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Freeze-out conditions and pion
spectrum in heavy ion collisions

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Умови „заморожування“ та спектр піонів у зіткненнях важких іонів

Проаналізовано численність піонів та енергетичні спектри у зіткненнях важких іонів при проміжних енергіях в моделі зі сферичним розширенням, вкладом розпаду резонансів та ефектами хімічної нерівноваги. Підганяючи дані для центральних $\text{La}+\text{La}$ зіткнень при $E_{\text{lab}} = 1.35 \text{ GeV/нукл}$ ми знаходимо, що фізично прийнятні умови заморожування вимагають сильного колективного руху, щоб описати піонний спектр, та додатного хімічного потенціалу, щоб описати повну численність піонів.

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Freeze-out conditions and pion spectrum in heavy ion collisions

Pion multiplicity and energy spectra in medium energy heavy ion collisions are analyzed in a model with spherical expansion, resonance decay contribution and chemical nonequilibrium effects. Fitting the data for central $\text{La}+\text{La}$ collision at $E_{\text{lab}} = 1.35 \text{ GeV/nucl}$, we find that the physically reasonable freeze-out condition require a strong collective motion to describe the pion spectrum and a positive value of pion chemical potential to fit the total pion multiplicity.

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1. INTRODUCTION

One of the central problems in the study of high energy heavy ion collisions is to deduce the state of the system, i.e., the energy density, entropy and temperature formed in the process, from the observed final particle properties. So far this can only be done by inference. This is because the hot and dense matter produced initially cools and expands considerably before freeze-out, namely, when the particles fly toward the detectors without further interaction. The freeze-out stage of the system is most directly connected to the experimental data. If the freeze-out characteristics of the system can be determined from the observed properties, the evolution history of the system may then be reconstructed with the help of appropriate model.

Pion production is a dominant feature in heavy ion collisions [1]. In the present study, we will limit ourself to the collisions with the incident energy in the region of $E_{lab} = (1 \div 2) \text{ GeV/nucleon}$, where measurements on both pion multiplicity and spectra exist. In this case we have rather complete experimental information: number of participating nucleons in the collision, energy per nucleon and number of produced pions per participating nucleons. These data permit us to use energy and baryonic number conservation laws and restrict strongly the possible set of the freeze-out parameters which one can use to fit the pion multiplicity and momentum spectra.

Glennan [2] showed that the thermodynamical model without collective motion overestimates the pion multiplicity per nucleon in the initial energy range $E_{lab} = (530 \div 1800) \text{ MeV/nucleon}$ and the expansion stage of hadron matter should be taken into account. He demonstrated, however, that with physically reasonable values for freeze-out baryonic density $\rho_b^f = (0.5 \pm 0.2)\rho_0$, ($\rho_0 \cong 0.16 \text{ fm}^{-3}$ is the normal nuclear matter density), one obtains after hydrodynamical expansion too small pion multiplicities [2]. This fact as will be shown in our study indicates the necessity of an introduction of non-zero values of pion chemical potentials (see also Ref.[3]).

The enhancement of pion spectra at low transverse momenta, which was observed in CERN experiments at the bombarding energy $E_{lab}/A = 200 \text{ GeV/nucleon}$ [4] and in AGS at $E_{lab}/A = 14.6 \text{ GeV/nucleon}$ [5] is also present at medium initial energies $E_{lab}/A = (0.5 \div 1.8) \text{ GeV/nucleon}$ [6,7]. Several different freeze-out characteristics have been proposed to account for this two-slope feature in the pion spectra. Collective flow effects [8-10] and resonance decays [12-14] have been considered with some controversial conclusions. Another possible physical origin of low- p_T pion enhancement, that pions are strongly out of chemical equilibrium, was suggested in Refs.[15,16]. It is supported by

the hydrochemical model calculation [17] and the relativistic quantum molecular dynamics calculation [18]. Unfortunately, no attempt has been made to study these three possible effects, namely the collective flow, resonance decays and pion chemical nonequilibrium, at the same time and for complete set of experimental observables.

