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Nonperturbative Dressing of Nucleons by Mesons

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Abstract

Following Feynman's treatment of the polaron problem we present an approximate nonperturbative method of dressing nucleons by scalar and vector mesons: After integrating out the meson fields in the path integral we perform a variational calculation for the effective two-time action of the nucleon by using the same ansatz as Feynman did for the polaron. New features arise from the vector mesons which lead to velocity-dependent terms. We obtain numerical results for the radius and the mean number of the mesons of the dressed particle and derive the effective action in the A nucleon case.

1 Introduction

How particles behave in a medium is a central question in many areas of physics. For example, an electron slowly moving in an ionic crystal has properties which are different from those in free space; in particular its energy and mass get renormalized so that its dispersion law reads

$$E(\mathbf{p}) = E_0 + \frac{\mathbf{p}^2}{2M^*} + \dots \quad (1)$$

The well-known Hamiltonian due to Fröhlich [1] is a simplified model for such a system: it describes the polarization of the crystal by means of an interaction between the single electron and (longitudinal optical) phonons of constant frequency. This interaction in turn acts back on the electron leading to a quasiparticle (“polaron”), i.e. a bare electron with a cloud of phonons around it. As a testing ground for non-relativistic field theories the Fröhlich (or optical) polaron has been the subject of a large number of investigations (for recent reviews see [2,3]). Among these Feynman’s path integral method [4] was particularly successful: he integrated out the phonons exactly and applied a variational approximation to the resulting two-time action for the electron. Recent Monte-Carlo calculations for the polaron problem [5] have confirmed that Feynman’s method is the best semi-analytical method available for both weak and strong coupling. Therefore it seems interesting to extend it to the strong interaction between nucleons and mesons having in mind possible applications in nuclear physics.

In the first step we ask how a single bare nucleon is changed by quantum fluctuations, i.e. the medium is the vacuum which allows for emission and absorption of mesons. Dressing of nucleons by mesons has, of course, been treated before, but mostly by using a coherent-state ansatz for the mesons (see e.g. [6,7]). In the polaron case it is well known that Feynman’s approach is superior to such an ansatz particularly for strong coupling. It is also worthwhile to mention that – in contrast to semiclassical or potential models of the nucleon – the present approach is fully quantum-mechanical in the sense that it includes loop effects to all orders – albeit in an approximate way. Preliminary results which did not include the vector mesons have already been published in refs. [8,9].

In this work we employ a simplified model due to Walecka [10] where the main features of the nucleon-nucleon interaction arise from the exchange of scalar and vector mesons. In order to apply Feynman’s ideas to this problem the main assumption we make is to consider the nucleons as much heavier than the mesons and treat them non-relativistically. This means that relativistic aspects of this model such as nucleon pair creation and renormalization are left out. Considering the Walecka model as an effective theory valid below approximately 1 GeV we also use form factors at the meson-nucleon vertices to cut down high-momentum components. If in addition the spin dependent part of the interaction is neglected the model then closely resembles the polaron model. The main difference and the new feature as compared to the Fröhlich polaron arises from the presence of the vector mesons: the two-time action now depends on the transverse velocities of the nucleons reflecting the transverse degrees of freedom of the vector mesons. We will mostly concentrate on the one-nucleon system where we call the bare nucleon with its cloud of scalar and vector mesons, a “nuclaron”. After minimizing the energy and fixing the variational parameters so that the nucleon mass is reproduced, we calculate the mean number of mesons and the root mean square radius for this dressed nucleon. However, we

also give the general formulae for the many-nucleon case and show how the static potential between two nucleons arises as a special case of our retarded action when retardation, self-energies and vertex corrections are neglected.

The paper is organized as follows: in section 2 we give the effective action for the one-nucleon system, and in section 3 we present the expressions for the energy and effective mass in the Feynman approximation. Observables like the mean number of scalar and vector mesons and the elastic form factor of the dressed particle are worked out in section 4. Our numerical results for the one-nucleon sector are given in section 5. Finally in the last section we present the main conclusions and future extensions. The appendices contain the details for the derivation of the many-nucleon effective action, the identification of the familiar static potential in our effective action as well as technical points about evaluation of the various path integrals and expectation values.

2 Effective action for the nucleons

2.1 The model

In the Walecka model (sometimes also called QHD-I) the interaction between nucleons (ψ) arises from the exchange of scalar mesons (Φ) which produce overall binding and vector mesons (V_μ) which are responsible for the short range repulsion. As is well known [11,12] the model is quite successful in describing properties of nuclear matter and finite nuclei. It is defined by the Lagrangian

$$\mathcal{L} = \mathcal{L}_0(\psi, \Phi, V_\mu) + g_S \bar{\psi} \psi \Phi - g_V \bar{\psi} \gamma_\mu \psi V^\mu. \quad (2)$$

where the free Lagrangian density $\mathcal{L}_0$ is given by

$$\begin{aligned} \mathcal{L}_0(\psi, \Phi, V_\mu) &= \bar{\psi}(x)(i\gamma_\mu \partial^\mu - M)\psi(x) + \frac{1}{2} \left(\partial_\mu \Phi(x) \partial^\mu \Phi(x) - m_S^2 \Phi^2(x) \right) \\ &- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_V^2 V_\mu(x) V^\mu(x). \end{aligned} \quad (3)$$

We have used the standard notation where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $\bar{\psi} = \psi^\dagger \gamma_0$. Since the conjugate momentum to $V^0(x)$ is zero, $V^0(x)$ is not a dynamical variable and one could eliminate it in favour of $\mathbf{V}(x)$ and the corresponding conjugate momenta $\pi_V = -\mathbf{E}$:

$$V^0(x) = -\frac{1}{m_V^2} \nabla \cdot \mathbf{E} + \frac{g_V}{m_V^2} \bar{\psi}(x) \gamma_0 \psi(x). \quad (4)$$

For our purpose here it is however more convenient to work in momentum space and expand V^μ in terms of *three* real polarization vectors.

To simplify the treatment we assume here that the nucleons are much heavier than the mesons

$$M \gg m_S, m_V \quad (5)$$

so that they can be treated *non-relativistically*. We will comment on the validity of this assumption when we discuss our results. However, the mesons are treated fully relativistically. As is well known (see, e.g. [14]) the nonrelativistic reduction of the QHD-I Lagrangian including terms up to order $1/M$ leads to the Pauli Hamiltonian for the nucleons. For an A -nucleon system the total Hamiltonian reads

$$H = \sum_{i=1}^A \frac{(\mathbf{p}_i - g_V \mathbf{V}(\mathbf{x}_i))^2}{2M} - \frac{1}{2M} \sum_{i=1}^A \boldsymbol{\sigma}_i \cdot (\nabla_i \times \mathbf{V}(\mathbf{x}_i)) - g_S \sum_{i=1}^A \Phi(\mathbf{x}_i) + g_V \sum_{i=1}^A V_0(\mathbf{x}_i) + H_0^{(V)} + H_0^{(S)}. \quad (6)$$

Here $H_0^{(V)}, H_0^{(S)}$ are the free Hamiltonians for the meson fields (see below). In addition we also neglect the spin-dependent terms in eq. (6) although they are of the same order $1/M$. The main reason for this is a technical one: we want to avoid the complications of spin in a path integral [15].

The main results in this paper are for the one-nucleon sector. The A -nucleon system may, however, be treated along similar lines and we give explicit expressions for the effective action also in this case (see Appendix A). Appendix B shows how the usual static potential between two nucleons is recovered from the effective action when self-energy corrections and retardation effects are neglected together with the velocity dependent terms. Under the stated simplifying assumptions our model Hamiltonian for $A = 1$ is given by

$$H = \frac{1}{2M} (\mathbf{p} - g_V \mathbf{V})^2 - g_S \Phi + g_V V_0 + H_0^{(S)} + H_0^{(V)} \quad (7)$$

where

$$H_0^{(S)} = \int d^3k \omega_S(k) a^\dagger(\mathbf{k}) a(\mathbf{k}) \quad (8)$$

$$H_0^{(V)} = \int d^3k \omega_V(k) \sum_{\lambda=1}^3 a^{(\lambda)\dagger}(\mathbf{k}) a^{(\lambda)}(\mathbf{k}) \quad (9)$$

are the free Hamiltonians for scalar and vector mesons. The meson fields are expanded as [16]

$$\Phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_S(k)}} \left[a(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}(t)} + \text{h.c.} \right] \quad (10)$$

$$V_\mu(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_V(k)}} \sum_{\lambda=1}^3 \epsilon_\mu^{(\lambda)}(\mathbf{k}) \left[a^{(\lambda)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}(t)} + \text{h.c.} \right] \quad (11)$$

where the creation and annihilation operators obey

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}') \quad (12)$$

for scalar mesons and

$$[a^{(\lambda)}(\mathbf{k}), a^{(\lambda')\dagger}(\mathbf{k}')] = \delta_{\lambda\lambda'} \delta(\mathbf{k} - \mathbf{k}') \quad (13)$$

for vector mesons. Furthermore

$$\omega_i(k) = \sqrt{m_i^2 + \mathbf{k}^2}; \quad i = S, V \quad (14)$$

denote the meson frequencies and $\epsilon_\mu^{(\lambda)}(\mathbf{k})$ are a set of three orthonormal polarization vectors which are orthogonal to the four-momentum k^μ ¹.

$$k^\mu \epsilon_\mu^{(\lambda)}(\mathbf{k}) = 0 \quad (15)$$

and fulfill

$$\sum_{\lambda=1}^3 \epsilon_\mu^{(\lambda)}(\mathbf{k}) \epsilon_\nu^{(\lambda)}(\mathbf{k}) = - \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_V^2} \right). \quad (16)$$

2.2 Integrating out the mesons

Compared to the polaron problem an essential difference is that the minimal coupling of the vector mesons to the nucleons introduces momentum-dependent interactions. In such a case the more general phase-space path integral for the partition function has to be used [17]

$$Z = \int \mathcal{D}[\text{meson fields}] \int \mathcal{D}[\mathbf{x}] \mathcal{D}[\mathbf{p}] \exp \left(i \int dt [\dot{\mathbf{x}} \cdot \mathbf{p} - H(\mathbf{p}, \mathbf{x}, \text{meson fields})] \right). \quad (17)$$

