HADRONIC DECAYS OF THE Z^0 AND QCD STUDIES

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1. INTRODUCTION

The Z^0 boson decays into hadrons with a probability of $\Gamma_h/\Gamma_Z = 70\%$. Since the start of LEP about 250 000 hadronic events have been registered by the four experiments ALEPH, DELPHI, L3, and OPAL, this talk summarizes the results available to date.

The physics analysis of hadronic distributions has been facilitated by two recent review articles on QCD at LEP by Kunszt and Nason¹ and on QCD generators by Sjöstrand². Figure 1 reminds of the structure of a typical event where we can distinguish four phases.

FIGURE 1
Structure of a hadronic event

Phase 1 is described by the well-known electroweak sector of the standard model. Phase 2 is the domain of perturbative QCD where either matrix element calculations in second order of the

running coupling α_s or parton shower models mainly in the leading logarithm approximation are available. While in the first case at most four partons are allowed, the typical number of partons produced by parton shower Monte Carlo calculations is about 10 at LEP energies. The fragmentation of coloured partons into hadrons (phase 3) can only be treated by phenomenological models like string or cluster fragmentation. Finally in phase 4 unstable hadrons decay into the detected particles. It is assumed that the main properties of the parton phase are still measurable in the hadron phase.

At LEP/SLC one expects hard QCD emission to become more important while hadronization effects become less important than at lower energies. Furthermore the perturbative expansion of infrared safe observables (like thrust, 3-jet rate, energy-energy correlation) in terms of the running coupling constant becomes a bit better behaved at higher energies.

2. GLOBAL EVENT STRUCTURE

Let me distinguish two rounds in the analysis and understanding of the global structure of hadronic events. The goals of round 1 were a test of the predictions of QCD-based event generators pre-tuned at lower energies and the search for unexpected phenomena. Dedicated papers have been presented by the Mark II³, ALEPH⁴, and

DELPHI⁵ Collaborations. Their results include distributions of

- a) global shape variables obtained from the eigenvalues of the momentum tensor like sphericity, aplanarity, etc.;
- b) shape variables linear in momentum like thrust, major and minor value, oblateness. These distributions are sensitive to low momenta and less sensitive to decays;
- c) inclusive single particle variables like rapidity, $x_p = p/p_{\text{beam}}$, p_T^{in} and p_T^{out} .

Already in this early stage of analysis complete Monte Carlo corrections for the geometrical detector acceptance, the detector resolution, inefficiency etc.,

for kinematic cuts, secondary interactions, and radiation of photons were performed. The size of the bin-by-bin correction function

$C(x) = (1/N \, dn/dx)_{\text{true}} / (1/N \, dn/dx)_{\text{accepted}}$ is $0.8 < C(x) < 1.2$ for most bins indicating the good quality of the detectors.

In Figure 2 the thrust and minor value distributions as obtained by the ALEPH Collaboration at a center of mass energy, $E_{\text{CM}} \approx M_Z$ are shown. For comparison data of the Mark II Collaboration taken at $E_{\text{CM}} = 29 \text{ GeV}$ are included. Obviously with increasing energy jets become narrower and events become more planar. The ALEPH data are compared with the predictions of three QCD-based event generators, good agreement is obtained with

FIGURE 2
Thrust and minor value distribution from ALEPH ($E_{\text{CM}} = 91 \text{ GeV}$), Mark II ($E_{\text{CM}} = 29 \text{ GeV}$) and predictions of event generators: JETSET parton-shower, JETSET matrix-element, and HERWIG.

the parton shower version of JETSET.

In Figure 3 the energy evolution of the mean values $\langle T \rangle$ and $\langle m \rangle$ of thrust and minor value are compared to the predictions of four event generators⁵. Again good agreement is obtained with the JETSET parton shower generator.

FIGURE 3

Energy dependence of the average values of (a) thrust $\langle T \rangle$ and (b) minor value $\langle m \rangle$ compared to Monte Carlo predictions⁵.

The mean charged particle multiplicity $\langle n_{ch} \rangle$ is sensitive to soft gluon radiation and thus contains information not contained in distributions of global shape variables. Figure 4(a) shows $\langle n_{ch} \rangle$ versus E_{CM} for several e^+e^- experiments³, Figure 4(b)

shows recent $\langle n_{ch} \rangle$ data measured at $E_{CM} \approx M_Z$ ^{4,5,6}. A new preliminary value obtained from a model independent unfolding procedure has been presented by the ALEPH Collaboration⁷

$$\langle n_{ch} \rangle = 20.36 \pm 0.06 \pm 0.79.$$

The first error is due to statistics, the second due to systematics. The average of all $\langle n_{ch} \rangle$ measurements at $E_{CM} \approx M_Z$ is

$$\langle n_{ch} \rangle = 20.9 \pm 0.4,$$

where the error includes statistical and systematic uncertainties. The corresponding Monte Carlo predictions are 21.2 for the JETSET parton shower model, and 20.8 for HERWIG (the numerical values are averages of the values quoted by the Mark II, ALEPH, DELPHI, and OPAL Collaborations). From the very good agreement we can conclude that the data are consistent with coherence of soft gluon radiation which is a result of gluon interference in the infrared region and a characteristic prediction of QCD.