It is hence of interest to pose the following question: what can be concluded from the experimental data on both the pion multiplicity and pion spectra about the role and relative importance of the above three physical effects at the freeze-out stage. As we will see the simultaneous consideration of both pion multiplicity and spectra, indeed, helps us to establish unambiguously the presence of collective flow and nonvanishing pion chemical potential at the freeze-out.

In Sec. 2, the formulae for the particle spectra when the freeze-out stage of the system is characterized by the presence of collective motion, resonance particles and chemical nonequilibrium are presented. Sec. 3 contains the results and discussions and we summarize in Sec. 4.

2. FORMULATION OF THE MODEL

We do not try in the present study to formulate the hydrodynamical evolution model for the pion production and find out the initial conditions for this evolution (one needs also to know the hadron matter equation of state to introduce it into the hydrodynamical equations). Instead, our aim is just to construct physically reasonable freeze-out conditions which include thermal and collective motion of produced pions, resonance decay contributions to pion spectra and possible chemical nonequilibrium effects. The main question of our study: what can be concluded from the experimental data on pion multiplicities and pion momentum spectra about the role and relative importance of the above three physical effects at the freeze-out stage.

Assuming local thermal equilibrium for hadron species we have for particle momentum spectrum (for hadron type "i") in the rest frame of the system element with volume ΔV^* at constant proper time ($dt^* = 0$):

$$\frac{dn_i}{d^3p^*} = \frac{\Delta V^*}{(2\pi)^3} \frac{d_i}{\exp\left[\frac{E^* - \mu_i}{T}\right] + \eta_i} \equiv \frac{\Delta V^* d_i}{(2\pi)^3} f_i(E^*), \quad (1)$$

where d_i is the number of internal degrees of freedom (degeneracy) of hadron "i", m_i is the hadron mass, $E^* = (m_i^2 + p^{*2})^{1/2}$ is its energy, T and μ_i are temperature and hadron "i" chemical potential, $\eta_i = 1, (-1)$ for fermions, (bosons).

We study medium energy heavy ion collisions at the initial bombarding energies $E_{lab} = (1 \div 2) GeV/nucleon$. In particular, we shall fit data [7] for central $La + La$ collisions at $E_{lab} = 1.35 GeV/nucleon$. Small values of the system temperature at these initial energies simplify the hadron chemical composition of the system: we take into account, apart from nucleons, only pions and delta resonances. All heavier mesons and baryons do not play an important role in the formation of the pion spectra and pion multiplicity and we leave them out. Small initial energy gives us one more simplification: simple spherical geometry of the final hadron state seems to be justified by the experimental data on particle spectra. We have $d_\pi = 3$, $d_N = 4$ and $d_\Delta = 16$ in Eq.(1). At the beginning of our study we do not consider an electrical part μ_e of the chemical potentials and, therefore, do not differ π^+ , π^0 , π^- isospin states of pion. The role of electrical chemical potential introduced in Ref.[3] will be estimated later. The delta finite decay width ($\Gamma \cong 115 MeV$) will be taken into account using the profile function (see [19]):

$$W(m) = \xi \theta(m - m_N - m_\pi) \frac{\Gamma/2}{(m - m_\Delta)^2 + \Gamma^2/4}, \quad (2)$$

where ξ is determined by the normalization condition for $W(m)$ as

$$\xi = \pi + \arctan \left(\frac{m_\Delta - m_N - m_\pi}{\Gamma/2} \right).$$

All quantities included delta particles will be integrated over resonance mass m with profile function (2). For example, particle number densities are

$$\rho_i(T, \mu_i) = \frac{d_i}{2\pi^2} \int_0^\infty p^2 dp f_i \left(\sqrt{m_i^2 + p^2} \right) \quad (3)$$

for $i = \pi, N$ and

$$\rho_\Delta(T, \mu_\Delta) = \frac{d_\Delta}{2\pi^2} \int_0^\infty W(m) dm \int_0^\infty p^2 dp f_\Delta \left(\sqrt{m^2 + p^2} \right) \quad (4)$$

for delta particle.