Performing the $\mathbf{p}$ -integration one obtains the usual Lagrangian path integral with the QED-like (real time) action

$$S[\mathbf{x}, \text{meson fields}] = \int dt \left(\frac{M}{2} \dot{\mathbf{x}}^2 + g_V \dot{\mathbf{x}} \cdot \mathbf{V} + g_S \Phi - g_V V_0 - H_0^{(S)} - H_0^{(V)} \right). \quad (18)$$

Inserting the field expansions we can now integrate out the meson fields since they appear at most quadratically in the action. This is done conveniently by using the coherent state path integral [5] and derived in detail in Appendix A for the many nucleon case. In the one nucleon case we obtain from eq. (A.20) the following retarded effective action

$$S_{\text{eff}} = \int_0^\beta dt \left\{ \frac{M}{2} \dot{\mathbf{x}}^2 - \int \frac{d^3 k}{8\pi^2} \int_0^\beta dt' \left[R_S(k, t-t') + R_V(k, t-t') \right. \right. \\ \left. \cdot \left(\frac{m_V^2}{k^2} - \dot{\mathbf{x}}_\perp(t) \cdot \dot{\mathbf{x}}_\perp(t') \right) \right] e^{i\mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))} + \int \frac{d^3 k}{4\pi^2} \omega_V(k) \left(\frac{1}{k^2} - \frac{1}{m_V^2} \right) R_V(k, 0) \right\} \quad (19)$$

with

$$R_i(k, \tau) = \frac{g_i^2}{4\pi} \frac{F_i^2(k)}{\omega_i(k)} \exp(-\omega_i(k) |\tau|) \quad , \quad i = S, V. \quad (20)$$

Here we have introduced form factors $F_i(k)$ at the meson-nucleon vertices since the model described by the Hamiltonian (7) is considered as an effective theory valid at energies up to the nucleon mass. As usual we parametrize the form factor by a monopole type with a common cut-off parameter Λ [18]

$$F_i(k) = \frac{\Lambda^2 - m_i^2}{\Lambda^2 + k^2} \quad (21)$$

Note that the scalar mesons give rise to an effective action which is very similar to the polaron action [4] whereas the vector mesons generate more complicated terms due to their velocity-dependent

¹Here the zeroth component is defined as $\omega_V(k)$. In addition, the magnitude of the three-momentum is always denoted as $k \equiv |\mathbf{k}|$

interaction. The velocity-dependence is explicit in the coordinate $\mathbf{x}_\perp$ which is transverse to the vector $\mathbf{k}$ while the longitudinal component has been eliminated by an appropriate integration by parts. This corresponds to the elimination of the longitudinal current by means of the continuity equation. For $m_V = 0$ the vector part is identical to what has been obtained for QED with nonrelativistic electrons (see e.g. ref. [19], eq. (9-64)) except for the last term in the curly bracket of eq. (19). This term is from the tadpole contribution (see Fig. 1 (b)) to the self-energy and does not depend on the relative time $t - t'$ between emission and absorption of the vector mesons. Therefore the last term in the vector meson effective action is a constant which is irrelevant for the dynamics of the system but which we keep for later comparison with perturbative calculations.

With the elimination of the meson degrees of freedom one is left with a one-body effective theory involving just the nucleon coordinate. However the effective action is highly non-local in time due to the retardation effects which reflect the original many-body nature of the problem. This prevents a Hamiltonian formulation and any further exact analytical treatment.

3 Feynman approximation

3.1 Energy

Following Feynman [4], it is possible to find a variational approximation for the effective action starting from some solvable trial action. Being less well known than the standard variational principle for Hamiltonians we discuss briefly the more general one for actions. It is based on the fact that the ground state energy E_0 of the system is obtained at large euclidean times from the partition function

$$Z(\beta) = \text{Tr} \left(e^{-\beta H} \right) \xrightarrow{\beta \rightarrow \infty} e^{-\beta E_0} + \dots \quad (22)$$

and on Jensen's inequality

$$\langle e^{-S} \rangle \geq e^{-\langle S \rangle} \quad (23)$$

for averages with normalized positive weight functions. To be more precise we project on good momentum of the moving "nuclaron" and consider the modified partition function

$$\tilde{Z}(\beta, \mathbf{p}) = \int d^3x e^{i\mathbf{p} \cdot \mathbf{x}} \langle \mathbf{x} | e^{-\beta H} | \mathbf{0} \rangle \quad (24)$$

in eq. (22). This also allows the easy evaluation of the momentum-dependent terms in eq. (1), i.e. the effective mass. Since the projected partition function can be written as

$$\tilde{Z}[\beta, \mathbf{p}] = \int d^3x e^{i\mathbf{p} \cdot \mathbf{x}} \int_{\mathbf{x}(0)=0}^{\mathbf{x}(\beta)=\mathbf{x}} \mathcal{D}[\mathbf{x}(\tau)] e^{-S[\mathbf{x}(\tau)]} \quad (25)$$

one easily derives for $\mathbf{p} = \mathbf{0}$

$$E_0 \leq E_F = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \left(-\log Z_F + \langle S - S_F \rangle_F \right) \quad (26)$$

where the averages are taken with respect to the trial action S_F . Because of the similarity to the polaron problem we take the same trial action as Feynman did, viz. a retarded harmonic oscillator

$$S_F[\mathbf{x}(t)] = \int_0^\beta dt \left\{ \frac{M}{2} \dot{\mathbf{x}}^2 + \frac{Mw}{4}(v^2 - w^2) \int_0^t dt' e^{-w(t-t')} |\mathbf{x}(t) - \mathbf{x}(t')|^2 \right\}. \quad (27)$$

Here v and w are variational parameters to be determined from minimizing the Feynman energy (26). As the trial action is quadratic all path integrals can be performed analytically. This is easily done in the Fourier path integral where the paths are expanded as [5]

$$\mathbf{x}(t) = \mathbf{x} \frac{t}{\beta} + \sum_{n=1}^{\infty} \frac{2\sqrt{M\beta}}{n\pi} \mathbf{b}_n \sin \left[\frac{n\pi t}{\beta} \right] \quad (28)$$

and the endpoint is written as

$$\mathbf{x} = \sqrt{2M\beta} \mathbf{b}_0. \quad (29)$$

The functional integral is now an infinite-dimensional integral over the Fourier coefficients $\mathbf{b}_n$ and the Feynman trial action for large β is simply

$$S_F[\mathbf{b}_n] = \sum_{n=0}^{\infty} A_n \mathbf{b}_n^2 = \sum_{n=0}^{\infty} \frac{v^2 \beta^2 + n^2 \pi^2}{w^2 \beta^2 + n^2 \pi^2} \mathbf{b}_n^2. \quad (30)$$

Compared to the polaron case the new features are due to the vector mesons whose treatment needs some extra care. In Appendix C we show that

$$E_F(v, w) = \frac{3}{4v}(v - w)^2 + C_F - \frac{1}{\pi} \int_0^\infty d\tau \int_0^\infty dk k^2 \left[R_S(k, \tau) + R_V(k, \tau) \right. \\ \left. \cdot \left(\frac{m_V^2}{k^2} + \frac{v^2 - w^2}{Mv} e^{-v\tau} \right) \right] \exp \left(-\frac{k^2}{2M} \mu^2(\tau) \right) \quad (31)$$

where

$$\mu^2(\tau) = \frac{w^2}{v^2} \tau + \frac{v^2 - w^2}{v^3} (1 - e^{-v\tau}). \quad (32)$$

The constant term

$$C_F = \frac{1}{\pi} \int_0^\infty dk k^2 R_V(k, 0) \left[\frac{1}{M} + \omega_V(k) \left(\frac{1}{k^2} - \frac{1}{m_V^2} \right) \right] \quad (33)$$

is independent of the variational parameters v, w but is essential for agreement with the perturbative result in the weak coupling limit $g_S, g_V \rightarrow 0$, $v \rightarrow w$ which we will discuss now.

The second-order contributions to the energy are shown in Fig. 1. For the scalar self-energy diagram (a) we obtain

$$\Delta E_S(\mathbf{p}) = g_S^2 \int d^3 p' d^3 k F_S^2(k) \langle \mathbf{p}, 0 | \Phi(\hat{\mathbf{x}}) | \mathbf{p}', 1_{\mathbf{k}} \rangle \langle \mathbf{p}', 1_{\mathbf{k}} | \Phi(\hat{\mathbf{x}}) | \mathbf{p}, 0 \rangle \frac{1}{E(\mathbf{p}, 0) - E(\mathbf{p}', 1_{\mathbf{k}})} \quad (34)$$

where $|\mathbf{p}, 1_{\mathbf{k}}\rangle$ is a state of a nucleon of momentum $\mathbf{p}$ and one meson of momentum $\mathbf{k}$ with total energy $E(\mathbf{p}, 1_{\mathbf{k}})$. $\hat{\mathbf{x}}$ denotes the quantum mechanical position operator. Eq. (34) can be worked

out by employing the expansion (10) for the scalar field. The perturbative self-energy at momentum $\mathbf{p} = 0$ is then given by

$$\Delta E_S(0) = - \int \frac{d^3k}{(2\pi)^3} F_S^2(k) \frac{g_S^2}{2\omega_S(k)} \frac{1}{k^2/2M + \omega_S(k)}. \quad (35)$$

For the vector mesons the self-energy is

$$\begin{aligned} \Delta E_V(\mathbf{p}) &= g_V^2 \int d^3p' d^3k F_V^2(k) \langle \mathbf{p}, 0 | V_0(\hat{\mathbf{x}}) - \frac{\hat{\mathbf{p}} \cdot \mathbf{V}(\hat{\mathbf{x}})}{2M} - \frac{\mathbf{V}(\hat{\mathbf{x}}) \cdot \hat{\mathbf{p}}}{2M} | \mathbf{p}', 1_{\mathbf{k}} \rangle \\ &\cdot \langle \mathbf{p}', 1_{\mathbf{k}} | V_0(\hat{\mathbf{x}}) - \frac{\hat{\mathbf{p}} \cdot \mathbf{V}(\hat{\mathbf{x}})}{2M} - \frac{\mathbf{V}(\hat{\mathbf{x}}) \cdot \hat{\mathbf{p}}}{2M} | \mathbf{p}, 0 \rangle \frac{1}{E(\mathbf{p}, 0) - E(\mathbf{p}', 1_{\mathbf{k}})} \end{aligned} \quad (36)$$

where $\hat{\mathbf{p}}$ denotes the momentum operator. Inserting the expansion (11) one obtains

$$\Delta E_V(0) = - \sum_{\lambda=1}^3 \int \frac{d^3k}{(2\pi)^3} F_V^2(k) \frac{g_V^2}{2\omega_V(k)} \frac{1}{k^2/2M + \omega_V(k)} \left(\epsilon_0^\lambda(k) + \frac{\mathbf{k} \cdot \boldsymbol{\epsilon}^\lambda(k)}{2M} \right)^2. \quad (37)$$