FIGURE 4

- (a) $\langle n_{ch} \rangle$ versus E_{CM} measured by several experiments,
 (b) $\langle n_{ch} \rangle$ measured at $E_{CM} \approx M_Z$, the line corresponds to the prediction of the HERWIG generator.

Round 2 of the investigation of the global event structure concerns the comparison of high statistics data with the predictions of various event generators and the tuning of free model parameters. While such work is performed by all collaborations, a detailed analysis has been presented by OPAL⁶ using two data sets:

1. Global shape distributions from measurements of energy clusters in the electromagnetic lead glass calorimeter. After cuts the sample contains 24.500 events. The data are fully unfolded for effects of finite detector resolution, acceptance, secondary interactions etc. by applying bin-by-bin correction constants.

2. Distributions obtained from charged tracks. In this case no unfolding correction is applied. Instead the measured distributions are compared to results of a full Monte Carlo simulation using the optimized Monte Carlo generators and including all detector effects.

For the optimization of the phenomenological parameters of QCD event generators, the major value distribution and the distribution of the second Fox-Wolfram moment, normalized to the zeroth moment, are selected. These two distributions do not constrain particle multiplicity nor do they completely constrain other unfolded distributions.

The models and parameters tuned are:

1. The parton shower version of JETSET 7.2 with Lund string fragmentation. As an example, Table 1 shows the parameters adjusted, their default value, and their optimized value.

The QCD scale parameter Λ_{QCD} determines the parton branching in the shower, the parameter Q_0 serves to terminate the shower evolution. The parameter σ_q determines the transverse momentum distribution of hadrons, a and b are the longitudinal fragmentation parameters.

Parameter	Default value	Optimized value
Λ_{QCD}	0.40 GeV	$0.29^{+0.02}_{-0.01}$ GeV
Q_0	1.0 GeV	$1.0^{+0.8}_{-0.3}$ GeV
σ_q	0.35 GeV	$0.37^{+0.03}_{-0.05}$ GeV
a	0.50	$0.18^{+0.12}_{-0.05}$
b	0.90 GeV^{-2}	0.34 GeV^{-2}

Table 1 Parameters of JETSET version 7.2 optimized by the OPAL Collaboration

2. The parton shower generator HERWIG version 3.4 involving cluster fragmentation. The optimized parameters are Λ_{QCD} , m_g , M_{max} . The formal mass of the gluon m_g serves as main perturbative cut-off parameter of the model, the parameter M_{max} controls the cluster fragmentation.

3. The generator ARIADNE version 3.1, which simulates parton showers originating from coherent gluon emission from a parton system or colour dipole. Parton fragmentation proceeds according to the Lund string model. The optimized parameters are Λ_{QCD} and p_T^{min} controlling the perturbative phase and σ_q .

4. A matrix element Monte Carlo using the calculations of Ellis, Ross, and Terrano⁸ in a version of JETSET. In this case mainly the fragmentation parameters are modified.

Distributions of global shape variables and of single particle inclusive variables computed with the optimized event generators are compared to the experimental distributions. Examples are presented in Figures 5 and 6. Figure 5 shows the sphericity and aplanarity distributions obtained from unfolded electromagnetic cluster data. The errors of the data points include the statistical and the full systematic error, which is about 3 to 4 times larger than the

FIGURE 5
Sphericity and aplanarity distributions measured by the OPAL Collaboration compared to predictions of optimized QCD models.

statistical error. The inset below the distributions shows the difference between the prediction of the optimized models and the data in units of standard deviations, calculated using the full error.

The differential multiplicity distribution for charged tracks obtained from measurements with the OPAL jet chamber is shown in Figure 6. The Monte Carlo predictions including a full detector simulation and initial state radiation agree well with the data.

The event generators optimized at 91 GeV also behave well at lower energies as can be seen in Figure 7, where the fully corrected thrust distribution measured at 91, 35, and 29 GeV is compared to

the predictions of JETSET 7.2.

To summarize this section let us note that all parton shower models describe the data very well. The fact that $\chi^2/\text{NDF} = 0.56$ is best for JETSET ($\chi^2/\text{NDF} = 0.95$ for HERWIG respectively 0.74 for ARIADNE) is related to performing the unfolding procedure with JETSET. After optimization of a few model parameters at $E_{\text{CM}} = 91$ GeV, no clear experimental distinction is possible at present. The $O(\alpha_s^2)$ matrix element generators can describe the data if the QCD and the fragmentation parameters are adjusted simultaneously. Here I refer to work performed by members of the DELPHI

FIGURE 6

Differential multiplicity measured by OPAL. The data are not unfolded for detector effects, the Monte Carlo predictions include a full detector simulation.

