Fermion scattering off dilatonic black holes

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Abstract

The scattering of massless fermions off magnetically charged dilatonic black holes is reconsidered and a violation of unitarity is found. Even for a single species of fermion it is possible for a particle to disappear into the black hole with its information content.

In recent years there has been a lot of interest in the physics of black holes. The issue which has engaged the attention of most workers is that of possible information loss. Matter falling into a black hole carries some information with it. This becomes inaccessible to the rest of the world, but may in principle be supposed to be stored inside the black hole in some sense. A problem arises when the black hole evaporates through the process of Hawking radiation. The information does seem to be lost now [1].

Although there have been attempts at studying this problem in its full complexity [2], most authors have considered simplified models of black holes as in [3, 4]; see [5] for a review. We shall consider the extremal magnetically charged black hole solution of dilatonic gravity. This is a four dimensional model involving an extra field – the dilaton – but for s-wave scattering of particles in the field of this black hole, the angular coordinates are not relevant and a two dimensional

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effective action can be used [6]. If the energies involved are not too high, the metric and the dilaton field can be treated as external classical quantities and an amusing version of electrodynamics emerges, where the kinetic energy of the gauge field has a position dependent coefficient [7].

The scattering of massless fermions has been considered in this context. The model admits a solution which is very close to the conventional solution of the Schwinger model, i.e., two dimensional massless electrodynamics. In this solution, there is a massive free particle, but in the present case its mass becomes position dependent [7, 8]. To be precise, the mass vanishes near the mouth of the black hole but increases indefinitely as one goes into the throat. (The dilatonic field increases linearly with distance in the throat.) This is interpreted to mean that massless fermions proceeding into the black hole cannot go very far and have to turn back with probability one. Thus the danger of information loss is averted very simply.

In this note, we reexamine the model by taking into account the possibility of alternative solutions. The Schwinger model possesses other solutions besides the conventional one, although this is not very well realized by everyone. These correspond to different quantum theories built from the same classical theory. Different quantum theories can be constructed without violating gauge invariance by changing the definition of the point split fermionic currents [9, 10]. By considering this freedom, we shall demonstrate that the problem of information loss can in fact appear even in the extremal magnetically charged black hole in a dilatonic background.

The model is described by the Lagrangian density [7, 8]

\[ \mathcal{L} = \overline{\psi}(i\slashed{D} + eA)\psi - \frac{1}{4}e^{-2\varphi(x)}F_{\mu\nu}F^{\mu\nu}, \]  

(1)

where the Lorentz indices take the values 0,1 corresponding to a \((1+1)\)-dimensional spacetime, \(e\) measures the coupling of the vector current corresponding to the massless fermion \(\psi\) to the gauge field \(A\), and there is a dilatonic background \(\varphi(x)\) whose dynamics we do not go into. It is clear that if \(\varphi(x)\) vanishes, we get the well-known Schwinger model [11, 12, 13]. The model with nonvanishing \(\varphi(x)\) has also been solved [7, 8] with the help of the usual scheme of bosonization. Here we discuss a solution in a different framework, which leads to vastly altered physical consequences.
In two dimensions we can always set

\[ A_\mu = - \frac{\sqrt{e}}{e} (\tilde{\partial}_\mu \sigma + \partial_\mu \tilde{\eta}), \]  

(2)

where

\[ \tilde{\partial}_\mu = \epsilon_{\mu \nu} \partial^\nu, \]  

(3)

with \( \epsilon_{01} = +1 \) and \( \sigma, \tilde{\eta} \) are scalar fields.

We shall restrict ourselves to the Lorentz gauge. From (2) we see that the field \( \tilde{\eta} \) can be taken as a massless field with \( \tilde{\eta} = 0 \). We introduce its dual through

\[ \tilde{\partial}_\mu \eta(x) = \partial_\mu \tilde{\eta}(x). \]  

(4)

These massless fields have to be regularized [12] but we shall not need the explicit form of the regularization.

The Dirac equation in the presence of the gauge field is

\[ [i \slashed{\partial} + e A] \psi(x) = 0. \]  

(5)

This equation is satisfied by

\[ \psi(x) = \pm e^{i \int \sqrt{\gamma} \gamma_0 [\gamma(x) + \eta(x)]} \psi^{(0)}(x), \]  

(6)

where, \( \psi^{(0)}(x) \) is a free fermion field satisfying \( i \slashed{\partial} \psi^{(0)}(x) = 0 \).

