \right\} \quad (5.16)$$

By definition, the class $[\underline{h}]_+$ is one out of the pair $\{[\underline{h}], [-\underline{h}]\}$ and can be chosen arbitrarily. The decomposition (5.6) implies

$$\begin{aligned} & \sum_{\underline{h} \in [\underline{h}]_+} \left\{ \cosh \frac{1}{\beta} (\mathcal{H}_\rho, C \underline{h}) - 1 \right\} \\ &= \sum_{\underline{u} \in \mathbb{Z}^3} \left\{ (\cosh X_\rho - 1) \cos Y_\rho + (\cos Y_\rho - 1) + i \sinh X_\rho \cdot \sin Y_\rho \right\} \end{aligned} \quad (5.17)$$

In the sums involving X_ρ the limit $L \nearrow \infty$ is easily controlled. Since $|X_\rho|$ is uniformly bounded in $(L, \underline{u})$, see (5.13), there are positive constants c_1, c_2 such that

$$\sum_{\underline{u}} |(\cosh X_\rho - 1) \cos Y_\rho| < \sum_{\underline{u}} c_1 X_\rho^2, \quad \sum_{\underline{u}} |\sinh X_\rho \sin Y_\rho| < \sum_{\underline{u}} c_2 |X_\rho| \quad (5.18)$$

is valid $\forall L$. The form (5.14) of X_ρ then implies finite limits

$$\lim_{L \nearrow \infty} \sum_{\underline{u} \in \mathbb{Z}^3} X_\rho^2 < \infty, \quad \lim_{L \nearrow \infty} \sum_{\underline{u} \in \mathbb{Z}^3} |X_\rho| < \infty \quad (5.19)$$

The remaining sum in (5.17) can be bounded by

$$\sum_{\underline{u}} (1 - \cos Y_\rho) \leq \frac{1}{2} \sum_{\underline{u}} Y_\rho^2 \quad (5.20)$$

Employing the representations (5.10–12) leads to

$$\sum_{\underline{u} \in \mathbb{Z}^3} Y_\rho^2 = \left[\frac{1}{8\alpha} \right]^2 (L+1) \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \left| \mathcal{K} p_{20}(e^{-i\mathbf{k}}) \right|^2 \Big|_{\mathbf{k}_1=0} + \mathcal{O}(L^0) \quad (5.21)$$

Since the inequality becomes an equality at $Y_\rho = 0$, the sum on the l.h.s. of (5.20) diverges linearly with L . In order to extract this dependence on L we observe that due to the polynomial

$$p_{20}(e^{ik_0}, e^{ik_1}, e^{ik_2}) = \sum_{\mathbf{q}} c_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{k}} \quad (5.22)$$

with $\mathbf{q} = (q_0, q_1, q_2)$, $q_\mu \in \mathbb{N} \cup \{0\}$, and real coefficients $c_{\mathbf{q}}$ the corresponding $Y = Y(\underline{u})$, (5.12) reads

$$Y(u) = \frac{1}{8\alpha} \sum_q c_q \sum_{n=1}^{L+1} \{ \delta(u, n\hat{1} + x - \hat{0} - q) - \delta(u, n\hat{1} + y - q) \} \quad (5.23)$$

Hence its support has a cylinder-like shape, growing linearly with L in the 1-direction. Returning to the sum still to be considered, the decomposition (5.10) implies

$$\sum_{u \in \mathbb{Z}^3} (1 - \cos Y_\rho) = \sum_{\ell=1}^{\infty} \frac{(-1)^{\ell+1}}{(2\ell)!} \sum_{u \in \mathbb{Z}^3} \{ Y^{2\ell} + 2\ell Y^{2\ell-1} \tilde{Y}_\rho + \dots + \tilde{Y}_\rho^{2\ell} \} \quad (5.24)$$

Due to the support of Y , (5.23), $\forall \ell \in \mathbb{N}$ holds

$$\sum_{u \in \mathbb{Z}^3} Y^{2\ell} = 2[L a_\ell + b_\ell] \quad (5.25)$$

with the constants a_ℓ, b_ℓ determined by p_{20} , i.e. the set $\{c_q\}$, not depending on L and T (for T big enough), however. (We remark, that the class of currents given as an example after the Lemma would lead in the case of $\mu = 0, \nu = 2$ to $a_\ell = b_\ell = (m/8\alpha)^{2\ell}$) Furthermore, because of the power law decay of $\tilde{Y}_\rho$, (5.15), and the support of $Y, \forall m, p \in \mathbb{N}$,

$$\lim_{L \nearrow \infty} \sum_{u \in \mathbb{Z}^3} Y^m \tilde{Y}_\rho^p < \infty. \quad (5.26)$$