FIGURE 7

Unfolded thrust distribution measured by OPAL, TASSO, and Mark II at 91, 35, and 29 GeV compared to predictions of the optimized JETSET 7.2 generator.

Collaboration⁹. The extrapolation to lower energies works very well for the JETSET parton shower version, also for the HERWIG and ARIADNE generators except that the predictions are too peaked towards high thrust values.

3. NEW DETERMINATIONS OF $\Lambda_{\overline{MS}}$ AND α_s

3.1 Jet production rates

The analysis of jet production rates allows direct tests of perturbative QCD since the influence of hadronization effects is very small. Most important, systematic uncertainties originating from theoretical problems can be studied in great detail.

Infrared safe cluster algorithms can be defined in

terms of resolution parameters based on the scaled invariant mass cut-off

$$y_{ab} = M_{ab}^2 / E_{CM}^2.$$

y_{ab} is requested to be larger than a given value y for all resolved partons or pseudopartons.

The parton rates $R_{i\text{-parton}}(y) = \sigma_{i\text{-parton}}(y)/\sigma_{\text{tot}}$ with $i = 2, 3, 4$ describing the fraction of final states with i partons at a given resolution y can then be written as

$$\begin{aligned} R_{2\text{-parton}}(y) &= 1 + C_1^{(2)}(y) \alpha_s + C_2^{(2)}(y, f) \alpha_s^2 \\ R_{3\text{-parton}}(y) &= C_1^{(3)}(y) \alpha_s + C_2^{(3)}(y, f) \alpha_s^2 \\ R_{4\text{-parton}}(y) &= C_2^{(4)}(y) \alpha_s^2. \end{aligned}$$

Here the strong coupling strength α_s is evaluated at renormalization scale μ^2 . Perturbation theory in second order does not specify the right scale μ^2 . A usual ansatz is $\mu^2 = f Q^2$ with $f \leq 10$, $Q^2 \equiv E_{CM}^2$.

A further complication arises from the freedom in the theoretical treatment of unresolved partons. In consequence, the coefficients $C_2^{(2)}$ and $C_2^{(3)}$ depend on the parton recombination scheme applied. Numerical values have been evaluated for the KL' scheme by Kramer and Lampe¹¹ and for the E, E₀, and p schemes by Kunszt and Nason¹.

In most experimental papers jet rates are reconstructed by using the jet finding algorithm introduced by the JADE Collaboration. The algorithm proceeds as follows: The particle pair with the smallest value

$$y_{ij} = 2 E_i E_j (1 - \cos \theta_{ij}) / E_{vis}^2$$

is selected. Here E_i, E_j are the energies, and θ_{ij} the angle between the momenta of the two particles. E_{vis} is the total visible energy of the event. The particle pair with the lowest value y_{ij} is replaced by a pseudoparticle with four momentum $p_i + p_j$. The procedure is repeated until all pairs of pseudoparticles or particles satisfy the condition $y_{ij} \geq y_{cut}$. The remaining pseudoparticles are called jets.

Though the resolution cut on the parton level y and the resolution cut on the hadron level y_{cut} are differently defined, there is a close correspondence between parton and jet rates for $y = y_{cut}$. This is illustrated in Figure 8 which can be explained as follows: For each hadronic event produced by a Monte Carlo jet generator one investigates at which value of the resolution cut y the parton configuration jumps from a resolved 3-parton event to a resolved 2-parton event. Call this $y_{3 \rightarrow 2}$. At the same time one investigates at which value of y_{cut} the hadronic event jumps from a resolved 3-jet configuration to a resolved 2-jet configuration. Call this

$y_{cut, 3 \rightarrow 2}$. Figure 8 then shows the distribution of the difference $\Delta y = y_{cut, 3 \rightarrow 2} - y_{3 \rightarrow 2}$. The solid histogram is for charged and neutral particles, the dotted for charged tracks only. For the case of charged tracks the average value is $\langle \Delta y \rangle = 0.002$ compared to a typical bin width of 0.01 or 0.02.

FIGURE 8

Distribution of $\Delta y = y_{cut, 3 \rightarrow 2} - y_{3 \rightarrow 2}$ computed by using a version of JETSET 6.3 with KL' matrix elements¹¹.

At this point one further problem for QCD fits needs to be discussed. The statistical errors of the experimental jet rate data are highly correlated. However, differential distributions defined for instance as

$$D_{2\text{-jet}}(y_{cut}) = (R_{2\text{-jet}}(y_c + \Delta y_c) - R_{2\text{-jet}}(y_c)) / \Delta y_c$$

or

$$D_{3\text{-jet}}(y_{cut}) = (R_{3\text{-jet}}(y_c + \Delta y_c) - R_{3\text{-jet}}(y_c)) / \Delta y_c + D_{2\text{-jet}}(y_c)$$

have uncorrelated errors. From fits to the differential jet rates $\Lambda_{\overline{MS}}$ (or α_s) and the scale factor $f = \mu^2/Q^2$ is therefore immediately obtained.