We can calculate the gauge invariant current using the point-splitting regularization. While constructing a gauge invariant bilinear of fermions which in the limit of zero separation would give the usual fermion current, we can generalize the conventional construction [11]. We take

\[ J^{\text{reg}}_\mu(x) = \lim_{\epsilon \to 0} \int_\gamma \left[ \psi(x + \epsilon) \gamma_\mu : e^{i \int e^{+} \phi \bar{\phi} \{ A_\mu(y) - 2 \epsilon \phi(y) \phi_{\text{reg}}(y) \} : \psi(x) \right] - \text{v.e.v.} \]  

(7)

where \( \Phi(y) \) is a nondynamical function of spacetime coordinates which we shall fix later on. The addition of a term containing this function in the exponent represents a generalization of Schwinger's regularizing phase factor [9, 10]. It preserves gauge invariance, Lorentz invariance and even the linearity of the theory. The explicit coordinate dependence of \( \Phi \) may come as a surprise, but it must be remembered that

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the model under discussion does not possess translation invariance because of the factor containing \( \varphi(x) \) in the Lagrangian density (1). In fact this freedom can be used to simplify the solution of the model enormously, as we shall see. Now using (2) and (6) together with

\[
F_{\mu \nu} = \frac{\sqrt{\pi}}{e} \epsilon_{\mu \nu} \sigma
\]

we obtain the current which, up to an overall wavefunction renormalization, is equal to

\[
J^{\rho \sigma}_\mu(x) \approx \bar{\psi}^{(0)}(x) \gamma_\mu \psi^{(0)}(x) : -i \sqrt{\pi} \lim_{\epsilon \rightarrow 0} \langle 0 | \bar{\psi}^{(0)}(x + \epsilon) \gamma_\mu ([\gamma_5 \epsilon \cdot \partial
+ \epsilon \cdot \tilde{\partial})(\sigma + \eta) + 2 \epsilon \cdot \tilde{\partial}(\Phi \sigma)] \psi^{(0)}(x) | 0 \rangle
+ \frac{1}{\sqrt{\pi}} \frac{\epsilon_{\mu \nu} - \epsilon_{\nu \mu}}{\epsilon^2} \tilde{\partial}^\nu (\sigma + \eta) + 2 \frac{\epsilon_{\nu \mu}}{\epsilon^2} \tilde{\partial}^\nu (\Phi \sigma),
\]

where we have used the identity

\[
\langle 0 | \bar{\psi}^{(0)}(x + \epsilon) \psi^{(0)}(x) | 0 \rangle = -i \frac{\tilde{\epsilon}_{\mu \nu}}{2\pi \epsilon^2}. \tag{11}\]

Now we take the symmetric limit i.e. average over the point splitting directions \( \epsilon \) and finally obtain

\[
J^{\rho \sigma}_\mu(x) = -\frac{1}{\sqrt{\pi}} \tilde{\partial}_\mu (\phi + \sigma + \Phi \sigma + \eta), \tag{12}\]

where \( \phi \) is a free massless bosonic field satisfying

\[
-\frac{1}{\sqrt{\pi}} \tilde{\partial}_\mu \phi = \bar{\psi}^{(0)}(x) \gamma_\mu \psi^{(0)}(x): \tag{13}\]

and thus representing the conventional bosonic equivalent of the free fermionic field \( \psi^{(0)} \) [14]. We find

\[
J^{\rho \sigma}_\mu(x) = \epsilon_{\mu \nu} J^{\nu \sigma}_\rho(x), \tag{14}\]

which implies that the anomaly in this regularization is

\[
\tilde{\partial}^\mu J^{\rho \sigma}_\mu = -\frac{1}{\sqrt{\pi}} (\phi + \eta + \sigma + \Phi \sigma). \tag{16}\]
Note now that Maxwell’s equation with sources, viz.,

$$\partial_\nu \left( \frac{F^{\nu \mu}}{g^2} \right) + e J^\mu_{\nu g} = 0, \quad (17)$$

where

$$g^2(x) = e^{2\nu(x)}, \quad (18)$$
can be converted to the pair of equations

$$\left[ \left( \frac{1}{g^2} + \frac{e^2}{\pi} \Phi \right) + \frac{e^2}{\pi} \right] \sigma = 0 \quad (19)$$

and

$$\phi + \eta = 0. \quad (20)$$

The first equation (19), which depends on the choice of $\Phi$, determines the spectrum of particles in the theory. The other equation (20), relating two massless free fields, has to be satisfied in a weak sense by imposing a subsidiary condition

$$(\phi + \eta) \mid_{phys}^{(+)} = 0 \quad (21)$$
to select out a physical subspace of states. One can ensure that $\phi + \eta$ creates only states with zero norm by taking $\eta$ to be a negative metric field, i.e., by taking its commutators to have the “wrong” sign. The subsidiary condition then separates out a subspace with nonnegative metric as usual.