Hence, from (5.24–26) follows

$$\sum_{u \in \mathbb{Z}^3} (1 - \cos Y_\rho) = 2L \sum_{\ell=1}^{\infty} \frac{(-1)^{\ell+1}}{(2\ell)!} a_\ell + F_\rho(T; L) \quad (5.27)$$

with $F_\rho(T; L)$ having a finite limit for $L \nearrow \infty$. Of course, the r.h.s. of (5.27) inherits from Y_ρ the dependence on p_{20} determined by the class $[\underline{h}]$ considered.

Collecting finally pieces, it follows from (5.16–19) and (5.27) that a renormalized version of (5.1), given by

$$\lim_{L \nearrow \infty} V_\Lambda^{(1)}(\mathcal{H}_\rho) + 2L \cdot 2 \sum_{[\underline{h}]_+} \tilde{z}(\underline{h}) \sum_{\ell=1}^{\infty} \frac{(-1)^{\ell+1}}{(2\ell)!} a_\ell([\underline{h}]_+) \quad (5.28)$$

allows to take $L \nearrow \infty$ resulting (term-wise) in a finite limit. Thus we observe, that in (2.7) an appropriate renormalization has to precede the removal of the countercharges. We propose to define the anyon–antianyon correlation function by the limit

$$\mathcal{G}(\mathbf{x}, \mathbf{y}; \omega_{(\infty)}, \mathbf{j}_{(\infty)}) = \lim_{L \nearrow \infty} \left\{ e^{2L\mathbf{f}(\tau)} \lim_{\Lambda \nearrow \mathbb{Z}^3} \frac{Z_{\Lambda}(\omega, \mathbf{j})}{Z_{\Lambda}} \right\} \quad (5.29a)$$

$$\mathbf{f}(\tau) = 2 \sum_{[\mathbf{h}]_+} \tilde{\mathbf{z}}(\mathbf{h}) \sum_{\ell=1}^{\infty} \frac{(-1)^{\ell+1}}{(2\ell)!} \mathbf{a}_{\ell}([\mathbf{h}]_+) + \text{higher order in the activities} \quad (5.29b)$$

Corresponding to the expansion (3.19) the renormalization factor (5.29a) has to be dealt with as a power series in our treatment. Then, by construction, the limit $L \nearrow \infty$ is guaranteed up to $V^{(1)}$ of (3.19). It would certainly be of interest to work out the renormalization explicitly up to $V^{(2)}$. We restrict to trace there the action the term renormalizing $V^{(1)}$.

Some effort is needed to rewrite $V_{\Lambda}^{(2)}$, (3.21), in a form that makes the infinite volume limit evident. Introducing for the sake of a concise presentation the notation

$$\begin{aligned} \tilde{\mathbf{z}}_i &\equiv \tilde{\mathbf{z}}(\mathbf{h}^{(i)}), \quad i = 1, 2 \\ \langle \rho | i \rangle &\equiv -\frac{1}{\beta} (\mathcal{H}_{\rho}, C_{\Lambda} \mathbf{h}^{(i)}), \quad i = 1, 2 \\ \langle 1 | 2 \rangle &\equiv -\frac{1}{\beta} (\mathbf{h}^{(1)}, C_{\Lambda} \mathbf{h}^{(2)}) \end{aligned} \quad (5.30)$$