There is also a tadpole contribution (see Fig. 1 b) for the vector mesons

$$\begin{aligned} \Delta E_t &= \frac{g_V^2}{2M} \langle \mathbf{p}, 0 | \mathbf{V}^2(\hat{\mathbf{x}}) | \mathbf{p}, 0 \rangle \\ &= \frac{g_V^2}{2M} \sum_{\lambda=1}^3 \int \frac{d^3k}{(2\pi)^3} F_V^2(k) \frac{1}{2\omega_V(k)} \langle 0, \mathbf{p} | \boldsymbol{\epsilon}^\lambda(k) \cdot \boldsymbol{\epsilon}^\lambda(k) | \mathbf{p}, 0 \rangle. \end{aligned} \quad (38)$$

Adding the self energy and tadpole contributions and performing the sum over polarizations with the help of eq. (16) we obtain for the total energy shift

$$\begin{aligned} \Delta E &= - \int \frac{d^3k}{(2\pi)^3} F_S^2(k) \frac{g_S^2}{2\omega_S(k)} \frac{1}{k^2/2M + \omega_S(k)} - \int \frac{d^3k}{(2\pi)^3} F_V^2(k) \frac{g_V^2}{2\omega_V(k)} \\ &\cdot \left[-\frac{1}{M} + \frac{1}{k^2/2M + \omega_V(k)} \left(\frac{k^2}{m_V^2} + \frac{\omega_V(k)k^2}{2Mm_V^2} - \frac{\omega_V(k)}{2M} \right) \right]. \end{aligned} \quad (39)$$

The above result is identical to what one obtains from the Feynman energy (31) when we set $v = w + \mathcal{O}(g_i^2)$ for the weak coupling limit. In this limit $\mu^2(\tau) = \tau$ and the τ -integration can be performed trivially. The agreement with perturbation theory serves as a useful check for the correctness of the variational calculation.

3.2 Effective mass

The effective mass of the quasiparticle is defined by the dispersion law (1). It determines the energy of the slowly moving “nuclaron” and can be calculated from [5]

$$M^* = M \lim_{\beta \rightarrow \infty} \frac{3\beta}{\langle \mathbf{x}^2 \rangle} \quad (40)$$

where the average is with respect to the full action. In first order of an expansion in powers of the difference between the exact action and the Feynman action one obtains [20]

$$M_F^* = \frac{3}{2} M \lim_{\beta \rightarrow \infty} \left[\frac{1}{\langle \mathbf{b}_0^2 \rangle} - \frac{\langle S - S_F \rangle}{\langle \mathbf{b}_0^2 \rangle} + \frac{\langle \mathbf{b}_0^2 (S - S_F) \rangle}{\langle \mathbf{b}_0^2 \rangle^2} \right]. \quad (41)$$

The zeroth order is given by

$$M_{(0)}^* = \frac{3}{2}M \lim_{\beta \rightarrow \infty} \frac{1}{\langle \mathbf{b}_0^2 \rangle} = M A_0 = M \frac{v^2}{w^2}. \quad (42)$$

The first order expression depends explicitly on the interaction and is worked out in Appendix D . The result is

$$\begin{aligned} M_F^*/M = 1 &+ \frac{2}{3\pi} \int_0^\infty d\tau \tau^2 \int_0^\infty dk k^2 \frac{k^2}{2M} \left[R_S(k, \tau) \right. \\ &+ \left. R_V(k, \tau) \left(\frac{4}{k^2 \tau^2} + \frac{m_V^2}{k^2} + \frac{v^2 - w^2}{Mv} e^{-v\tau} \right) \right] \exp \left(-\frac{k^2}{2M} \mu^2(\tau) \right) \end{aligned} \quad (43)$$

Again for $g_S, g_V \rightarrow 0$, i.e. $v \rightarrow w$ we recover the perturbative one-loop result which is obtained by expanding eqs. (34, 36) in powers of $\mathbf{p}^2$.

4 Observables

4.1 Mean number of mesons

After the Feynman parameters have been determined it is possible to evaluate other properties of the “nuclaron” in the Feynman approximation and to compare it with the known properties of the nucleon. This is achieved by evaluating the expectation value of the corresponding operator $\mathcal{O}$ from the general expression

$$\langle \mathcal{O} \rangle = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \int_0^\beta dt \frac{\int \mathcal{D}[\text{meson fields}] \int \mathcal{D}[\mathbf{x}] \mathcal{O}(t) \exp(-S)}{\int \mathcal{D}[\text{meson fields}] \int \mathcal{D}[\mathbf{x}] \exp(-S)}. \quad (44)$$

Since the quantum mechanical operator $\hat{\mathcal{O}}(a^\dagger, a, t)$ will, in general, be given in terms of the meson creation and annihilation operators its path integral expectation value can be easily obtained by adding appropriate source terms to the original meson-nucleon action and differentiating with respect to these sources. For scalar mesons, for example we replace the action in the coherent state path integral by

$$S \longrightarrow S - \int_0^\beta dt \int d^3k [\sigma(\mathbf{k}, t) \phi_{\mathbf{k}}^*(t) + \tilde{\sigma}(\mathbf{k}, t) \phi_{\mathbf{k}}(t)] \quad (45)$$

and integrate out the mesons as before. The corresponding operator is then obtained by an appropriate functional differentiation with respect to the sources after which the sources are set to zero. Thus any observable can be evaluated from the general expression

$$\langle \mathcal{O} \rangle = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \int_0^\beta dt \frac{\int \mathcal{D}[\mathbf{x}] \mathcal{O}(\delta/\delta\sigma, \delta/\delta\tilde{\sigma}, t) \exp(-S_{\text{eff}})}{\int \mathcal{D}[\mathbf{x}] \exp(-S_{\text{eff}})} \Big|_{\sigma=\tilde{\sigma}=0}. \quad (46)$$

Application to the evaluation of the mean number of scalar and vector mesons is described in detail in Appendix E. In the lowest order of a systematic expansion in powers of $S - S_F$ we replace the

exact effective action simply by S_F . We note that the zeroth order effective mass (42) already is quite a good approximation [20] and therefore we expect that for most other observables the zeroth order is also sufficiently accurate. However, there are some subtleties here which do not arise in the evaluation of energy and effective mass: one has to be careful not to combine different terms of the effective action by using symmetries under the exchange $\mathbf{k} \rightarrow -\mathbf{k}$. This cannot be used in the presence of external sources where, for instance, $\tilde{\sigma}(\mathbf{k}, t)$ is unrelated to $\sigma(-\mathbf{k}, t)$ since both are independent sources. Taking into account these caveats it is shown in Appendix E that one obtains ²

$$\langle N_S \rangle \equiv \langle \int d^3k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \rangle \simeq \langle N_S \rangle_F = \frac{1}{\pi} \int_0^\infty d\tau \tau \int_0^\infty dk k^2 R_S(k, \tau) \exp\left(-\frac{k^2}{2M} \mu^2(\tau)\right). \quad (47)$$

The corresponding expression for the mean number of vector mesons is more involved but can be obtained similarly

$$\begin{aligned} \langle N_V \rangle_F = \frac{1}{\pi} \int_0^\infty d\tau \tau \int_0^\infty dk k^2 R_V(k, \tau) & \left[\frac{k^2}{m_V^2} \left(1 + \frac{\omega_V(k)}{2M} \frac{\partial \mu^2(\tau)}{\partial \tau} \right)^2 \right. \\ & \left. + \frac{v^2 - w^2}{Mv} e^{-v\tau} \left(1 + \frac{\omega_V^2(k)}{2m_V^2} \right) \right] \exp\left(-\frac{k^2}{2M} \mu^2(\tau)\right). \end{aligned} \quad (48)$$

Again we have checked that these formulae reduce to the perturbative ones in the weak-coupling limit.

4.2 Form factor and radius

The root-mean square radius $\langle r^2 \rangle$ can be calculated from the form factor

$$F_{\mathbf{p}}(q) = \langle \mathbf{p}', \text{g.s.} | \hat{\rho}(\mathbf{x} = \mathbf{0}) | \mathbf{p}, \text{g.s.} \rangle \quad (49)$$

in the limit $q \rightarrow 0$. Here $\mathbf{p}' = \mathbf{p} + \mathbf{q}$ and $\hat{\rho}(\mathbf{x})$ is the density operator. As the theory is neither Galilei- nor Lorentz-invariant the form factor will be frame-dependent in general which is indicated by the subscript $\mathbf{p}$. As in eq. (24) one can project out the lowest energy state of a moving “nuclaron” by

$$| \mathbf{p}, \text{g.s.} \rangle = \lim_{\beta \rightarrow \infty} C_{\mathbf{p}}(\beta) \int d^3x e^{i\mathbf{p} \cdot \mathbf{x}} e^{-\beta H} | \mathbf{x}, \varphi \rangle \quad (50)$$

where φ is an arbitrary meson configuration. The normalization constant is determined by requiring

$$\langle \mathbf{p}, \text{g.s.} | \mathbf{p}', \text{g.s.} \rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}') \quad (51)$$

which is the normalization used in eq. (49). Using the path integral representation (25) we then obtain

$$F_{\mathbf{p}}(q) = \lim_{\beta \rightarrow \infty} \frac{f_{\beta}(\mathbf{q}, \mathbf{p}')}{[f_{\beta}(\mathbf{0}, \mathbf{p}) f_{\beta}^*(\mathbf{0}, \mathbf{p}')]^{1/2}} \quad (52)$$

with

$$f_{\beta}(\mathbf{q}, \mathbf{p}) = \int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle \mathbf{x}, \varphi | e^{-\beta H} e^{i\mathbf{q} \cdot \mathbf{x}} e^{-\beta H} | \mathbf{0}, \varphi \rangle \quad (53)$$

$$= \text{const} \int d^3x e^{-i\mathbf{p} \cdot \mathbf{x}} \int_{\mathbf{x}(0)=0}^{\mathbf{x}(\beta)=\mathbf{x}} \mathcal{D}[\mathbf{x}(t)] \frac{1}{\beta} \int_0^\beta dt e^{i\mathbf{q} \cdot \mathbf{x}(t)} e^{-S_{\text{eff}}[\mathbf{x}]}. \quad (54)$$

²Eq. (56) in ref. [8] is incorrect although the numerical values are not changed very much.

