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Abstract

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MULTIPARTON ANGULAR CORRELATIONS IN e^+e^- INTERACTIONS

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ABSTRACT

Theoretical predictions on angular correlations using the Double Log Approximation (DLA) of QCD are compared to experimental results from e^+e^- annihilation into hadrons at the LEP collider (DELPHI-Experiment).

1. Introduction

A description of multihadron production in e^+e^- reactions using QCD is difficult in the low energy nonperturbative region. For instance the parton cascade which can be handled with Leading Log Approximations (LLA) has to be cut off at some scale $Q_0 \leq 1 \text{ GeV}$. Although the additional introduction of some phenomenological models (strings, etc) has yielded good results, it has been suggested to extend the parton evolution down to a lower mass scale (if possible to the pion mass scale). Using this concept the multihadron final states can be directly compared to the multiparton final states [1]. It is important that the multiparton variables adopted have to be infrared safe. The possibility that perturbative QCD has also some applicability in the low energy regime has led in the mid eighties to the concept of Local Parton Hadron Duality [2].

Our interest is the study of correlations between hadrons produced in e^+e^- annihilations using recent tool of Correlation Integrals (CI) [3]. The main theoretical effort for evaluating multiparton correlations in the framework of QCD has been performed by applying the Double Log Approximation (DLA) [4]. Recently detailed prescriptions for multiparton angular correlations in cones, rings etc. using DLA were brought forward [5], [6], [7]. We give in the following a short outline of the procedure by W.Ochs and J.Wosiek, who computed the correlation integral for the relative angle θ_{12} of two particles.

Starting from an angular ordered parton cascade using several approximations (neglecting $\bar{q}q$ production, neglecting recoil of radiated partons, small angle radiation, large energies etc) one obtains for the probability of a gluon radiation: $M(k)d^3k = c_a a^2 \frac{dk}{k} \frac{d\Theta}{\Theta} \frac{d\phi}{2\pi}$, k being the gluon momentum, $a^2 = 6\alpha_s/\pi$ and $c_a = 1$ (for gluons). The generating functional for the multiparton variables under study - here the Correlation Integral - is set up and the master equation is solved for radiated gluons. This is done by solving a system of integral equations (with kernels $M(k)$) for the density functions $\rho^{(n)}(\Omega_1, \dots, \Omega_n)$ by using the pole approximation. Using instead of P (beam momentum) and Θ (opening angle of a cone around the sphericity axis) the "natural" variables $\epsilon = \frac{\ln(\Theta/\theta_{12})}{\ln(P\Theta/\Lambda)}$ and $\zeta = \frac{1}{\beta\sqrt{\ln(P\Theta/\Lambda)}}$ (with $\beta=1.25$ and $\Lambda=0.15$) the solution can be represented in a

power expansion of $\omega_j(\epsilon, n)\zeta^j$. In the high energy limit they obtain a scaling function $\omega(\epsilon, n) = \omega_0(\epsilon, n)$, which is determined by a set of equations [5], which can be approximated at the 1% level of accuracy by

$$\omega(\epsilon, n) = n\sqrt{1-\epsilon}\left(1 - \frac{1}{2n^2}\ln(1-\epsilon)\right) \quad (1)$$

In chapter 2 we will confront some predictions stemming from the asymptotic solution above with experimental data. In chapter 3 the experimental results will be summarized.

2. Confrontation with the theoretical predictions

The data sample used for the analysis was about 170k events measured by the DELPHI experiment (e^+e^- interactions at $\sqrt{s} = 91 \text{ GeV}$). The standard cuts for hadronic events have been applied [8]. Some comparisons have been done using the LUND jetset MC [9], which has a good record in describing multiparticle correlation phenomena [10]. We define the sphericity axis as jet axis.

a) Correlations in the Relative Angle θ_{12} in Cones

The 2-parton angular correlation function is defined in the following way [3]:

$$r^{(2)}(\theta_{12}) = \frac{1}{Norm} \int_{\Theta} d\Omega_1 d\Omega_2 \rho^{(2)}(\Omega_1 \Omega_2) \delta(\theta_{12} - \theta(\Omega_1 \Omega_2)) \quad (2)$$

where Θ is a cone angle around the jet axis and θ_{12} is the angle between 2 particles within this cone. Norm denotes the density function containing the purely statistical fluctuations only (obtained by event mixing [11]).