The "chemical reactions" in our $\pi N \Delta$ system are

$$\pi + N \leftrightarrow \Delta, \quad N + N \leftrightarrow \Delta + N. \quad (5)$$

Assuming the chemical equilibrium in both these reactions one obtains

$$\mu_\pi + \mu_N = \mu_\Delta, \quad \mu_N + \mu_N = \mu_\Delta + \mu_N \quad (6)$$

and consequently $\mu_\pi = 0$. These conditions can be, however, violated because of either the small rate of the second reaction in (6) at small baryonic densities (see Refs. [2, 17]), or because of the chemical nonequilibrium initial conditions for the expansion process (see the detail discussion of this point in Ref. [18]). In what follows we admit the possibility of nonzero values of μ_π (it means the deviation from the chemical equilibrium). It will be shown that $\mu_\pi > 0$ is necessary to explain the experimental data on pion multiplicity for physically reasonable freeze-out conditions. For delta chemical potential we assume $\mu_\Delta = \mu_N + \mu_\pi$ (it means the chemical equilibrium for the first reaction in Eq.(5)) and discuss other possibilities later.

Assuming the thermal particle spectra (1) in the rest frame of the system elements we superimpose the spherical collective (hydrodynamical) motion in the form

$$v(r) = v_0 \left(\frac{r}{R} \right)^n \quad (7)$$

with linear ($n = 1$) or quadratic ($n = 2$) dependence of the collective velocity v on the radius r ($0 \leq r \leq R$). Rewriting Eq.(1) in the relativistic-invariant form ($E = (p^2 + m_i^2)^{1/2}$, $\gamma = (1 - v^2)^{-1/2}$ and $\mathbf{v} = v\hat{\mathbf{r}}$)

$$E \frac{dn_i}{d^3p} = E^* \frac{dn_i}{d^3p^*} = \frac{\Delta V}{(2\pi)^3} (E - \mathbf{p}\mathbf{v}) f_i [\gamma(E - \mathbf{p}\mathbf{v})] ,$$

we obtain then for the total (thermal) particle spectra from spherically expanding fireball with velocity function (7) in the center mass fireball frame:

$$E \frac{dN_i^{th}}{d^3p} = \frac{d_i}{4\pi^2} \int_0^R r^2 dr \int_{-1}^1 dx \frac{E - pvx}{\exp \left[\frac{\gamma(E - pvx) - \mu_i}{T} \right] + \eta_i} . \quad (8)$$

We call these spectra "thermal" (they include, of course, also the collective motion) to distinguish them from an additional contributions ("decay") to pion and nucleon spectra from delta decays.

The general formula for particle spectra in the hydrodynamical model looks like the following [20]:

$$E \frac{dN_i}{d^3p} = \frac{d_i}{(2\pi)^3} \int_{\Sigma_f} d\sigma^\nu p_\nu f_i(u^\lambda p_\lambda) , \quad (9)$$

where $u^\nu = \gamma(1, \mathbf{v})$ is the 4-velocity of the fluid element and $d\sigma^\nu$ is the 4-vector normal to the freeze-out hypersurface Σ_f . It was pointed out in Ref. [10] that the hypersurface Σ_f in the hydrodynamical model calculations should be

closed to the initial condition hypersurface $\Sigma_0 = (t = 0, 0 \leq r \leq R_0)$ and it always consists of both space-like and time-like parts. However, the problem of the physical treatment of time-like parts of Σ_f is not yet solved in the hydrodynamical model. As it is seen from Eq.(9) they give negative (nonphysical) contributions for $d\sigma'' p_\nu < 0$ to particle number spectra. It means that some of emitted particles from time-like parts of the freeze-out hypersurface "return" into the fluid. Therefore, Eq.(9) for the particle emission from time-like parts of Σ_f should be modified. Besides, this emission influences also the hydrodynamical evolution, and this influence should be included selfconsistently into hydrodynamical equations. Assuming (1) in the rest frame of the fluid elements (at $dt^* = 0$) and velocity functions (7) we obtain our particle spectra (8) which correspond to Eq.(9) with unclosed freeze-out hypersurfaces which are completely space-like ones.