In lowest order approximation (i.e. replacing the exact action by the Feynman action (30)) this can be easily worked out and the result for the form factor is

$$F_{\mathbf{p}}(q) = \lim_{\beta \rightarrow \infty} \exp \left[-\frac{1}{2A_0} \frac{\beta}{2M} (p'^2 - p^2) \right] \frac{1}{\beta} \int_0^\beta dt \exp \left[\frac{\mathbf{p}' \cdot \mathbf{q}}{MA_0} t - \frac{q^2}{4M} \mu^2(2t) \right]. \quad (55)$$

This only has a reasonable large β -limit in the *Breit frame*

$$\mathbf{p} = -\frac{\mathbf{q}}{2}, \quad \mathbf{p}' = \frac{\mathbf{q}}{2} \quad (56)$$

in which there is no energy transfer ($p'^2 = p^2$). Considering the lack of Galilei invariance it is not unexpected that a consistent result can only be obtained when initial and final states are treated symmetrically. It is easily seen that terms linear in t in the exponent of eq. (55) cancel and the final result is a gaussian form factor

$$F_F(q) = \exp \left(-\frac{1}{6} q^2 \langle r^2 \rangle_F \right) \quad (57)$$

with

$$\langle r^2 \rangle_F = \frac{3}{2M} \frac{v^2 - w^2}{v^3}. \quad (58)$$

This radius is due to the recoil of the nucleon when absorbing the momentum $\mathbf{q}$ while mesons have been emitted. It should be pointed out that the same result is obtained in the picture [4] where the bare particle is coupled harmonically to a fictitious particle which simulates the effects of the meson cloud:

$$H = \frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2M_2} + \frac{k}{2} (\mathbf{r}_1 - \mathbf{r}_2)^2. \quad (59)$$

In this picture the effective mass is simply the combined mass $M + M_2$ of the system and the spring constant is related to the retardation frequency in the Feynman ansatz by $k = M_2 w^2$. These identifications allow to express M_2, k in terms of the Feynman parameters v, w :

$$\begin{aligned} M_2 &= M \frac{v^2 - w^2}{w^2} \\ k &= M (v^2 - w^2). \end{aligned} \quad (60)$$

A simple quantum mechanical calculation then gives the internal wave function and exactly the formfactor (57) with rms radius (58) for the nucleon.

One can also define a “cloud radius” which measures the extension of the meson cloud around the bare nucleon and which is given by the expectation value of the square of the relative coordinate $\mathbf{r}_1 - \mathbf{r}_2$ calculated with the intrinsic wavefunction from eq. (59). One obtains [8]

$$\langle r^2 \rangle_{\text{cloud}} = \frac{3}{2M} \frac{v}{v^2 - w^2}. \quad (61)$$

Since $v > w$ the cloud radius is always larger than $\langle r^2 \rangle_F^{1/2}$ approaching it for large couplings when $v \rightarrow \infty$. The cloud radius is closer to the charge radius of the proton (≈ 0.86 fm) if the mesons are charged whereas the radius (58) corresponds to a “core” radius of the nucleon.

5 Results and discussion

We have minimized the Feynman energy E_F with respect to the variational parameters v, w and fixed the bare mass M so that the effective mass equals the physical mass $M_N = 938.9$ MeV. This was most easily achieved by minimizing not E_F but

$$\chi^2(v, w, M) = \bar{E}_F(v, w, M) + \lambda \left[M_F^*(v, w, M) - M_N \right]^2 / M_N \quad (62)$$

where $\bar{E}_F = E_F - C_F$ and λ is a suitable weighting factor. It was chosen large enough so that the nucleon mass was reproduced to better than 20 keV. It should be pointed out that in a non-covariant calculation like the present one the Feynman energy E_F simply shifts the energy scale but is not related to the mass of the particle.

The minimization was performed with the CERN subroutine MINUIT while the expressions for the energy and the effective mass were evaluated numerically with typically 2×72 Gauss-Legendre points. The numerical calculations have been performed with the values

$$\frac{g_S^2}{4\pi} = 6.52 \quad m_S = 520 \text{ MeV} \quad (63)$$

$$\frac{g_V^2}{4\pi} = 10.84 \quad m_V = 783 \text{ MeV} \quad (64)$$

which were determined from a fit to spherical nuclei all over the periodic table [21]. It is obvious that with this parameter set the meson masses are not small enough to justify the condition (5), i.e. the nucleon recoil momentum due to the emission and absorption of these massive mesons may be comparable to the bare nucleon mass itself. This is an obvious drawback of the present approach which, however, is mitigated by the introduction of phenomenological meson-nucleon form factors which suppress large virtual momenta. The price to be paid is the strong dependence on the cut-off parameter in the form factors.

The results of our calculations are shown in Table 1. It can be seen that with increasing cut-off the vector mesons dominate the properties of the “nuclaron” and push the bare nucleon mass M to such low values that the nonrelativistic treatment is not valid anymore. The reason why the vector mesons become dominant in the variational calculation for large virtual momenta can be deduced from the explicit expressions (31), and (43) for the Feynman energy and the effective mass respectively. Comparing the relative contributions of scalar and vector mesons in the integrands for large k we see that for the energy the weighting is similar but not for the effective mass. Since the relative time τ has to be counted as $\mathcal{O}(1/k^2)$ ³ the term $4/(k^2\tau^2)$ which multiplies the vector contribution in eq. (43) is the leading term for $k \rightarrow \infty$ and is responsible for the relative enhancement of vector mesons compared to scalar mesons when the cut-off parameter in the form factors is increased. The presence of this term in the vector meson contribution is, of course, due to the velocity-dependent interaction between vector mesons and nucleons which grows stronger when the recoil momentum of the nucleons increases.

³This is due to the damping factor $\exp(-\mu^2(\tau) k^2/2M)$ and the fact that $\mu^2(\tau) \rightarrow \tau$ for $\tau \rightarrow 0$.

We also notice from Table 1 that with increasing cut-off the core radius becomes larger and tends to the Compton wavelength of the mesons which is 0.38 fm for the scalar meson and 0.25 for the vector meson. As pointed out before, the core radius is too small compared with the empirical charge radius. The mean number of scalar mesons only grows modestly in the range of Λ -parameters which we consider while the mean number of vector mesons is a rapidly increasing function of that parameter. It is amusing (but of course misleading) to think of the mesons in the cloud as real particles. In analogy to nuclear masses one then can define a “mass defect”

$$\Delta M \equiv M + \langle N_S \rangle m_S + \langle N_V \rangle m_V - M_N . \quad (65)$$

From the figures of Table 1 one finds that this quantity grows rapidly from about 70 MeV for $\Lambda = 900$ MeV to more than 1.3 GeV for $\Lambda = 1400$ MeV . Again “mass defects” of the order of the nucleon mass are inconsistent with the nonrelativistic treatment of the nucleon dynamics.

Finally it should be noted that the cut-off parameters which we consider as reasonable are smaller than the ones which are fitted in one-boson exchange models of NN scattering, e.g. in ref. [18]. A direct comparison is, however, not very meaningful, since in these models (even if they contain relativistic features) the nucleons are considered as pointlike, i.e. self-energies are neglected. In addition, many more mesons and different couplings are needed in these models to give a good description of the NN interaction and the form factor cut-off is just one of the many parameters fitted to the experimental data.

6 Conclusions

With a non-perturbative variational method we have calculated properties of the nucleon in a modified Walecka model by integrating out the scalar and vector mesons and utilising the similarity of the resulting effective action for non-relativistic nucleons to the polaron problem. This method has some attractive features: no truncation of the Hilbert space for mesons is necessary and it is expected to work also for strong coupling.

The new aspect in the present work is the inclusion of vector mesons which have a velocity-dependent coupling to the nucleons. However, with typical parameter values of the model the inclusion of massive vector mesons has unwanted consequences for the self-energy: we found both analytically and numerically that the vector mesons start to dominate the nucleon properties if higher virtual meson momenta are not cut down sufficiently by phenomenological form factors. This is, of course, caused by the velocity-dependence in the interaction of these vector mesons with nucleons. The basic assumption that the (bare) nucleon is much heavier than the mesons also breaks down as the cut-off in the vector meson-nucleon form factor increases. It would have been possible, of course, to cure superficially these problems by introducing different cut-offs for scalar and vector mesons or by taking a parametrization of the vector meson form factor which falls off more rapidly with momentum. We have refrained from doing that in order to keep the number of parameters reasonably small. Another

outcome of the numerical calculations was that the radius of the dressed nucleon was rather small (of the order of the Compton wavelength of the mesons). Again this can be traced back to the fact that the mesons in the model are too heavy or, equivalently, are not the most important degrees of freedom for the long-range dressing of the nucleon.

Therefore the present work is not to be taken as a quantitative description but rather as an exploratory method of taking into account non-perturbative dressing of the nucleon. As such it can also be applied to the two-nucleon system in a similar manner as it is done for bipolarons [22]. The resulting “binuclaron” can be considered as a rough model for the deuteron and it will be interesting to see how self energy and vertex corrections which are left out from traditional nuclear physics calculations affect the binding energy. We already have made the first step for addressing the many nucleon problem along similar lines by deriving the effective (retarded) action for a system of A nucleons in Appendix A. Actually, since the Feynman trial action is quadratic it is possible to generalize the variational calculation in a rather straight-forward way (except for antisymmetrization) to the case of A nucleons. This opens up the interesting possibility for studying the dressing of nucleons *inside* a nucleus within the present approach. Results will be reported elsewhere [23].

In the pion-nucleon system the assumption of heavy nucleons is much better fulfilled and it would be a more realistic case for describing the long-range dressing of nucleons by the present approach. The $\pi - N$ interaction, however, is strongly spin and isospin dependent and we need a generalisation of Feynman’s ansatz to include spin and isospin degrees of freedom. One can incorporate spin into the path integral using coherent spin states [24]. This makes possible to treat, for instance, the more realistic Chew-Low model [25]. The resulting effective action [9] is, in general, complex and a better understanding is needed before results of similar accuracy as those in the polaron problem are to be obtained for systems with spin and isospin degrees of freedom.