In the following some predictions stemming from the considerations above are compared with the experimental data:

- $r^{(2)}(\theta_{12})$ rises with $\ln \frac{\Theta}{\theta_{12}}$ and ϵ and levels off for small θ_{12} . For sufficiently large angles θ_{12} (i.e. small ϵ 's) the following power law behaviour is expected [5]:

$$r(\theta_{12}) = \left(\frac{\Theta}{\theta_{12}}\right)^{0.5a} \quad (3)$$

In Fig. 1 this dependence is investigated for several cone opening angles (60, 45, 30 and 15 degrees). Fitting the parameter a (defined in section 1) in the θ_{12} range between 5.7 and 13 degrees, values of 0.1 to 0.16 are obtained for α_s . As expected the values of α_s become smaller with bigger cone openings.

- For asymptotically high energies

$$\ln(r(\theta_{12})) \sim \frac{2}{\zeta}(\omega(\epsilon, 2) - 2\sqrt{1-\epsilon}) \quad (4)$$

is expected for any choice of cone opening angle Θ ([5]). In Fig.2 $\ln(r(\theta_{12}))$ is given as a function of ϵ for several cone opening angles (data). It does not depend much

on Θ (in agreement with the predictions above). This scaling with respect to the variable ϵ is especially good for broader cones; for smaller Θ 's uncertainties of the determination of the jet axis are expected to cause deviations. The shape is similar to the predicted curve though lying flatter even below the numerical calculation for finite energy ($\sqrt{s} = 91$ GeV) [12] and also "bending down" earlier. Thus at LEP energies the asymptotic behaviour is, as expected, not yet reached, and the approximations used for the analytic calculations [12] cause significant deviations. Nevertheless, some predicted features show up already, in particular the ϵ -scaling property.

- In Fig.3 a comparison with Lund parton shower Monte Carlo calculations is performed. $\ln(r(\theta_{12}))$ for a cone opening angle of 45° (data, full circles) is compared to the MC (accounting for detector response, stars) and to the MC (generated quantities only, open triangles). The agreement is very satisfying. The MC for $P=900$ GeV/c (generated quantities only, open squares), however, has, for $\epsilon > 0.45$, a much steeper slope similar to the asymptotic curve (dotted line); for $\epsilon < 0.4$, however, scaling also in energy is indicated.

- Defining

$$\tilde{r}(\epsilon) = \frac{\rho^{(2)}(\epsilon, P, \Theta)}{\bar{n}^2(P, \Theta)} \quad (5)$$

then

$$-\frac{\ln(\tilde{r}(\epsilon)/b)}{2\sqrt{\ln(\frac{P\Theta}{\Lambda})}} = 2\beta(1 - 0.5\omega(\epsilon, 2)) \quad , \quad b = 2\beta\sqrt{\ln(\frac{P\Theta}{\Lambda})} \quad (6)$$

is expected to hold asymptotically [5] ($\bar{n}$ being the mean total charged multiplicity). In Fig.4 it is shown that this relation agrees well with the DELPHI data (closed circles) as well as with the LUND parton shower MC (open triangles) and also with a HERWIG MC (only generated partons without hadronization [5]). The agreement with the Monte Carlo calculations on both the partonic and hadronic level is in support of parton hadron duality. The data (and the MC comparison) also indicate that the expression defined above with $\tilde{r}(\epsilon)$ is scaling already at LEP energies. Note that no arbitray normalization has been applied in Fig.4.