we eventually arrive at

$$\begin{aligned} V_{\Lambda}^{(2)}(\mathcal{H}_{\rho}) &= \frac{1}{2} \sum_{\mathbf{h}^{(1)}} \sum_{\mathbf{h}^{(2)}} \tilde{\mathbf{z}}_1 \tilde{\mathbf{z}}_2 \left\{ e^{\langle 1 | 2 \rangle} + \langle \rho | 1 \rangle + \langle \rho | 2 \rangle \right. \\ &\quad \left. - e^{\langle \rho | 1 \rangle} - e^{\langle \rho | 2 \rangle} - e^{\langle 1 | 2 \rangle} + 2 \right\} \\ &\quad - \frac{1}{2} \sum_{\substack{\mathbf{h}^{(1)} \\ \mathbf{h}^{(1)} \cap \mathbf{h}^{(2)} \neq \emptyset}} \sum_{\mathbf{h}^{(2)}} \tilde{\mathbf{z}}_1 \tilde{\mathbf{z}}_2 e^{\langle 1 | 2 \rangle} \left\{ e^{\langle \rho | 1 \rangle} + \langle \rho | 2 \rangle - 1 \right\} \\ &\quad - \sum_{\substack{\mathbf{h}^{(1)} \\ \mathbf{h}^{(1)} \cap \mathbf{h}_{\rho} \neq \emptyset}} \tilde{\mathbf{z}}_1 e^{\langle \rho | 1 \rangle} \sum_{\mathbf{h}^{(2)}} \tilde{\mathbf{z}}_2 \left\{ e^{\langle \rho | 2 \rangle} + \langle 1 | 2 \rangle - 1 \right\} \\ &\quad + \frac{1}{2} \left[\sum_{\substack{\mathbf{h}^{(1)} \\ \mathbf{h}^{(1)} \cap \mathbf{h}^{(2)} \neq \emptyset}} \sum_{\substack{\mathbf{h}^{(2)} \\ (\mathbf{h}^{(1)} \cup \mathbf{h}^{(2)}) \cap \mathbf{h}_{\rho} \neq \emptyset}} - \sum_{\substack{\mathbf{h}^{(1)} \\ \mathbf{h}^{(1)} \cap \mathbf{h}_{\rho} \neq \emptyset}} \sum_{\substack{\mathbf{h}^{(2)} \\ \mathbf{h}^{(2)} \cap \mathbf{h}_{\rho} \neq \emptyset}} \right] \tilde{\mathbf{z}}_1 \tilde{\mathbf{z}}_2 e^{\langle 1 | 2 \rangle} + \langle \rho | 1 \rangle + \langle \rho | 2 \rangle \end{aligned} \quad (5.31)$$

A similar argument as before in the case of $V_{\Lambda}^{(1)}$, using (5.3), establishes the (term–wise) existence of the infinite volume limit, L fixed. One should notice,, that in the bracket $\{\dots\}$ of the first term upon expansion of the exponential functions the zeroth and first powers cancel. Moreover, adding and subtracting $\exp [<\rho|1> + <\rho|2>]$ in the first bracket $\{\dots\}$ and $\exp <\rho|2>$ in the third one, we can bring (5.31) in the form

$$V_{\Lambda}^{(2)}(\mathcal{H}_{\rho}) = \frac{1}{2} \left[V_{\Lambda}^{(1)}(\mathcal{H}_{\rho}) \right]^2 + \tilde{V}_{\Lambda}^{(2)}(\mathcal{H}_{\rho}). \quad (5.32)$$

Herein $\tilde{V}^{(2)}$ contains further contributions diverging linearly with L , with weights, however, which are homogeneous of order two in the activities. The renormalization (5.29) requires just this form of $V^{(2)}$.

6. CONCLUDING REMARKS

Within the lattice field theory considered we have derived a series representation for the two–point function of transmuted matter fields. It emerged from a particular gauge invariant four–point function of the basic matter field, given as a Euclidean functional integral, by a succession of operations: subsequent integrations over the matter field and the gauge field, performing a quotient of power series, infinite volume limit term by term, removal of two countercharges to infinity. The precise mathematical status of the series remains an open question. The unbounded localization of the transmuted matter field, to be generated by shifting a countercharge to infinity, demands an appropriate renormalization, (5.29), which has been worked out explicitly to low orders. It is quite remarkable that this necessity is caused only by vacuum polarization; neglecting it, i.e. in a "quenched approximation" that only takes into account fluctuations of the internal current lines h_{ρ} , (3.19), the interplay between the Dirac–Mandelstam string j and the disorder insertion ω leads to a cancellation of the terms diverging with the geometrical size of their supports, (4.7). An extension of the approach to correlation functions involving two transmuted matter fields together with the observables, i.e. gauge invariant localized products of a basic field, seems feasible. Although the analysis presented is not mathematically rigorous, it is supposed to shed some light on dynamical properties of non–local quantum fields with Abelian braid (group) statistics.

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APPENDIX

Here some details in evaluating the large L behaviour of the functional (4.5) of section 4 are collected.