Another extension of the present approach would be the field-theoretical one, i.e., to treat the nucleons fully relativistically. This would not only remove the constraint that the nucleon has to be much heavier than the mesons but would also give covariant results for the form factor and other observables. In the present approach calculation of the form factor only made sense in the Breit frame because of the lack of Galilei or Lorentz invariance which is inherent in a non-relativistic field theory. Also the strong dependence on the cut-off in the phenomenological meson-nucleon form factors is eliminated in a renormalizable field theory. Progress in this direction has been made recently in the scalar Wick-Cutkosky model [26].

As a final conclusion we would like to point out that the dictum “Yet this was minor work” [27] on Feynman’s polaron approach seems unfounded to us considering the many applications it has in different fields and the remarkable accuracy of the method in the dressing of electrons by phonons which is unsurpassed by any analytical method up to now.

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References

- [1] H. Fröhlich, *Phil. Mag. Suppl.* **3** (1954) 325
- [2] T. K. Mitra, A. Chatterjee and S. Mukhopadhyay, *Phys. Rep.* **153** (1987) 91
- [3] B. Gerlach and H. Löwen, *Rev. Mod. Phys.* **63** (1991) 63
- [4] R. P. Feynman, *Phys. Rev.* **97** (1955) 660
- [5] C. Alexandrou and R. Rosenfelder, *Phys. Rep.* **215** (1992) 1
- [6] M. Bolsterli, in: *Adv. Nucl. Phys.*, vol. **11**, Plenum (1979), p. 367
- [7] P. K. Panda, S. P. Misra and P. Sahu, *Phys. Rev. C* **45** (1992) 2079
- [8] R. Rosenfelder, in: *Dirkfest 92*, World Scientific (1992), p. 165
- [9] C. Alexandrou, in: *Polarons and Applications*, International Workshop, Pushchino, May 1992, Manchester press, PSI-PR-92-18.
- [10] J. D. Walecka, *Ann. Phys. (N.Y.)* **83** (1974) 491
- [11] B. D. Serot and J. D. Walecka, *Adv. Nuc. Phys.* **16** (1986) 1
- [12] B. D. Serot, *Rep. Progr. Phys.* **55** (1992) 1855
- [13] C.J. Horowitz and B. D. Serot, *Nucl. Phys. A* **368** (1981) 503
- [14] J. D. Bjorken and S. D. Drell: *Relativistic Quantum Mechanics*, p. 51, McGraw-Hill (1964)
- [15] L. Brink, P. DiVecchia and P. Howe, *Nucl. Phys. B* **118** (1977) 76
 E. Fradkin and M. Stone, *Phys. Rev. B* **38** (1988) 7215
 E. S. Fradkin and D. M. Gitman, *Phys. Rev. D* **44** (1991) 3230
 M. Bergeron, *Fortschr. Phys.* **40** (1992) 119
- [16] C. Itzykson and J.-B. Zuber: *Quantum Field Theory*, p. 134, McGraw-Hill (1980)
- [17] L. S. Schulman: *Techniques and Applications of Path Integration*, John Wiley (1981), chapter 4
- [18] F. Gross, J. W. van Orden and K. Holinde, *Phys. Rev. C* **45** (1992) 2094
- [19] R. P. Feynman and A. R. Hibbs: *Quantum Mechanics and Path Integrals*, p. 251, eq. (9-64), McGraw-Hill (1965)
- [20] Yang Lu and R. Rosenfelder, *Phys. Rev. B* **46** (1992) 5211
- [21] C. J. Horowitz and B. D. Serot, *Nucl. Phys. A* **368** (1981) 503

- [22] V. M. Fomin and M. A. Smondyrev: Fundamentals of the large bipolaron theory, Dubna preprint (1992)
- [23] C. Alexandrou and Ph. Diakonou, in preparation
- [24] J. R. Klauder, Phys. Rev. **D 19** (1979) 2349
- [25] G. F. Chew and F. E. Low, Phys. Rev. **101** (1956) 1570
- [26] R. Rosenfelder and A. W. Schreiber, preprint PSI-PR-94-07 (March 1994)
- [27] J. Gleick: Genius: The Life and Science of Richard Feynman, Pantheon (1992), p. 349
- [28] J. W. Negele and H. Orland: Quantum Many-Particle Systems, [Frontier in Physics, vol. 68], Addison-Wesley (1988), p. 69
- [29] Handbook of Mathematical Functions, eds. M. Abramowitz and I. A. Stegun, Dover (1965)

Appendix

A Integrating out the mesons

In this Appendix we derive the effective action in the A – nucleon sector by integrating out the mesons. If the spin-dependent terms in eq. (6) are neglected the Hamiltonian describing A non-relativistic nucleons is given by

$$H = \sum_{i=1}^A \frac{(\mathbf{p}_i - g_V \mathbf{V}(\mathbf{x}_i))^2}{2M} - g_S \sum_{i=1}^A \phi(\mathbf{x}_i) + g_V \sum_{i=1}^A V_0(\mathbf{x}_i) + H_{(V)}^0 + H_{(S)}^0. \quad (\text{A.1})$$

Here $H_{(V)}^0$ and $H_{(S)}^0$ are the free Hamiltonians for vector and scalar mesons respectively given in eqs. (8,9). The Lagrangian path integral for the partition function is obtained in the A -nucleon case as in eq. (18). We eliminate the vector and scalar mesons by using the coherent state representation of the functional integral for the meson fields. In euclidean time we then can write the partition function in the following form [28]

$$Z = \int \left(\prod_{j=1}^A \mathcal{D}[\mathbf{x}_j] \right) \mathcal{D}[\phi_{\mathbf{k}}^* \phi_{\mathbf{k}}] \mathcal{D}[V_{\mathbf{k}}^{\lambda*} V_{\mathbf{k}}^{\lambda}] \exp(-S[\mathbf{x}_j, \text{meson fields}]) \quad (\text{A.2})$$

with

$$\begin{aligned} S[\mathbf{x}_j, \text{meson fields}] &= \int_0^\beta dt \left\{ \sum_{j=1}^A \left[\frac{M}{2} \dot{\mathbf{x}}_j^2 \right. \right. \\ &\quad - i g_V \dot{\mathbf{x}}_j \cdot \int d^3k \sum_{\lambda} \epsilon^{\lambda}(\mathbf{k}) \left(f_{\mathbf{k}}^V(\mathbf{x}_j) V_{\mathbf{k}}^{\lambda*}(t) + f_{\mathbf{k}}^{V*}(\mathbf{x}_j) V_{\mathbf{k}}^{\lambda}(t) \right) \\ &\quad - g_S \int d^3k \left(f_{\mathbf{k}}^S(\mathbf{x}_j) \phi_{\mathbf{k}}^*(t) + f_{\mathbf{k}}^S(\mathbf{x}_j) \phi_{\mathbf{k}}(t) \right) \\ &\quad + g_V \int d^3k \sum_{\lambda} \epsilon_0^{\lambda}(\mathbf{k}) \left(f_{\mathbf{k}}^V(\mathbf{x}_j) V_{\mathbf{k}}^{\lambda*}(t) + f_{\mathbf{k}}^{V*}(\mathbf{x}_j) V_{\mathbf{k}}^{\lambda}(t) \right) \Big] \\ &\quad + \int d^3k \phi_{\mathbf{k}}^*(t) \left(\frac{\partial \phi_{\mathbf{k}}(t)}{\partial t} - \omega_S(k) \phi_{\mathbf{k}}(t) \right) \\ &\quad + \int d^3k \sum_{\lambda} V_{\mathbf{k}}^{\lambda*}(t) \left(\frac{\partial V_{\mathbf{k}}^{\lambda}(t)}{\partial t} - \omega_V(k) V_{\mathbf{k}}^{\lambda}(t) \right) \Big\}. \end{aligned} \quad (\text{A.3})$$

Here the expansions (10,11) for the meson fields have been used and we have introduced the abbreviations

$$f_{\mathbf{k}}^{S,V}(\mathbf{x}_j) = \frac{1}{\sqrt{2\omega_{S,V}(k)}} \frac{e^{-i\mathbf{k}\cdot\mathbf{x}_j}}{(2\pi)^{3/2}}. \quad (\text{A.4})$$

For later use we also define

$$h_{\mathbf{k}}^{\lambda}(\mathbf{x}_j) = f_{\mathbf{k}}^V(\mathbf{x}_j) \left(i \dot{\mathbf{x}}_j \cdot \epsilon^{\lambda}(\mathbf{k}) - \epsilon_0^{\lambda}(\mathbf{k}) \right). \quad (\text{A.5})$$

and

$$\tilde{h}_{\mathbf{k}}^{\lambda}(\mathbf{x}_j) = f_{\mathbf{k}}^{V*}(\mathbf{x}_j) \left(i \dot{\mathbf{x}}_j \cdot \epsilon^{\lambda}(\mathbf{k}) - \epsilon_0^{\lambda}(\mathbf{k}) \right). \quad (\text{A.6})$$

Note that the velocity-dependent interaction term becomes imaginary in euclidean time and that $\tilde{h}^\lambda$ is not the complex conjugate of h^λ . The phenomenological meson-nucleon form factors (21) have been left out for notational simplicity but may be re-introduced at any stage of the calculation.