- As indicated above the MC models used agree well with the data. There are possible deviations at small angular differences less than few degrees (not shown here). In general the Jetset Parton Shower MC lies closer to the data than the Herwig MC.

b) Multiplicity Moments in Rings

Next we consider particle correlations in a ring $\Theta \pm \theta$ around the jet axis. For the normalized cumulant moments [5] or the factorial moments [6] [7] the following prediction has been made:

$$C^{(n)}(\Theta, \theta) \sim \left(\frac{\Theta}{\theta}\right)^{\phi_n} \quad , \quad \phi_n = D - 2a(n - \omega(\epsilon, n))/\epsilon \quad , \quad D = 1 \quad (7)$$

- From Equ.7 the following simple relation can be deduced:

$$- \tilde{C}^{(n)} = - \frac{\ln \left[\left(\frac{\vartheta}{\Theta} \right)^{D(n-1)} C^{(n)} \right]}{n \sqrt{\ln(P\Theta/\Lambda)}} \sim 2\beta \left(1 - \frac{\omega(\epsilon, n)}{n} \right) \quad (8)$$

As it can be seen in Fig.5, the data agree with this prediction only in shape for $\epsilon \geq 0.3$. For smaller ϵ 's the data points lie far away from the asymptotic prediction. Nevertheless it has to be noted that for the two orders of the cumulants ($n=2,3$) the respective asymptotic curves for $n=2,3$ are approached separately. If we use the factorial moments instead of the cumulants for the left hand side of Equ.8, $-F^{(n)}$, we find, as seen from Fig.6, qualitative agreement of the shapes with the asymptotic predictions over the whole ϵ range, though the data are a bit flatter. It should be noted that the predictions are determined up to a possible shift in the logarithmic plots.

- In Fig.7 the cumulants of orders $n=2,3$ normalized by $C^{(n)}(0)$ are shown. From Equ.7 we deduce:

$$\ln \left(\frac{C^{(n)}(\epsilon)}{C^{(n)}(0)} \right) \sim \ln \left(\frac{P\Theta}{\Lambda} \right) \epsilon \left(D(n-1) - 2a \frac{n - \omega(\epsilon, n)}{\epsilon} \right) \quad (9)$$

Here the agreement between data and prediction is very bad: even with a shift in the log-plot the prediction does not resemble the data. In Fig.8 we present again, instead of cumulants, the factorial moments (left hand side of Equ.9). There is even an almost quantitative agreement over the whole ϵ range considered with the asymptotic prediction, especially for order $n=2$.

- It has to be pointed to the fact that the asymptotic predictions for the respective expressions $\tilde{r}$ in Fig.4 and C in Fig.5 (and F in Fig.6) are the same.

3. Summary and outlook

We compared for the first time experimental data (e^+e^- annihilations into hadrons at $\sqrt{s} = 91 \text{ GeV}$) to theoretical predictions [5] [6] [7] [12] on angular correlations using the DLA of QCD. Some qualitative predictions for 2 body angular correlations in cones around the jet axis are supported by the data, in particular: the dependence of " α_s " on Θ , that, apart from the weak α_s dependence, the particle correlation function r_2 is only a function of ϵ (" ϵ -scaling", no further dependence on Θ) and that $\tilde{r}$ has a shape already similar to the analytical asymptotic prediction, agreeing well with Monte Carlo calculations at low and high energies. Since the predictions concern the correlations only on the parton level, the Monte Carlo Calculations have been performed both on partonic and hadronic levels. The close similarity of partonic and hadronic angular correlation functions in the various MC calculations (see [5]) suggests that hadronization and especially resonance production might not destroy the pattern. Thus, from this point of view, direct comparison of hadron correlations observed experimentally with theoretical predictions on parton level might have some meaning.

For correlations in a ring region ($25^\circ \pm \vartheta$) the asymptotic prediction containing cumulants $C^{(n)}$ does not agree with the data (beside a similarity of shape for larger values of ϵ), the equivalent expression containing the factorial moments $F^{(n)}$ are agreeing well over the whole range of ϵ . It has been commented [13] that the difference between the predictions containing cumulants [5] and factorial moments [6] [7] is non leading in the asymptotic formulae. It should become negligible only at very high energies.

We want, however, to stress that the above experimental analysis at LEP energy are not expected to agree in all details with the analytical calculations (beside the fact that the data are far away from the asymptotic energy also various approximations have been used for the predictions) and therefore the sizable deviations found in our comparisons are not surprising. More checks on more refined predictions are desirable. We think that the observed deviations provide valuable information, which can be used to improve further QCD calculations.

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Figures

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7

Fig.8

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