From particle spectra (8) we calculate total particle numbers and energies as (for delta particle we have additional integration over m with probability $W(m)$):

$$\tilde{N}_i = \int d^3p \left(\frac{dN_i^{th}}{d^3p} \right), \quad \tilde{E}_i = \int d^3p E \left(\frac{dN_i^{th}}{d^3p} \right). \quad (10)$$

To calculate total pion momentum spectra we add delta decay contribution to thermal pion spectra (8). Each delta with mass m ($m > m_N + m_\pi$) in the final state decays into nucleon plus pion. In the delta rest frame the decay products are distributed isotropically and pion energy and momentum are

$$E_\pi^* = \frac{m^2 - m_N^2 + m_\pi^2}{2m}, \quad p_\pi^* = (E_\pi^{*2} - m_\pi^2)^{1/2}.$$

Pion spectrum from delta decays can be then calculated after Lorentz-transformation of the pion 4-momenta to the c.m. system of the expanding fireball and the integration of delta thermal spectrum (8) over variable delta mass m and delta energy $E_\Delta = (m^2 + p_\Delta^2)^{1/2}$. We obtain

$$\frac{dN_\pi^{dec}}{d^3p} = \frac{1}{2E_p} \int_0^\infty \frac{dm}{p_\pi^*} W(m) \int_{E_-}^{E_+} dE_\Delta \left(E_\Delta \frac{dN_\Delta^{th}}{d^3p_\Delta} \right), \quad (11)$$

where

$$E_\mp = \frac{m}{m_\pi^2} (EE_\pi^* \mp pp_\pi^*)$$

are defined by the decay kinematics. Similar expressions we have for the delta decay contribution to the nucleon spectra.

3. RESULTS OF THE MODEL AND COMPARISON WITH EXPERIMENTAL DATA

In this section we analyze the experimental data for $La + La$ collisions at bombarding energy 1350 MeV/nuc (this corresponds to 292 MeV/nuc kinetic energy per nucleon in the c.m. system). The experimental value for the pion number per participating nucleon is 0.18 [7]. For fixed value of freeze-out baryonic density ρ_b^f we obtain then the following system of equations

$$\frac{\tilde{E}_N + \tilde{E}_\pi + \tilde{E}_\Delta}{\tilde{N}_N + \tilde{N}_\Delta} - m_N = 292 \text{ MeV} , \quad (12)$$

$$\frac{\tilde{N}_\pi + \tilde{N}_\Delta}{\tilde{N}_N + \tilde{N}_\Delta} = 0.18 , \quad (13)$$

$$\rho_b^f = \rho_N + \rho_\Delta . \quad (14)$$

In Eq.(13) we take into account that each delta after freeze-out decays into nucleon plus pion.

The total pion spectrum in our model is

$$\frac{dN_\pi}{d^3p} = \frac{dN_\pi^{th}}{d^3p} + \frac{dN_\pi^{dec}}{d^3p} \quad (15)$$

The experimental data for the pion (π^-) spectrum are parametrized by two exponential functions [7] (see Fig.1):

$$\frac{dN_\pi}{d^3p} = A_1 \exp\left(-\frac{E_{kin}}{T_1}\right) + A_2 \exp\left(-\frac{E_{kin}}{T_2}\right) , \quad (16)$$

with $T_1 \cong 39 \text{ MeV}$, $T_2 \cong 80 \text{ MeV}$ and $A_1/A_2 \cong 3.65$.