Since the action given in eq. (A.3) is at most quadratic in the meson fields the stationary phase approximation is exact and we can use the equations of motion for the classical fields

$$\frac{\partial \phi_{\mathbf{k}}^{\text{cl}}(t)}{\partial t} + \omega_S(k) \phi_{\mathbf{k}}^{\text{cl}}(t) = g_S \sum_{i=1}^A f_{\mathbf{k}}^S(\mathbf{x}_i(t)) \quad (\text{A.7})$$

$$\frac{\partial V_{\mathbf{k}}^{\lambda \text{cl}}(t)}{\partial t} + \omega_V(k) V_{\mathbf{k}}^{\lambda \text{cl}}(t) = g_V \sum_{i=1}^A h_{\mathbf{k}}^\lambda(\mathbf{x}_i(t)) \quad (\text{A.8})$$

to eliminate the meson degrees of freedom. The prefactor arising from the quadratic fluctuations of the fields is an irrelevant constant which can be absorbed in the normalization of the path integral. Apart from this overall constant factor the partition function is then simply given by $\exp(-S_{\text{cl}})$. In terms of the classical fields $\phi_{\mathbf{k}}^{\text{cl}}(t)$ and $V_{\mathbf{k}}^{\lambda \text{cl}}(t)$ the classical action is given by

$$S_{\text{cl}} = \sum_{j=1}^A \int_0^T dt \left\{ \frac{M}{2} \dot{\mathbf{x}}_j^2 + g_S \int d^3k f_{\mathbf{k}}^{S*}(\mathbf{x}_j) \phi_{\mathbf{k}}^{\text{cl}}(t) + g_V \int d^3k \sum_{\lambda} \tilde{h}_{\mathbf{k}}^{\lambda*}(\mathbf{x}_j) V_{\mathbf{k}}^{\lambda \text{cl}}(t) \right\}. \quad (\text{A.9})$$

The solutions of the equations of motion (A.7, A.8) with periodic boundary conditions are

$$\phi_{\mathbf{k}}^{\text{cl}}(t) = \frac{g_S}{1 - e^{-\omega_S \beta}} \sum_{i=1}^A \left\{ \int_t^\beta dt' e^{-\omega_S(\beta+t-t')} f_{\mathbf{k}}^S(\mathbf{x}_i(t')) + \int_0^t dt' e^{-\omega_S(t-t')} f_{\mathbf{k}}^S(\mathbf{x}_i(t')) \right\} \quad (\text{A.10})$$

$$V_{\mathbf{k}}^{\lambda \text{cl}}(t) = \frac{g_V}{1 - e^{-\omega_V \beta}} \sum_{i=1}^A \left\{ \int_t^\beta dt' e^{-\omega_V(\beta+t-t')} h_{\mathbf{k}}^\lambda(\mathbf{x}_i(t')) + \int_0^t dt' e^{-\omega_V(t-t')} h_{\mathbf{k}}^\lambda(\mathbf{x}_i(t')) \right\} \quad (\text{A.11})$$

Inserting eq. (A.10) into eq. (A.9) we obtain for the action arising from the elimination of the scalar mesons

$$S_{\text{cl}}^S = -g_S^2 \sum_{i,j=1}^A \int_0^\beta dt \int_0^\beta dt' \int d^3k \frac{1}{1 - e^{-\omega_S \beta}} f_{\mathbf{k}}^S(\mathbf{x}_i(t')) f_{\mathbf{k}}^{S*}(\mathbf{x}_j(t)) \cdot \left[\theta(t' - t) e^{-\omega_S(\beta+t-t')} + \theta(t - t') e^{-\omega_S(t-t')} \right]. \quad (\text{A.12})$$

Using the relation $f_{\mathbf{k}}^{S*} = f_{-\mathbf{k}}^S$ this can be also written as

$$S_{\text{cl}}^S = -\frac{g_S^2}{2} \sum_{i,j=1}^A \int_0^\beta dt \int_0^\beta dt' \int d^3k f_{\mathbf{k}}^S(\mathbf{x}_i(t')) f_{\mathbf{k}}^{S*}(\mathbf{x}_j(t)) \frac{\cosh[\omega_S(k)|t - t'| - \omega_S(k)\beta/2]}{\sinh[\omega_S(k)\beta/2]}. \quad (\text{A.13})$$

Equivalently elimination of the vector mesons yields

$$S_{\text{cl}}^V = -g_V^2 \sum_{i,j=1}^A \int_0^\beta dt \int_0^\beta dt' \int d^3k \frac{1}{1 - e^{-\omega_V \beta}} \sum_{\lambda} \tilde{h}_{\mathbf{k}}^{\lambda}(\mathbf{x}_i(t')) h_{\mathbf{k}}^{\lambda}(\mathbf{x}_j(t)) \cdot \left[\theta(t' - t) e^{-\omega_V(\beta+t-t')} + \theta(t - t') e^{-\omega_V(t-t')} \right]. \quad (\text{A.14})$$

The summation over the polarizations λ in eq. (A.14) can be performed using eq. (16) and one obtains

$$\begin{aligned}
S_{cl}^V &= -g_V^2 \sum_{i,j=1}^A \int_0^\beta dt \int_0^\beta dt' \int d^3k \frac{1}{1 - e^{-\omega_V \beta}} f_{\mathbf{k}}^V(\mathbf{x}_i(t')) f_{\mathbf{k}}^{V*}(\mathbf{x}_j(t)) \\
&\cdot \left[\theta(t' - t) e^{-\omega_V(\beta+t-t')} + \theta(t - t') e^{-\omega_V(t-t')} \right] \\
&\cdot \left[\frac{k^2}{m_V^2} - \dot{\mathbf{x}}_{\perp(i)}(t') \cdot \dot{\mathbf{x}}_{\perp(j)}(t) - \frac{\omega_V^2}{m_V^2 k^2} \mathbf{k} \cdot \dot{\mathbf{x}}_i(t') \mathbf{k} \cdot \dot{\mathbf{x}}_j(t) - i \frac{\omega_V}{m_V^2} (\mathbf{k} \cdot \dot{\mathbf{x}}_i(t') + \mathbf{k} \cdot \dot{\mathbf{x}}_j(t)) \right]
\end{aligned} \tag{A.15}$$

where $\mathbf{x}_{\perp}$ are the transverse components with respect to the $\mathbf{k}$ -vector. Note that the action is not symmetric under $\mathbf{k} \rightarrow -\mathbf{k}$ and therefore the θ -functions cannot be combined at this stage. The last two terms in eq. (A.16) involving the longitudinal components, $\mathbf{x}_{\parallel}$, can be eliminated by integration by parts. For this purpose it is convenient to define the following function

$$F_{ij}(t, t') \equiv e^{-\omega_V(t-t')} e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t'))} \tag{A.16}$$

in terms of which we obtain the following relations

$$\begin{aligned}
(i\mathbf{k} \cdot \dot{\mathbf{x}}_i(t) + i\mathbf{k} \cdot \dot{\mathbf{x}}_j(t')) F_{ij}(t, t') \theta(t' - t) &= \left(\frac{d}{dt} - \frac{d}{dt'} \right) F_{ij}(t, t') \theta(t' - t) \\
&+ 2\omega_V F_{ij}(t, t') \theta(t' - t) + 2F_{ij}(t, t') \delta(t' - t)
\end{aligned} \tag{A.17}$$

and

$$\begin{aligned}
\mathbf{k} \cdot \dot{\mathbf{x}}_i(t) \mathbf{k} \cdot \dot{\mathbf{x}}_j(t') F_{ij}(t, t') \theta(t' - t) &= \frac{d}{dt} \frac{d}{dt'} [F_{ij}(t, t') \theta(t' - t)] + \frac{d}{dt'} [F_{ij}(t, t') \delta(t' - t)] \\
&- \omega_V \left(\frac{d}{dt} - \frac{d}{dt'} \right) [F_{ij}(t, t') \theta(t' - t)] - \omega_V^2 F_{ij}(t, t') \theta(t' - t) \\
&- (i\mathbf{k} \cdot \dot{\mathbf{x}}_i(t) + \omega_V) F_{ij}(t, t') \delta(t' - t)
\end{aligned} \tag{A.18}$$

with similar expressions for $t > t'$. Substituting in S_{cl}^V and using periodic boundary conditions ($\mathbf{x}_i(0) = \mathbf{x}_i(\beta)$ for all i) which eliminate end-dependent terms we end up with

$$\begin{aligned}
S_{cl}^V &= -\frac{g_V^2}{2} \sum_{i,j=1}^A \int \frac{d^3k}{(2\pi)^3} \int_0^\beta dt \int_0^\beta dt' \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t'))}}{2\omega_V(k)} \left[\frac{m_V^2}{k^2} - \dot{\mathbf{x}}_{\perp(i)}(t) \cdot \dot{\mathbf{x}}_{\perp(j)}(t') \right] \\
&\cdot \frac{\cosh[\omega_V(k)|t - t'| - \omega_V(k)\beta/2]}{\sinh[\omega_V(k)\beta/2]} + g_V^2 \sum_{i,j=1}^A \int \frac{d^3k}{(2\pi)^3} \int_0^\beta dt \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t))}}{2\omega_V(k)} \left(\frac{m_V^2 - k^2}{m_V^2} \right) \frac{\omega_V(k)}{k^2}.
\end{aligned} \tag{A.19}$$

Note that the vector effective action now explicitly is real. In the limit $\beta \rightarrow \infty$ the hyperbolic functions can be replaced by exponentials and we obtain

$$\begin{aligned}
S_{\text{eff}} &= \int_0^\beta dt \left\{ \sum_{i=1}^A \frac{M}{2} \dot{\mathbf{x}}_i^2 \right. \\
&- \frac{g_S^2}{2} \sum_{i,j=1}^A \int_0^\beta dt' \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t'))}}{2\omega_S(k)} e^{-\omega_S(k)|t-t'|}
\end{aligned}$$

$$\begin{aligned}
& - \frac{g_V^2}{2} \sum_{i,j=1}^A \int_0^\beta dt' \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t'))}}{2\omega_V(k)} \left[\frac{m_V^2}{k^2} - \dot{\mathbf{x}}_{\perp(i)}(t) \cdot \dot{\mathbf{x}}_{\perp(j)}(t') \right] e^{-\omega_V(k)|t-t'|} \\
& + g_V^2 \sum_{i,j=1}^A \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t))}}{2\omega_V(k)} \left(1 - \frac{k^2}{m_V^2} \right) \frac{\omega_V(k)}{k^2} \Big\} . \tag{A.20}
\end{aligned}$$

The effective action can be split up into a self-energy part and a part which describes the interaction with other nucleons:

$$S_{\text{eff}} = \sum_{i=1}^A S_{\text{eff}}(i) + \sum_{i \neq j}^A S_{\text{eff}}(i, j) . \tag{A.21}$$

The latter contains the usual static two-body potential if retardation effects and velocity dependent terms are neglected (see Appendix B). Note that the vector mesons also generate a single-time term in the effective action. For $i = j$ this term is independent of the nucleon co-ordinate and contributes to the constant C_F in the Feynman energy (31) which does not depend on the variational parameters.