Results have been presented by the Mark II, OPAL, DELPHI, and L3 Collaborations, all using slightly different techniques.

Mark II¹²

The collaboration uses the same detector at two center of mass energies at PEP, $E_{CM} = 29$ GeV and at SLC, $E_{CM} \approx M_Z$. Events are selected by requiring that the sum of charged and neutral energy E_{vis} is greater than $0.50 E_{CM}$ at SLC (391 events) or $0.55 E_{CM}$ at PEP (7348 events).

A differential jet multiplicity distribution g_3 is defined as follows: For each event particles are assigned to 3 jets, and the value y_3 is determined at which the transition from 3 jets into resolved 2 jets occurs. Then

$$g_3(y_3) \Big|_{y_3 = y_{cut}} = \partial/\partial y_{cut} R_{2-jet}(y_{cut}).$$

A full Monte Carlo correction of g_3 for detector effects and initial state radiation has been performed. However, no corrections are applied for fragmentation effects. For the region $0.04 \leq y_{cut} \leq 0.14$ Mark II evaluates that $|g_3^{partons} - g_3^{hadrons}|$ is 3 - 5% at SLC, and 5 - 10% at PEP by using the JETSET parton shower or matrix element generator. This uncertainty is included in the systematic error.

In Figure 9 the corrected g_3 distributions measured at the two energies are shown together with QCD predictions for three $\Lambda_{\overline{MS}}$ values.

Performing a fit to g_3 for $0.04 \leq y_{cut} \leq 0.14$, the lower limit being chosen rather high in order to avoid large hadronization effects, and setting $\mu^2 = Q^2$ yields:

$$\Lambda_{\overline{MS}} = 290^{+170}_{-120} {}^{+110}_{-60} \text{ MeV for the SLC data,}$$

FIGURE 9
 $g_3(y_3)$ measured at SLC and at PEP.
The curves indicate QCD predictions for three values of $\Lambda_{\overline{MS}}$; $\mu^2 = Q^2$.

$\Lambda_{\overline{MS}} = 280 \pm 20 \pm 70$ MeV for the PEP data.

The corresponding values of α_s are

$$\alpha_s = 0.123 \pm 0.009 \pm 0.005 \text{ at SLC,}$$

$$\alpha_s = 0.149 \pm 0.002 \pm 0.007 \text{ at PEP.}$$

OPAL^{13,14}

The production rates of multijet hadronic events are analyzed using the information from the central

jet chamber and the barrel and end cap lead-glass calorimeters¹³. Events are selected by requiring at least five charged tracks and three neutral clusters. The visible energy has to exceed $0.4 E_{CM}$, and the absolute value of the vector sum of the momenta of all particles must be less than 40% of the visible energy. After cuts 10.385 events at a mean center of mass energy of 91 GeV remain.

As the investigation of hadronization effects shows, most event generators predict a reduction of the 3-jet rate relative to the 3-parton rate by 2% to 6% for $y_{cut} \geq 0.03$. In consequence a full Monte Carlo correction is only applied for all detector effects and for initial state radiation, but no correction is performed for hadronization. The corresponding uncertainty is contained in the systematic error.

Data are presented for the jet rates R_{2-jet} to R_{5-jet} and for the differential distribution D_{2-jet} covering $0.005 \leq y_{cut} < 0.20$, the total error being dominated by the statistical error. Figure 10 shows D_2 versus y_{cut} compared to the results of two $O(\alpha_s^2)$ QCD fits. The solid line indicates the result of a fit for $y_{cut} \geq 0.01$ leaving f free, the dotted line shows the result for $y_{cut} \geq 0.05$ with $f = 1$.

Including systematic uncertainties on the experimental jet rates, on hadronization effects and variations of the results if determined in different y -intervals, the final results quoted in¹³ are:

$$\Lambda_{\overline{MS}} = 80 \text{ MeV} - 180 \text{ MeV}$$

$$f = 0.001 - 0.003.$$

For fits with $f = 1$ the results are $\Lambda_{\overline{MS}} = 200 - 450$ MeV or $\alpha_s = 0.124 \pm 0.008$.

In a second important study¹⁴ (preliminary), the OPAL Collaboration investigates the systematic uncertainty due to different recombination schemes for the jet finding y -algorithm and due to variations of

FIGURE 10
Differential distribution D_2 measured by the OPAL compared to the results of two $O(\alpha_s^2)$ QCD fits.

the renormalization scale. The new feature is the strict correspondence between jet algorithms on the hadron level and on the parton level. Both the theoretical jet rates on the parton level and the experimental jet rates on the hadron level depend strongly on the recombination scheme applied. The E_0 scheme has been introduced before (JADE algorithm). The E , p , and p_0 schemes, also applied by OPAL, evaluate the scaled invariant mass and the four momenta of the pseudopartons or pseudoparticles differently. For details I refer to¹. In the study the data statistics is updated and the jet rates are corrected for detector acceptance and for hadronization.