We analyze the system (12-14) for three possible values of ρ_b^f : $0.3\rho_0, 0.5\rho_0, 0.7\rho_0$. At fixed ρ_b^f we solve the system of equations (12-14) numerically and find the sets of thermodynamical parameters T, μ_N, μ_π for given velocity distribution (7) with given v_0 . Then we can calculate pion spectrum (15) and compare it with experimental one (16) to find the specific values of v_0 and corresponding sets of the thermodynamical parameters which give the best fits of the experimental pion spectrum.

As the first step of our analysis we show the necessity of the collective motion to explain experimental data. Without collective motion (i.e. $v_0 = 0$) the solutions of the system (12-14) are presented in Table 1.

Table 1: Freeze-out parameters without collective motion ($v_o = 0$ in Eq.(7)).

ρ_b^f/ρ_o	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	128.16	471.69	-302.47
0.5	129.41	529.10	-264.52
0.7	130.29	567.31	-243.14

One observes from Table 1 very large negative values of pion chemical potentials which should be in the system to fit pion multiplicity data (Eq.(13)). The chemical equilibrium ($\mu_\pi = 0$) thermodynamical model without collective motion greatly overestimates the pion number per nucleon. But with the correct total pion number (admitting negative μ_π), freeze-out parameters of Table 1 give completely wrong pion energy spectra: $T \cong 130 \text{ MeV}$ in the thermodynamical system is much larger than pion experimental slope parameters in Eq.(15). It becomes evident that experimental pion slope parameters have to be the results of both thermal and collective motion in the hadron gas system.

With the presence of the collective motion, the best choices of the freeze-out parameters (which are the solutions of Eqs.(12-14)) for different freeze-out baryonic densities are shown in Tables 2 and 3.

Table 2: Freeze-out parameters for the linear velocity function ($n = 1$ in Eq.(7)).

ρ_b^f/ρ_o	v_o	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.635	53.81	819.52	40.00
0.5	0.625	56.39	838.11	44.90
0.7	0.620	57.52	853.73	48.66

Table 3: Freeze-out parameters for the quadratic velocity function ($n = 2$ in Eq.(7)).

ρ_b^f / ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.750	51.94	826.65	46.70
0.5	0.740	54.21	845.54	52.19
0.7	0.735	55.18	861.09	56.16

The comparison of pion spectrum (15) with the experimental one (16) is shown in Figs.2(a-c) for quadratic velocity function. Similar fits are obtained with the linear velocity function. Representing total particle (pion and nucleon) spectra in the exponential form

$$\frac{dN}{d^3p} = \text{const} \exp \left(-\frac{E_{kin}}{T_{eff}} \right), \quad (17)$$

we introduce an "effective temperature" $T_{eff} = T_{eff}(E_{kin})$ which defines the particle spectrum slope as a function of E_{kin} . The results for $T_{eff}^\pi(E_{kin})$ and $T_{eff}^N(E_{kin})$ are shown in Fig.3 for the freeze-out parameters from Tables 2 and 3 at $\rho_b^f = 0.5\rho_0$. One observes that T_{eff}^N is essentially larger than T_{eff}^π , (the collective motion affects the heavier nucleons much stronger than pions), and T_{eff}^N has different behaviour as a function on E_{kin} for linear ($n = 1$) and quadratic ($n = 2$) velocity function (7).

From Figs.2(a-c) one can see that an agreement with data for the pion spectrum can be achieved with different values for the freeze-out baryonic density ρ_b^f . The relative importance of delta decay contribution increases with increasing ρ_b^f . The ratios of total number of thermal pions (N_π) to pion number from delta decays (N_Δ) are 1.27 at $0.3\rho_0$, 0.79 at $0.5\rho_0$ and 0.57 at $0.7\rho_0$. As it is seen from Figs.2(a-c) we have also indicate that the delta decay contributions to the pion spectrum enhancement at low E_{kin} become more important with increasing ρ_b^f .