We first state the Fourier transforms of the currents $j^{(L)}$ and $\delta\omega$. Recalling the definition of the Fourier transform (5.4) we obtain from (2.11)

$$\widehat{j^{(L)}}_{\mu}(k) = \delta_{\mu 1} \frac{1-e^{ik_1 L}}{1-e^{ik_1}} \frac{1}{4} e^{ik_1} (1+e^{ik_0}) (1+e^{ik_2}) \chi \quad (\text{A.1})$$

with the shorthand

$$\chi = e^{ik(x-\hat{0})} - e^{iky}. \quad (\text{A.2})$$

Because of the particular form of ω , (2.8), we have

$$\begin{aligned} (\delta\omega)_0(z) &= \omega_{20}(z) - \omega_{20}(z-\hat{2}) \\ (\delta\omega)_1(z) &= 0 \\ (\delta\omega)_2(z) &= -[\omega_{20}(z) - \omega_{20}(z-\hat{0})] \end{aligned} \quad (\text{A.3})$$

and hence

$$\begin{aligned} \widehat{(\delta\omega)}_0(k) &= (1-e^{ik_2}) \frac{1-e^{ik_1(L+1)}}{1-e^{ik_1}} e^{ik_1} \chi \\ \widehat{(\delta\omega)}_1(k) &= 0 \\ \widehat{(\delta\omega)}_2(k) &= -(1-e^{ik_0}) \frac{1-e^{ik_1(L+1)}}{1-e^{ik_1}} e^{ik_1} \chi \end{aligned} \quad (\text{A.4})$$

The terms of the real part of (4.5) that grow individually with L are

$$I_1 \equiv (\omega, \omega) - (\delta\omega, C^{(1)} \delta\omega) \quad (\text{A.5})$$

$$I_2 \equiv \left[\frac{8\alpha}{\beta} \right]^2 (j^{(L)}, C^{(1)} j^{(L)}) \quad (\text{A.6})$$

$$I_3 \equiv -16 \frac{\alpha}{\beta} (j^{(L)}, C^{(2)} \delta\omega) \quad (\text{A.7})$$

Using (4.3–4) together with (A.1), (A.4) they can be written in the following form, with $\lambda \equiv \lambda(\mathbf{k})$ and $M \equiv M(\mathbf{k})$,

$$I_1 = \int \frac{d^3k}{(2\pi)^3} \frac{|M|^2}{\lambda + |M|^2} \left| \frac{1 - e^{ik_1(L+1)}}{1 - e^{ik_1}} \right|^2 |\kappa|^2$$

$$+ 4 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\lambda + |M|^2} [1 - \cos k_1(L+1)] [1 - \cos k_0 T] \quad (\text{A.8})$$

$$I_2 = \int \frac{d^3k}{(2\pi)^3} \frac{\left| \frac{2\alpha}{\beta} (1 + e^{ik_0})(1 + e^{ik_2}) \right|^2}{\lambda + |M|^2} \left| \frac{1 - e^{ik_1 L}}{1 - e^{ik_1}} \right|^2 |\kappa|^2 \quad (\text{A.9})$$

$$I_3 = -2 \int \frac{d^3k}{(2\pi)^3} \frac{\frac{2\alpha}{\beta} (1 + e^{ik_0})(1 + e^{ik_2}) M}{\lambda + |M|^2} \frac{e^{ik_1} - e^{ik_1(L+1)}}{1 - e^{ik_1}} \frac{1 - e^{-ik_1(L+1)}}{1 - e^{-ik_1}} |\kappa|^2$$

$$+ 2 \int \frac{d^3k}{(2\pi)^3} \frac{\frac{2\alpha}{\beta} (1 + e^{ik_0})(1 + e^{ik_2}) M}{(\lambda + |M|^2)\lambda} [e^{ik_1} - e^{ik_1(L+1)}] [1 - e^{-ik_1(L+1)}] |\kappa|^2 \quad (\text{A.10})$$

Although I_1 , I_2 and I_3 depend linearly on L for large L , their sum has a finite limit, as inspection of (A.8–10) shows

$$\lim_{L \rightarrow \infty} (I_1 + I_2 + I_3) = a(T) \quad (\text{A.11})$$

where $a(T)$ is a well defined function of the distance T .

The dangerously looking terms in the imaginary part of (4.5), however, all vanish

$$(j^{(L)}, C^{(1)} \delta\omega) = (j^{(L)}, C^{(2)} j^{(L)}) = (\delta\omega, C^{(2)} \delta\omega) = 0 \quad (\text{A.12})$$

because of the particular form of their constituents.

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Fig. 1: The supports of the Dirac–Mandelstam string j , (2.10), and the disorder insertion ω , (2.8), depicted by broad lines and hatched plaquettes, respectively.