Examples of the differential distribution D_2 evaluated in the E_0 scheme and in the E scheme are shown in Figure 11a) and b). Note the scheme

FIGURE 11
Differential distribution D_2 obtained by OPAL with updated statistics from an analysis in the E_0 recombination scheme (a) and the E recombination scheme. The curves indicate $O(\alpha_s^2)$ QCD fits.

dependence of the data. The data are compared to two $O(\alpha_s^2)$ QCD fits, in one case f is left free (solid curves), in the other case f is fixed to one (dotted curves). Table 2 summarises the results obtained for $\alpha_s(M_Z^2)$ when applying the four recombination schemes. The second column in Table 2 is for the case $f = 1$, the third for the case where f is left free. It has to be pointed out that hadronization corrections are very small only for the E_0 scheme, they are large for the E scheme and sizable for the p_0 and p schemes. Including all possible experimental and theoretical uncertainties the new OPAL result is:

$$\alpha_s(M_Z^2) = 0.116 \pm 0.016.$$

Scheme	$\alpha_s(M_Z^2)$		Hadronization correction
	$f = 1$	f free	
E_0	0.106	0.123	v. small
E	0.105	0.132	large
p_0	0.110	0.120	sizable
p	0.111	0.119	sizable

Table 2 Recombination scheme dependence of $\alpha_s(M_Z^2)$

DELPHI¹⁵

Jet production rates and differential jet rates are determined using the information from charged

tracks. The event selection requests a visible energy greater than 15 GeV and an absolute value of the vector sum of the particle momenta of less than 30 GeV/c. After applying the selection criteria 6717 events remain. Monte Carlo corrections include all detector effects as well as photon radiation. The collaboration also performs a correction for hadronization effects according to

$$R_{i\text{-jet}}(y_{\text{cut}}) = \sum_{j=2}^4 M_{ij}(y_{\text{cut}} = y) R_{i\text{-parton}}(y).$$

Here $R_{i\text{-parton}}$ is evaluated from the $O(\alpha_s^2)$ expansion of the parton rates using KL' coefficients. The transition matrix M_{ij} is calculated by Monte Carlo using the KL' matrix elements and Lund string fragmentation. The free parameters of the Monte Carlo have previously been tuned to the measured thrust, major value, sphericity, and aplanarity distributions.

A first series of $O(\alpha_s^2)$ QCD fits is performed to $D_{2\text{-jet}}$ for $0.05 \leq y_{\text{cut}} \leq 0.25$ and to $R_{2\text{-jet}}(y_{\text{cut}} = 0.05)$. Thus the maximum information contained in the data is used without introducing additional correlations. In the selected y_{cut} region the contribution from 4-jets is negligible and hadronization effects are very small. Setting $f = \mu^2/Q^2 = 1$ yields

$$\Lambda_{\overline{\text{MS}}} = 180^{+33+43}_{-30-38} \text{ MeV}$$

where the first error is due to statistics and the second due to systematic uncertainties of the experiment. The measured jet rates are plotted in Figure 12 as a function of y_{cut} and compared to the results of the above mentioned fit.

The collaboration also studied the variation of $\Lambda_{\overline{\text{MS}}}$ and of α_s due to a change of the renormalization scale μ^2 and the ambiguity due to various parton recombination schemes. The dependence on the renormalization scale is shown in Figure 13, where $\Lambda_{\overline{\text{MS}}}$ values obtained from fits with various fixed

FIGURE 12

Jet production rates measured by DELPHI compared to the result of an $O(\alpha_s^2)$ QCD fit with f fixed to one to the range $0.05 \leq y_{\text{cut}} \leq 0.25$.

FIGURE 13

$\Lambda_{\overline{\text{MS}}}$ obtained from $O(\alpha_s^2)$ QCD fits using KL' coefficients with different fixed values of f . The curve indicates χ^2/NDF of the various fits.

values of $f = \mu^2/Q^2$ are plotted. $\Lambda_{\overline{\text{MS}}}$ changes by about a factor of two if f is varied between 1 and 0.001. The curve in Figure 13 represents χ^2/NDF and indicates that no significant difference in the quality of the fits is observed when varying f .

The parton recombination scheme dependence is investigated by repeating the fits using second order

coefficients $C_2^{(2)}$ and $C_3^{(2)}$ as evaluated in the E_0 , E , and p schemes¹. Hadronization corrections are again included. The resulting changes of $\Lambda_{\overline{MS}}$ are summarized in Table 3.

Scheme	$\Delta \Lambda_{\overline{MS}}$ (MeV)
E_0	+ 10
E	- 90
p	+ 160

Table 3 Change of $\Lambda_{\overline{MS}}$ value for different parton recombination schemes with respect to the value obtained for the KL' scheme.