It has been suggested in Ref.[21] that it should be no delta decay contributions to pion spectrum. The argument for this was that pion experimental spectrum did not show the "delta decay peak". We find that delta collective motion and delta finite width smearing out the pion energy spectrum from delta decays and, therefore, we do not see any "delta decay peak" in our calculations. On the other hand, there are also no indications from experimental

data on the necessity of delta decay contributions to pion spectrum. To check this in more details we solve our equations (12-14) excluding delta particles (we put $d_\Delta \equiv 0$). The results for the freeze-out parameters (the solutions of (12-14)) which fit pion spectrum (16) are presented in Table 4. The pion

Table 4: Freeze-out parameters for quadratic velocity function ($n = 2$ in Eq.(7)) without delta particles at the freeze-out.

ρ_b^f / ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.76	49.02	841.87	83.51
0.5	0.755	49.99	865.31	102.87
0.7	0.745	52.14	877.36	111.93

spectrum (dN_π^{th}/d^3p in this case) for $\rho_b^f = 0.5\rho_0$ is shown in Fig.4, where the agreement with data is excellent. We conclude, therefore, that the comparison of our model calculations with experimental data gives no definite indications of the presence of delta particles at the freeze-out.

In the physical picture without delta particles at the freeze-out we have essentially larger values of μ_π (compare values of μ_π in Table 4 with those in Tables 2 and 3). Therefore, the effects of Bose statistics and the possibility of different shapes of π^- and π^+ spectra suggested in Ref.[3] should be studied. To do this we introduce electrical chemical potential μ_e and define

$$\mu_{\pi^+} = \mu_\pi + \mu_e, \quad \mu_{\pi^0} = \mu_\pi, \quad \mu_{\pi^-} = \mu_\pi - \mu_e, \quad \mu_{prot} = \mu_N + \mu_e, \quad \mu_{neut} = \mu_N. \quad (18)$$

We solve now the system (12-14) ($d_{\pi^+} = d_{\pi^0} = d_{\pi^-} = 1$, $d_{prot} = d_{neut} = 2$) excluding delta particles ($d_\Delta \equiv 0$) adding one more equation

$$\frac{\tilde{N}_{prot} + \tilde{N}_{\pi^+} - \tilde{N}_{\pi^-}}{\tilde{N}_{prot} + \tilde{N}_{neut}} = Q/B = \frac{57}{139} \quad (19)$$

for the ratio of the total electric charge Q to the total baryonic number B (this ratio coincides with its initial value for La nuclei). Freeze-out parameters T , μ_N , μ_π , μ_e (the solutions of Eqs.(12-14,19)) which give the best agreement with experimental π^- spectrum (15) are presented in Table 5.

In Fig.5 we show π^- spectrum for the freeze-out parameters in Table 5 at $\rho_b^f = 0.5\rho_0$ and compare it with its Boltzmann approximation ($\eta \equiv 0$ in Eq.(1)). Fig.6 shows the Bose enhancement factor (see also Ref.[3]) which

Table 5: Freeze-out parameters for the system of equations (12-14,19) for quadratic velocity function ($n = 2$ in Eq.(7)) without delta particles at the freeze-out.

ρ_b^f/ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)	μ_c (MeV)
0.3	0.76	49.04	847.45	82.28	-11.90
0.5	0.75	51.22	867.23	98.53	-12.31
0.7	0.74	53.36	879.79	107.75	-12.65

is the ratio of Bose to Boltzmann pion spectra of Fig.5. We observe that the essential part of pion spectrum enhancement at low E_{kin} is now due to Bose-statistic effects.