B The static two-body potential

The potential due to the exchange of scalars is of Yukawa type and attractive. It is straightforward to obtain it from the $i \neq j$ part of the scalar effective action (A.20)

$$S_{\text{eff}}^{(S)}(i, j) = -2g_S^2 \int_0^\beta dt \int_0^t dt' \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot (\mathbf{x}_i(t) - \mathbf{x}_j(t'))}}{2\omega_S(k)} e^{-\omega_S(k)|t-t'|} . \tag{B.1}$$

if the retardation is neglected. In eq. (B.1) the t' -integration has been restricted to the range $t' \leq t$ which gives a factor 2 and the summation over pairs $i \neq j$ has given another factor 2. Introducing total and relative times $t - t' = \tau$ and $(t + t')/2 = T$ we have

$$S_{\text{eff}}^{(S)}(i, j) = -2g_S^2 \int_0^\beta d\tau \int_{\tau/2}^{\beta-\tau/2} dT \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_S(k)} e^{-\omega_S \tau} \exp[i\mathbf{k} \cdot (\mathbf{x}_i(T + \tau/2) - \mathbf{x}_j(T - \tau/2))] . \tag{B.2}$$

The main contribution to the above integral comes from small τ -values and we may therefore take the limit $\tau \rightarrow 0$ for the limits in the integration over dT . Furthermore, we approximate the exponent $\mathbf{x}_i(T + \tau/2) - \mathbf{x}_j(T - \tau/2) \sim \mathbf{x}_i(T) - \mathbf{x}_j(T)$. Performing the τ integral we then obtain

$$S_{\text{eff}}^{(S)}(i, j) \simeq -2g_S^2 \int_0^\beta dT \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_S^2(k)} e^{i\mathbf{k} \cdot (\mathbf{x}_i(T) - \mathbf{x}_j(T))} . \tag{B.3}$$

In a single-time action this should equal

$$\int_0^\beta dT \int \frac{d^3k}{(2\pi)^3} \tilde{V}^{(S)}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{x}_i(T) - \mathbf{x}_j(T))} . \tag{B.4}$$

from which we deduce the familiar attractive static two-body potential

$$\tilde{V}^{(S)}(\mathbf{k}) = - \frac{g_S^2}{k^2 + m_s^2} \tag{B.5}$$

arising from scalar exchange.

We now discuss the static potential arising due to the exchange of vector mesons and show that the usual repulsive potential emerges from our effective action under similar approximations. In this case the two-time part of the vector effective action given in eq. (A.20) becomes

$$S_{\text{eff}}^{(V)}(i, j) \simeq -2g_V^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\beta dT \frac{1}{2\omega_V^2(k)} \frac{m_V^2}{k^2} \exp[i\mathbf{k} \cdot (\mathbf{x}_i(T) - \mathbf{x}_j(T))] \quad (\text{B.6})$$

where in addition we dropped the velocity dependent terms. Combining with the single-time term in eq. (A.20) we obtain

$$S_{\text{eff}}^{(V)}(i, j) \simeq -2g_V^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\beta dT \left(\frac{m_V^2}{2k^2\omega_V^2(k)} - \frac{m_V^2 - k^2}{2m_V^2 k^2} \right) \exp[i\mathbf{k} \cdot (\mathbf{x}_i(T) - \mathbf{x}_j(T))] . \quad (\text{B.7})$$

Thus we end up with

$$\begin{aligned} V^V(\mathbf{x}) &= -g_V^2 \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} \left(\frac{1}{m_V^2} - \frac{1}{k^2 + m_V^2} \right) \\ &= -g_V^2 \left(\frac{1}{m_V^2} \delta^3(x) - \frac{e^{-m_V|\mathbf{x}|}}{4\pi|\mathbf{x}|} \right) . \end{aligned} \quad (\text{B.8})$$

Apart from the attractive contact interaction the potential is the usual repulsive Yukawa potential typical for vector meson exchange. The above result can also be derived from the full Hamiltonian before elimination of the meson degrees of freedom using perturbation theory which provides an useful check. The T-matrix in Born approximation reads

$$T_{1,2 \rightarrow 1',2'} = - \langle \mathbf{p}'_1, \mathbf{p}'_2 | H_{\text{in}} \frac{1}{H_0^{(V)}} H_{\text{in}} | \mathbf{p}_1, \mathbf{p}_1 \rangle \quad (\text{B.9})$$

where

$$H_{\text{in}} = g_V \sum_{i=1}^2 V_0(\mathbf{x}_i) - \frac{g_V}{2M} \sum_{i=1}^2 (\hat{\mathbf{p}}_i \cdot \mathbf{V}(\mathbf{x}_i) + \mathbf{V}(\mathbf{x}_i) \cdot \hat{\mathbf{p}}_i) . \quad (\text{B.10})$$

Using conservation of energy and working in the center of mass frame one obtains in leading order in $1/M$

$$T_{1,2 \rightarrow 1',2'}(\mathbf{q} = \mathbf{p}'_1 - \mathbf{p}) = \frac{g_V^2}{\omega_V^2(q)} - \frac{g_V^2}{m_V^2} . \quad (\text{B.11})$$

The Fourier transform of $T(q)$ reproduces the result (B.8) including the contact term.

C Evaluation of the Feynman energy

To evaluate the Feynman energy E_F from eq. (26) the main task is to average the effective action (19) with the Feynman trial action. It is useful to divide the effective action into a free part and an interaction part

$$S_{\text{eff}} = S_0 + S_1 \quad (\text{C.1})$$

with

$$S_0 = \sum_{n=0}^{\infty} b_n^2. \quad (\text{C.2})$$

Averaging over the free and the trial action is trivial and gives

$$\begin{aligned} -\log Z_F + \langle S_0 - S_F \rangle_F &= \frac{3}{2} \sum_{n=0}^{\infty} \left[\log A_n - 1 + \frac{1}{A_n} \right] \\ &\xrightarrow{\beta \rightarrow \infty} \beta \frac{3(v-w)^2}{4v} \end{aligned} \quad (\text{C.3})$$

where in the last line the explicit form of the coefficients A_n in eq. (30) has been used for performing the sum.

We now concentrate on the interaction part S_1 of the effective action. For scalar mesons the required averaging amounts to calculating the standard expression

$$G(k, t, t') \equiv \left\langle e^{i\mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))} \right\rangle_F. \quad (\text{C.4})$$

With the Fourier expansion (28) and the trial action (30) this is just an infinite product of gaussian integrals. Therefore we obtain

$$G(k, t, t') = \exp \left(-\frac{k^2}{2M} \mu^2(t, t') \right) \quad (\text{C.5})$$

where

$$\mu^2(t, t') = \beta \sum_{n=0}^{\infty} \frac{\lambda_n^2(t, t')}{A_n} \quad (\text{C.6})$$

with

$$\lambda_0(t, t') = \frac{t - t'}{\beta} \quad (\text{C.7})$$

$$\lambda_n(t, t') = \frac{\sqrt{2}}{n\pi} \left[\sin \left(\frac{n\pi t}{\beta} \right) - \sin \left(\frac{n\pi t'}{\beta} \right) \right] \quad n = 1, 2, \dots \quad (\text{C.8})$$

We are only interested in the limit $\beta \rightarrow \infty$ which projects out the ground state. In this limit it can be shown [5] that $\mu^2(t, t') \rightarrow \mu^2(t - t')$ where the last quantity is given explicitly in eq. (32). Therefore

$$\lim_{\beta \rightarrow \infty} G(k, \tau = t - t') = \exp \left(-\frac{k^2}{2M} \mu^2(\tau) \right). \quad (\text{C.9})$$

With the help of eq. (C.9) the scalar part of the Feynman energy (31) is then obtained.

In the vector meson case one has to evaluate the new expression

$$H_i(k, t, t') \equiv \left\langle \dot{x}_i(t) \dot{x}_i(t') e^{i\mathbf{k} \cdot (\mathbf{x}(t) - \mathbf{x}(t'))} \right\rangle_F. \quad (\text{C.10})$$

where $i = 1, 2$ denote the cartesian coordinates perpendicular to $\mathbf{k} = k(0, 0, 1)$. Note that this expression cannot be obtained by a differentiation of eq. (C.4) with respect to the times t and t' .

However, it can be evaluated again by inserting the Fourier expansion (28) of the paths and performing the gaussian integrals. One then obtains

$$H_i(k, t, t') = \frac{2}{\beta M} \sum_{n, n'=0}^{\infty} \alpha_n(t) \alpha_{n'}(t') \left[\frac{\delta_{nn'}}{2A_n} - \frac{\lambda_n(t, t') \lambda_{n'}(t, t')}{A_n A_{n'}} \beta \frac{k_i^2}{2M} \right] \exp \left(-\frac{k^2}{2M} \mu^2(t, t') \right) \quad (\text{C.11})$$

where

$$\alpha_0(t) = 1 \quad (\text{C.12})$$

$$\alpha_n(t) = \sqrt{2} \cos \left(\frac{n\pi t}{\beta} \right) \quad n = 1, 2, \dots \quad (\text{C.13})$$

Since we only need the perpendicular components the last term in the square bracket of Eq. (C.11) does not contribute. In contrast to eq. (C.6) the sum over α_n^2/A_n generates a δ -function since the higher modes are not damped by inverse powers of $n\pi$. This is, of course, due to the time-derivatives in eq. (C.10). Explicit summation or use of the Euler-MacLaurin formula [29] gives in the limit $\beta \rightarrow \infty$

$$H(k, \tau = t - t') \equiv \lim_{\beta \rightarrow \infty} \sum_{i=1}^2 H_i(k, t, t') = \frac{2}{M} \delta(\tau) - \frac{v^2 - w^2}{Mv} e^{-v\tau} \exp \left(-\frac{k^2}{2M} \mu^2(\tau) \right). \quad (\text{C.14})$$

By using eq. (C.14) the vector part for the Feynman energy (31) follows. The δ -function gives another contribution to the constant C_F .