A second series of QCD fits is performed to $D_{2\text{-jet}}$ and $D_{3\text{-jet}}$ for $0.02 \leq y_{\text{cut}} \leq 0.25$, to $R_{2\text{-jet}}(y_{\text{cut}} = 0.02)$ and $R_{3\text{-jet}}(y_{\text{cut}} = 0.25)$. At $y_{\text{cut}} = 0.02$ the 4-jet rate contributes about 10%. In this case fits with $f = 1$ do not describe the data. A simultaneous fit of $\Lambda_{\overline{MS}}$ and f yields $\Lambda_{\overline{MS}} = 92 \pm 10 \pm 6 \pm 5$ MeV and $f = 0.001 \pm 0.0005 \pm 0.0002$. Figure 14 compares the jet production rates with the result of this fit.

FIGURE 14

Jet rates measured by DELPHI compared to the result of a QCD fit with f free to the range $0.02 \leq y_{\text{cut}} \leq 0.25$.

Summarizing the results of this investigation, the DELPHI Collaboration finds

$$\alpha_s(M_Z^2) = 0.114 \pm 0.003 \pm 0.004 \text{ (syst.)} \pm 0.012 \text{ (theor.)}$$

where the first error is due to statistics, the second due to the systematic uncertainty of the experiment, and the third due to theoretical ambiguities.

L3¹⁶

The L3 Collaboration presents jet rates based on an analysis of cluster hits from 36.700 hadronic events in the electromagnetic detector and the hadron calorimeter. The polar angular coverage of the hadron calorimeter is $5^\circ < \theta < 175^\circ$, therefore detector corrections are very small.

After applying hadronization corrections $\Lambda_{\overline{MS}}$ is evaluated from the jet rates measured at the fixed value of $y_{\text{cut}} = 0.08$. Fixing the renormalization scale to $f = \mu^2/Q^2 = 0.08$ results in

$$\Lambda_{\overline{MS}} = 190 \pm 60 \text{ (exp.)} \pm 170 \text{ (theor.) MeV.}$$

Here the first error includes systematical uncertainties of the experiment. The theoretical error includes uncertainties due to fragmentation, to recombination scheme dependence, and to renormalization scale. The corresponding value of α_s is $\alpha_s(M_Z^2) = 0.115 \pm 0.005 \text{ (exp.)} \pm 0.012 \text{ (theor.)}$. Figure 15 compares the measured jet rates with the QCD prediction for $f = 0.08$ and $\Lambda_{\overline{MS}} = 190$ MeV.

The energy dependence of the measured 3-jet rates allows a further test of perturbative QCD. At fixed y_{cut} , R_3 is predicted to depend only on the running coupling constant α_s as long as hadronization corrections are unimportant. For instance, at $y_{\text{cut}} = 0.08$

$$R_{3\text{-parton}} = 1.074 \alpha_s (1 + 3.15 \alpha_s)$$

for $\mu^2/Q^2 = 1$. In Figure 16 the data available at $y_{\text{cut}} = 0.08$ are collected^{17, 18, 19, 20, 14, 15, 16}. A QCD prediction with $\Lambda_{\overline{MS}} = 250$ MeV reproduces well the measured energy evolution.

FIGURE 15
Jet rates measured by L3 compared to a QCD prediction.

FIGURE 16
Energy dependence of the 3-jet rate measured at $y_{\text{cut}} = 0.08$ compared to a QCD prediction for $\Lambda_{\overline{\text{MS}}} = 250 \text{ MeV}$ and $\mu^2 = Q^2$.

Summarizing we can conclude:

- The jet rates measured by OPAL, DELPHI, and L3 agree within errors.
- R_3 decreases with E_{CM} at fixed y_{cut} suggesting that α_s is running.
- The value of $\Lambda_{\overline{\text{MS}}}$ as determined by experiments depends on the recombination scheme applied in the jet analysis and on the selection of the renormalization scale. Including all theoretical uncertainties

$$80 < \Lambda_{\overline{\text{MS}}} < 460 \text{ MeV.}$$

3.2 Energy-energy correlation

The energy-energy correlation (EEC) and the asymmetry of the energy-energy correlation (AEEC) are considered as good observables for the determination of $\Lambda_{\overline{\text{MS}}}$ due to their insensitivity to hadronization effects¹. From an experimental point of view the EEC function is defined as the histogram of all angles between all pairs of particles weighted by their energies:

$$\text{EEC}(\chi) = \frac{2}{N \Delta\chi} \sum_1^N \sum_{i>j} \frac{E_i E_j}{E_{\text{vis}}^2} \delta(\theta_{ij} - \chi)$$

where E_i, E_j are the energies of the particle pair and θ_{ij} is the angle between the particles.

Due to gluon radiation an asymmetry is observed around 90° :

$$\text{AEEC}(\chi) = \text{EEC}(180^\circ - \chi) - \text{EEC}(\chi), \quad 0^\circ < \chi < 90^\circ.$$

It is important to note that the asymmetry is insensitive to the tuning of fragmentation parameters which only change the EEC in a symmetric way. In $O(\alpha_s^2)$:

$$\text{EEC}_{\text{parton}}(\chi) = \alpha_s/\pi F(\chi) (1 + \alpha_s/\pi R(\chi))$$

with F and R being known functions. The α_s^2 term contributes about 30% to the EEC and about 15% to the AEEC.