Bose-statistic effects lead also to some difference in the shape of π^- and π^+ spectra because of the difference between μ_{π^-} and μ_{π^+} (see Eq.(18)). In the Boltzmann approximation we would obtain only the different π^- and π^+ multiplicities but the same shape of their spectra. The ratio of π^- to π^+ momentum spectra is shown in Fig.7 for the freeze-out parameters from Table 5 at $\rho_b^f = 0.5\rho_0$. The asymptotic value of this ratio at large E_{kin} can be calculated within Boltzmann approximation and equals $\exp(-2\mu_\pi/T) \cong 1.62$. It is seen from Fig.7 that this ratio increases at small E_{kin} . As it has been suggested in Ref.[3], we observe a larger enhancement at small kinetic energies for π^- than that for π^+ .

4. SUMMARY

In summary, we have set out to determine the freeze-out conditions of the fireball formed in the high energy heavy ion collisions from the experimental data. For this purpose, we have formulated a model for the freeze-out state of the system which includes three physical ingredients: collective flow, resonance decays and chemical nonequilibrium effects. These features of the freeze-out states are likely to be common to most of the heavy ion collisions taken place at a wide range of initial energies. The model is used to analyze medium energy central collisions at $E_{lab} = (1 \div 2) GeV/nucleon$ for the following reasons. First, at these energies the data exist for both the total pion numbers per participating nucleons and the pion momentum spectra. This enables us to determine essentially all the physically admissible sets of the freeze-out parameters. Second, the freeze-out stage of the system at these initial energies

is supposed to be relatively simple. We can restrict our consideration to just $\pi N \Delta$ gas and neglect all other heavier mesons and baryons. Furthermore, the system can be approximated with spherically symmetrical geometry for the case of central collisions.

In the physically reasonable region of freeze-out baryonic density $\rho'_b = (0.5 \pm 0.2)\rho_0$, we find it impossible to explain the spectrum in $La + La$ collisions at $E_{lab} = 1.35 \text{ GeV/nucleon}$ [7] unless a strong collective flow is present. The presence of this collective flow further requires the positive values of the pion chemical potential in order to obtain the correct pion multiplicity. It also leads to a strong difference between the pion and nucleon "effective temperatures", as shown in Fig.(3).

On the contribution of Δ decay to the pion spectrum, we do not find any definite indication from the data about its relative importance or even about its presence in the pion momentum spectra. We find that it is still possible to give excellent fit to the data in a model excluding the Δ 's, as seen in Fig. 4. It requires instead a large value of μ_π for the freeze-out stage. The enhancement in the pion spectrum at small energies is, in this case, connected with the Bose statistics. When the electrical chemical potential is included in the analysis, a difference in the shape of π^- and π^+ spectra at small energies develops, as was predicted in Ref. 3.

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FIGURE CAPTIONS

Fig.1. Experimental pion spectrum of Ref. [7]. Dashed lines are exponential functions of Eq.(16).

Fig.2. Solid lines correspond to pion spectrum (15) for the freeze-out parameters from Table 3. Short dashed lines are thermal pion spectra (8) and long dashed lines are delta decay contributions (11) to pion spectra.

The solid triangles represent the experimental data as parametrized by Eq.(16). In (a)-(c), $\rho_b^f = 0.3\rho_0$, $0.5\rho_0$ and $0.7\rho_0$ respectively.

Fig.3. Effective "temperatures" (spectrum slopes) for pion (lower lines) and nucleon (upper lines) spectra. The solid and lines correspond to freeze-out parameters of Table 3 and 2 respectively at $\rho_b^f = 0.5\rho_0$.

Fig.4. The solid line is the pion spectrum (8) with the freeze-out parameters from Table 4 at $\rho_b^f = 0.5\rho_0$.

Fig.5. The solid line is π^- spectrum (8) with the freeze-out parameters from Table 5 at $\rho_b^f = 0.5\rho_0$.

Fig.6. The Bose enhancement factor for π^- spectrum.

Fig.7. The solid line is the ratio of π^- to π^+ momentum spectra for the freeze-out parameters from Table 5 at $\rho_b^f = 0.5\rho_0$. The dashed line corresponds to the asymptotic value of this ratio which coincides with its Boltzmann approximation.

Fig.1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig.6

Fig.7

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