D Evaluation of the effective mass

We calculate the effective mass of the “nuclaron” from eq. (41) by noting that

$$\langle \mathbf{b}_0^2 \rangle_F = \frac{3}{2A_0} \quad (\text{D.1})$$

where $A_0 = v^2/w^2$. Furthermore the factor $\mathbf{b}_0^2$ within averages can be generated simply by differentiating the corresponding average with respect to A_0 . For example, for the interaction part S_1 of the action one obtains

$$\langle \mathbf{b}_0^2 S_1 \rangle_F = -\frac{\partial}{\partial A_0} \langle S_1 \rangle_F + \frac{3}{2A_0} \langle S_1 \rangle_F \quad (\text{D.2})$$

whereas the difference between free and Feynman actions weighed by $\mathbf{b}_0^2$ has the average

$$\langle \mathbf{b}_0^2 (S_0 - S_F) \rangle_F = \frac{15}{4} \frac{1 - A_0}{A_0^2} + \frac{9}{4A_0} \sum_{n=1}^{\infty} \frac{1 - A_n}{A_n}. \quad (\text{D.3})$$

Combining all pieces one obtains the simple expression

$$\frac{M_F^*}{M} = \lim_{\beta \rightarrow \infty} \left[1 - \frac{2A_0^2}{3} \frac{\partial}{\partial A_0} \langle S_1 \rangle_F \right]. \quad (\text{D.4})$$

The evaluation of the effective mass is thus reduced to the evaluation of the contributions of scalar and vector mesons to the energy followed by a simple differentiation. In this way eq. (43) is obtained.

E Evaluation of the mean numbers of mesons

To obtain the average numbers of scalar and vector mesons surrounding the nucleon we use the effective action due to scalar mesons, eq. (A.13) and due to vector mesons, eq. (A.14) derived in Appendix A. As discussed in section 4.1, we introduce source terms in the meson-nucleon Hamiltonian as in eq. (45) to calculate expectation values. This amounts to the replacement by

$$\begin{aligned} f_{\mathbf{k}}^S(t) &\rightarrow f_{\mathbf{k}}^S(t) - \sigma(\mathbf{k}, t) \\ f_{\mathbf{k}}^{S*}(t) &\rightarrow f_{\mathbf{k}}^{S*}(t) - \bar{\sigma}(\mathbf{k}, t) \\ h_{\mathbf{k}}^\lambda(t) &\rightarrow h_{\mathbf{k}}^\lambda(t) - \eta^\lambda(\mathbf{k}, t) \\ \tilde{h}_{\mathbf{k}}^\lambda(t) &\rightarrow \tilde{h}_{\mathbf{k}}^\lambda(t) - \tilde{\eta}^\lambda(\mathbf{k}, t) \end{aligned} \quad (\text{E.1})$$

where $\sigma(\mathbf{k}, t)$, $\bar{\sigma}(\mathbf{k}, t)$ and $\eta^\lambda(\mathbf{k}, t)$, $\tilde{\eta}^\lambda(\mathbf{k}, t)$ are the sources coupled to the scalar and vector mesons respectively.

The mean number of scalar mesons is given by

$$\langle N_S \rangle = \langle \int d^3k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \rangle = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \int_0^\beta dt \int d^3k \langle \mathcal{O}_{\mathbf{k}}(t) \rangle \quad (\text{E.2})$$

with

$$\begin{aligned} \mathcal{O}_{\mathbf{k}}(t) &= \left. \frac{\delta^2 e^{-S_{\text{eff}}}}{\delta \sigma(\mathbf{k}, t) \delta \bar{\sigma}(\mathbf{k}, t)} \right|_{\sigma=\bar{\sigma}=0} \\ &= \frac{1}{(1 - e^{-\omega_S \beta})^2} \int_0^\beta dt_1 \int_0^\beta dt_2 f_{\mathbf{k}}^{S*}(t_1) f_{\mathbf{k}}^S(t_2) \left[\theta(t_1 - t) e^{-\omega_S(t_1-t)} + \theta(t - t_1) e^{\omega_S(t-t_1-\beta)} \right] \\ &\quad \cdot \left[\theta(t - t_2) e^{-\omega_S(t-t_2)} + \theta(t_2 - t) e^{\omega_S(t_2-t-\beta)} \right]. \end{aligned} \quad (\text{E.3})$$

The time average of the quantity which appears in eq. (E.2) can be simplified in the large β limit by noting that

$$\frac{1}{\beta} \int_0^\beta dt \left[\theta(t_1 - t) e^{-\omega_S(t_1-t)} + \theta(t - t_1) e^{-\omega_S(\beta+t_1-t)} \right] \cdot \left[\theta(t - t_2) e^{-\omega_S(t-t_2)} + \theta(t_2 - t) e^{-\omega_S(\beta+t-t_2)} \right] \quad (\text{E.4})$$

has the limit

$$\frac{1}{\beta} \theta(t_1 - t_2) (t_1 - t_2) e^{-\omega_S(t_1-t_2)}. \quad (\text{E.5})$$

Combined with the average (C.9) we obtain eq. (47) for the mean number of scalar mesons in the leading order Feynman approximation.

For vector mesons, a similar derivation leads to

$$\begin{aligned} \langle N_V \rangle &= g_V^2 \int \frac{d^3k}{(2\pi)^3} \frac{F_V^2(k)}{2\omega_V(k)} \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \int_0^\beta dt_1 \int_0^\beta dt_2 \theta(t_1 - t_2) (t_1 - t_2) e^{-\omega_V(t_1-t_2)} \\ &\quad \cdot \left\langle e^{i\mathbf{k} \cdot (\mathbf{x}(t_1) - \mathbf{x}(t_2))} \left[\frac{k^2}{m_V^2} - \dot{\mathbf{x}}_\perp(t_1) \cdot \dot{\mathbf{x}}_\perp(t_2) \right. \right. \\ &\quad \left. \left. - \frac{\omega_V^2}{m_V^2 k^2} \mathbf{k} \cdot \dot{\mathbf{x}}(t_1) \mathbf{k} \cdot \dot{\mathbf{x}}(t_2) - i \frac{\omega_V}{m_V^2} (\mathbf{k} \cdot \dot{\mathbf{x}}(t_1) + \mathbf{k} \cdot \dot{\mathbf{x}}(t_2)) \right] \right\rangle \end{aligned} \quad (\text{E.6})$$

which should be compared with eq. (A.16). In the leading order of the Feynman approximation, we need to calculate the following four averages with the trial action

$$G(k, t_1, t_2) = \left\langle e^{i\mathbf{k} \cdot (\mathbf{x}(t_1) - \mathbf{x}(t_2))} \right\rangle_F \quad (\text{E.7})$$

$$\begin{aligned} G'(k, t_1, t_2) &= \left\langle (\mathbf{k} \cdot (\dot{\mathbf{x}}(t_1) + \dot{\mathbf{x}}(t_2))) e^{i\mathbf{k} \cdot (\mathbf{x}(t_1) - \mathbf{x}(t_2))} \right\rangle_F \\ &= \frac{1}{i} \left(\frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2} \right) G(k, t_1, t_2) \end{aligned} \quad (\text{E.8})$$

$$\begin{aligned} G''(k, t_1, t_2) &= \left\langle \mathbf{k} \cdot \dot{\mathbf{x}}(t_1) \mathbf{k} \cdot \dot{\mathbf{x}}(t_2) e^{i\mathbf{k} \cdot (\mathbf{x}(t_1) - \mathbf{x}(t_2))} \right\rangle_F \\ &= \frac{\partial}{\partial t_1} \frac{\partial}{\partial t_2} G(k, t_1, t_2) \end{aligned} \quad (\text{E.9})$$

$$H(k, t_1, t_2) = \left\langle \dot{\mathbf{x}}_{\perp}(t_1) \cdot \dot{\mathbf{x}}_{\perp}(t_2) e^{i\mathbf{k} \cdot (\mathbf{x}(t_1) - \mathbf{x}(t_2))} \right\rangle_F. \quad (\text{E.10})$$

In the limit of $\beta \rightarrow \infty$ all averages are functions of $\tau = t_1 - t_2$ only. $G(k, \tau)$ is given explicitly in eq. (C.9). From this result the averages G' and G'' can be easily obtained by differentiation

$$\begin{aligned} G'(k, \tau) &= \frac{2}{i} \frac{d}{d\tau} G(k, \tau) \\ &= i \frac{k^2}{M} \left(\frac{w^2}{v^2} + \frac{v^2 - w^2}{v^2} e^{-v\tau} \right) \exp \left(-\frac{k^2}{2M} \mu^2(\tau) \right) \end{aligned} \quad (\text{E.11})$$

$$\begin{aligned} G''(k, \tau) &= -\frac{d^2}{d\tau^2} G(k, \tau) \\ &= -\left[\frac{k^2}{2M} \frac{v^2 - w^2}{v} e^{-v\tau} + \left(\frac{k^2}{2M} \right)^2 \left(\frac{w^2}{v^2} + \frac{v^2 - w^2}{v^2} e^{-u\tau} \right)^2 \right] \exp \left(-\frac{k^2}{2M} \mu^2(u) \right) \end{aligned} \quad (\text{E.12})$$

The average H has already been obtained in eq. (C.14). Employing these expressions in eq. (E.6) we obtain in the leading order Feynman approximation the average number of vector mesons as given in eq. (49).

Table 1 :

Variational results for different values of the cut-off parameter Λ in the phenomenological meson-nucleon form factors. The effective mass of the “nuclaron” has been fixed at 938.9 MeV and the meson masses and coupling constants which are used are given in eq. (64). The variational parameters in the Feynman trial action are v and w while the bare mass of the nucleon is denoted by M . The subtracted Feynman energy $\bar{E}_F \equiv E_F - C_F$ (see eq. (31)) is listed in the fifth column. The last three columns give the radius $\langle r^2 \rangle_F^{1/2}$ of the dressed nucleon and the mean number of scalar and vector mesons respectively in the leading order Feynman approximation.

Λ [MeV]	v [MeV]	w [MeV]	M [MeV]	$\bar{E}_F$ [MeV]	$\langle r^2 \rangle_F^{1/2}$ [fm]	$\langle N_S \rangle_F$	$\langle N_V \rangle_F$
900	1669	1599	853.9	-284	0.058	0.180	0.079
1000	2007	1818	749.8	-517	0.084	0.237	0.268
1100	2440	2041	624.3	-783	0.107	0.287	0.573
1200	3019	2265	493.1	-1086	0.131	0.329	0.998
1300	3817	2417	363.1	-1486	0.159	0.368	1.56
1400	5237	2280	224.8	-2167	0.201	0.420	2.30

Figure caption

Fig. 1 :

Second-order contributions to the energy shift of the nucleon. (a) self-energies due to the exchange of scalar (S) and vector (V) mesons. (b) tadpole diagram for vector mesons.

Fig. 1