$\Phi$ is as yet undetermined. We shall consider a few possible choices. The conventional choice [7, 8] is zero. (19) then becomes

$$\left[ + \frac{e^2 g^2}{\pi} \right] \sigma = 0. \quad (22)$$

This describes a particle of mass $\frac{e g(x)}{\sqrt{\pi}}$. Now $g$ is related to $\varphi$, which is taken to vary linearly with distance in the throat of the black hole. The situation envisaged is that $g$ vanishes at the mouth of the black hole, but rises indefinitely as one proceeds into the interior. The effect is that the mass of the particle vanishes at the mouth but rises indefinitely inside the throat. Since massless scalars are equivalent to massless fermions in two dimensions, it follows that one can think of an initial condition where a massless fermion starts at the mouth of the black hole and proceeds inwards. The fact that the mass involved
in the equation of motion rises indefinitely means that the fermion cannot go arbitrarily far and is reflected back with unit probability. Thus the scattering of the fermion off the black hole is unitary and information is not lost.

On the other hand, if $\Phi$ is chosen to satisfy the condition

$$\frac{e^2}{\pi} \Phi = g^2,$$  \hspace{1cm} (23)

(19) simplifies to

$$\left[ + \frac{e^2}{\pi(g^2 + g^{-2})} \right] \sigma = 0. \hspace{1cm} (24)$$

The mass of the particle now vanishes not only at the mouth, but also asymptotically in the interior of the black hole. In fact, the mass has a maximum somewhere in between. Therefore it is possible for a massless fermion to exist both at the mouth and in the interior, and the height of the barrier being finite, there is a finite amplitude for the fermion to go in and get lost. Thus the danger of information loss is not averted in this case.

A somewhat mundane case is when $\Phi$ is such that

$$\left( \frac{1}{g^2} + \frac{e^2}{\pi} \Phi \right) = 1,$$ \hspace{1cm} (25)

and (19) simplifies to

$$\left[ + \frac{e^2}{\pi} \right] \sigma = 0. \hspace{1cm} (26)$$

This means that the usual massive free scalar field of the Schwinger model is recovered. The modified Schwinger model thus accommodates the unmodified solution with this altered definition of currents.

A more dramatic case is when $\Phi$ is allowed to go to infinity. In this case the free scalar field becomes exactly massless. So the fermion is massless at all positions. This fermion travels freely into the black hole and all information is lost.

To summarize, we have looked at the extremal magnetically charged black hole in a dilatonic background using a generalized construction of fermion bilinears. We point - split the current which is formally defined as the product of two fermionic operators. Schwinger has prescribed the insertion of an exponential of a line integral of the gauge field to make the product gauge invariant. However, his choice was
only one of many possible choices; see, e.g. [9, 10]. We have inserted an extra factor which involves the field strength of the gauge field and a nondynamical function of spacetime coordiantes and therefore does not interfere with the gauge invariance of the product. This is not the most general gauge invariant regularization possible in this approach, but is enough to illustrate the range of possibilities. By varying the regularization, the equations of motion of the Schwinger model can be converted to free field equations with the mass exactly as in the usual case, or going to zero at both ends of the spatial axis or even vanishing everywhere. In the first case, there is no fermion in the spectrum at all and the question of scattering does not arise. In the other two case, the massless fermion is not totally reflected, so that the problem of information loss appears unless further gravitational effects can change the scenario.

There is one question which may arise here. Have we, in changing the definition of the current, changed the model? To be more specific, the introduction of the $\Phi$- dependent term in addition to just $A_\mu$ in the phase factor entering the point-split current may be suspected to amount to the addition of an extra interaction. This is not really the case, as the equations of motion of the dilatonic Schwinger model itself are satisfied. The change is only in the definition of fermion bilinears as composite operators and this is well known to have a lot of flexibility. Formally, in the limit $\epsilon \to 0$, the phase factor does reduce to unity, so that the definition of the bilinears adopted in this paper cannot be thought of as changing the underlying classical theory. Only the quantum theory, which is not fully defined until the definition of composite operators is specified, is altered. This alteration takes the form of a renormalization of the effective coupling constant in the theory. The dilatonic field, which entered the model through this coupling constant, can thus be said to get effectively transformed in the quantum theory. However, this change is not a real one as far as the dilatonic field is concerned. This can be seen by considering the kinetic energy term of the dilaton field, which does not get altered. However, in the approximation made by us following [7, 8], this term is neglected and the dilaton appears purely as a background field.

Lastly, it should be mentioned that if several species of fermions are included, the problem of information loss appears automatically [7]. This is in keeping with our finding that magnetically charged black holes do not necessarily behave like elementary particles in scattering.
incident fermions. Therefore, the S-matrices envisaged by [2, 8] cannot always be constructed. The two dimensional model considered here has no horizon, but the four dimensional model from which it is derived does have one. There, the passage of the fermion into the black hole amounts to a loss of unitarity.

References