The DELPHI Collaboration provides fully corrected distributions of the EEC and the AEEC. Bin-by-bin Monte Carlo corrections are small and amount to less than 10% for all bins entering the $\Lambda_{\overline{\text{MS}}}$ analysis. Figure 17 compares the measured distributions with the predictions obtained from a retuned version of the JETSET 7.2 generator involving ERT-matrix elements.

$\Lambda_{\overline{\text{MS}}}$ has been determined at a fixed renormalization scale $\mu^2 = 0.002 Q^2$ by two different methods. In the first case the measured integral

$$\int_{28.8^\circ}^{90^\circ} \text{AEEC}(\chi) d\chi = 0.0246 \pm 0.0023$$

is compared with QCD expectations yielding $\Lambda_{\overline{\text{MS}}} = 108 \pm 30 \pm_{40}^{45} \pm 30 \text{ MeV}$. The first error is due to statistics, the second due to uncertainties of fragmentation parameters, and the third represents an estimate of the uncertainty caused by the renormalization scale problem.

In the second case a fit of Monte Carlo distributions to the measured $\text{AEEC}(\chi)$ is performed for $28.8^\circ \leq \chi \leq 90^\circ$. Here the ERT-matrix element version of JETSET is used to generate the AEEC on the hadron level. A fit results in

$$\Lambda_{\overline{\text{MS}}} = 104 \pm 25 \pm_{40}^{45} \pm 30 \text{ MeV}$$

(the meaning of the errors is as indicated above), which is in good agreement with the corresponding result of the DELPHI jet analysis.

4. ANGULAR CORRELATIONS IN 4-JET EVENTS

The investigation of angular correlations in 4-jet events provides a new test of QCD sensitive to contributions from the triple gluon vertex.

DELPHI²²

The two variables relevant to the analysis are the

FIGURE 17
Energy-energy correlations and asymmetry of the energy-energy correlations measured by DELPHI compared to a QCD matrix element Monte Carlo calculation.

generalized Nachtmann-Reiter angle²³ θ_{NR} , which is defined as the angle between the two vectors $\vec{p}_1 - \vec{p}_2$ and $\vec{p}_3 - \vec{p}_4$, and the angle α_{34} between $\vec{p}_3$ and $\vec{p}_4$. Here $\vec{p}_1$ to $\vec{p}_4$ are the energy ordered total momenta of jets 1 to 4.

The two dimensional differential multiplicity distribution in terms of these angles depends on the QCD

Data

FIGURE 18

Double differential distribution of 4-jet events in $|\cos \theta_{NR}|$ and $\cos \alpha_{34}$ as measured by DELPHI together with the two projections. The histogram in the one dimensional distributions represents the result of a fit with N_C/C_F and T_R/C_F free in which all detector corrections are included.

colour factors C_F, N_C, T_R ⁸ and allows an investigation of contributions originating from:

- the triple gluon vertex, weight $C_F N_C$;
- double gluon bremsstrahlung, weight C_F^2 for planar contributions, and weight $C_F(C_F - 1/2 N_C)$ for non-planar contributions;
- four quark production;

$$\frac{1}{N} \frac{d^2 n}{d(\cos \theta_{NR}) d(\cos \alpha_{34})} \propto C_F^2 \left[A + \left(1 - \frac{1}{2} \frac{N_C}{C_F}\right) B + \frac{N_C}{C_F} C + \frac{T_R}{C_F} D \right]$$

with A, B, C, D being known functions of θ_{NR} and α_{34} .

In the DELPHI analysis 884 clean 4-jet events are selected from a total of 21024 hadronic events. The two dimensional distribution in terms of $|\cos \theta_{NR}|$ and $\cos \alpha_{34}$ is given in Figure 18 together with the two projections. Leaving the ratios N_C/C_F and T_R/C_F free in a fit to the distributions of Figure 18 results in

$$N_C/C_F = 2.05 \pm 0.40 \pm_{-0.40}^{+0.80}$$

$$T_R/C_F = 0.1 \pm 1.7$$

The first error is due to statistics. The second error of N_C/C_F gives an estimate of the systematic uncertainty mainly due to Monte Carlo simulation and due to hadronization effects. In QCD the ratios are $N_C/C_F = 2.25$, $T_R/C_F = 1.875$. No acceptable fit is obtained with $N_C/C_F = 0$. Therefore, the contribution from the triple gluon vertex is demonstrated without comparison to a specific non-QCD model.

OPAL²⁴

Properties of 4-jet events are studied in terms of the Bengtsson-Zerwas angle²⁵ χ_{BZ} which is defined as the angle between the two planes spanned by the vectors $\vec{p}_1, \vec{p}_2$ respectively $\vec{p}_3, \vec{p}_4$ and in terms of the generalized Nachtmann-Reiter angle θ_{NR} .

The analysis is based on a sample of about 50.000 hadronic events. After correction for detector effects and for hadronization, the differential distributions in χ_{BZ} and $\cos \theta_{NR}$ are compared to a range of QCD predictions for various values of the jet resolution parameter y_{cut} . Figure 19 shows the results in a comprehensive form. The measured mean values of χ_{BZ} are plotted as a function of y_{cut} together with the predictions obtained from QCD generators and from generators based on an abelian vector gluon gauge theory. In both cases calculations in second order perturbation theory and in terms of parton shower models are included using various options of JETSET.

In case of the QCD parton shower prediction, gluon spin and closest neighbour interference effects are considered. The systematical uncertainties of the two alternative theories are indicated by the hatched areas. The data are fully

FIGURE 19

Mean value of χ_{BZ} as a function of the jet resolution parameter y_{cut} as measured by OPAL compared to predictions of QCD and of the abelian theory.

compatible with QCD and clearly disagree with predictions of abelian models.

5. INTERMITTENCY

Density fluctuations in rapidity or pseudorapidity distributions have been found in cosmic ray events, in hadron-hadron and nucleus-nucleus collisions and in e^+e^- annihilation into hadrons. For a recent review see²⁶.

Scaled factorial moments are introduced as suitable measure of such fluctuations²⁷. Given a distribution of N particles in the rapidity interval $-Y/2$ to $Y/2$, the interval is divided into M equal subintervals of size $\delta y = Y/M$. Bin m contains n_m charged particles. Factorial moments of rank q corrected for statistical noise are then defined as

$$F_q(\delta y) = \frac{M^{q-1}}{\langle N \rangle^q} \left\langle \sum_{m=1}^M n_m (n_m - 1) \dots (n_m - q + 1) \right\rangle.$$

For random uncorrelated particle production F_q is expected to be constant for all q . For self-similar random cascade structures, however, Bialas and Peschanski predict in the limit of δy small

$$F_q \propto (1/\delta y)^{f_q}$$

with $f_q = q(q-1)f_2/2$ for all q .

The DELPHI Collaboration studied factorial moments of ranks 2 to 5²⁸ in the rapidity interval $Y = 4$. In Figure 20 moments evaluated from uncorrected data are compared to predictions of the JETSET parton shower Monte Carlo including full detector simulations. As a consequence of the good agreement between uncorrected data and Monte Carlo, DELPHI also evaluated factorial moments after applying the full correction of all detector effects and of photon radiation.

FIGURE 20

In F_q versus $\ln M$, uncorrected data are compared to predictions obtained from the JETSET parton shower Monte Carlo after including the full detector simulation.

In order to judge whether or not intermittent behaviour is observed, two critical points have to be considered: The numerical values f_q depend on the selection of the M -range used for the fit and on the careful treatment of all correlations between the data. Figure 21 shows the quantity α_q , defined as f_q divided by $q(q-1)/2$, as determined from fits to the corrected factorial moments for $6 \leq M \leq 32$.

FIGURE 21

Dependence of $\alpha_q = 2 f_q / [q(q-1)]$ on the rank q of the moment. Results from fits to the corrected DELPHI data are compared to predictions of QCD event generators.

The data are compared to predictions obtained from the JETSET parton shower, JETSET ERT-matrix element, and HERWIG parton shower Monte Carlo. The following conclusions can be drawn: For any selection of the M region used in the fit, the experimental f_q values agree with the predictions of parton shower generators. Approximate agreement is also observed with the matrix element Monte Carlo after retuning the generator parameters. Bose-Einstein correlations exist but are unimportant for the f_q fit. The dependence of f_q on the rank q is as expected by models with self-similar cascade structure.

6. CONCLUSIONS

All hadronic distributions measured at LEP/SLC are accurately described by QCD-based multihadron production models. The best overall agreement is achieved by parton shower models. Models involving QCD matrix element calculations in second order of α_s also reproduce the data after retuning of free model parameters. Till now, no surprising phenomenon has been observed in the studies of hadronic decays of the Z^0 boson.

New determinations of α_s include careful evaluations of all theoretical uncertainties due to fragmentation models, the dependence on the recombination scheme used in the jet finding algorithm and the ambiguity of the renormalization scale. QCD fits to the measured jet production rates result in the following values of $\alpha_s(M_Z^2)$:

Mark II	0.123 ± 0.010 (exp.)	
OPAL	0.116	± 0.016 (theor.)
DELPHI	0.114 ± 0.005 (exp.)	± 0.012 (theor.)
L3	0.114 ± 0.005 (exp.)	$^{+0.012}_{-0.010}$ (theor.).

The experimental error contains statistical and systematical uncertainties. The error quoted by OPAL is dominated by theoretical uncertainties. From an analysis of energy-energy correlations, DELPHI obtains $\alpha_s(M_Z^2) = 0.106 \pm 0.007$ (exp.) ± 0.003 (theor.). Summarizing the new results,

$$\alpha_s(M_Z^2) = 0.115 \pm 0.016.$$

Measurements of the angular correlations inside 4-jet events provide a further sensitive confirmation of the spin and colour structure of QCD.

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