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Casimir Force Between two Parallel Plates With Small Distortions
of Different Types

by

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Abstract

We investigate the corrections to the Casimir force between two parallel plates covered with distortions described by periodic functions of two variables. The periods of the distortions in both coordinates are suggested to be much smaller than the sizes of the plates. A general expression is obtained for the Casimir force in the form of a perturbation expansion with respect to the amplitude of the distortions divided by the distance between the plates. The coefficients of this expansion are expressed in terms of the Fourier coefficients of the distortion function up to fourth order. It is shown that the perturbative expansion starts from the second order and does not depend on the period of the distortions. Some characteristic examples are calculated of both longitudinal and hillock-type distortions. It is shown that the contribution of the distortions to the Casimir force may achieve some tens percents. So it must be taken into account in precision Casimir force measurements.

1 Introduction

During last years the Casimir effect attracted much attention in different branches of physics (see, e.g., the reviews [1, 2] and the monograph [3]). This effect plays a significant role in statistical physics [4], in the MIT bag model [5], in Kaluza-Klein theories [6], in cosmology [7]. Actual interest comes from obtaining restrictions on the constants of hypothetical long-range interactions [8], from applications to thin films [9] or wetting processes on alkali metals [10].

The usual way to calculate the Casimir force is the separation of variables in the wave equation and construction of the Green's function for the geometry under consideration. This can be done only for several simplest configurations (such as two plane parallel plates, a sphere, a small ball over a plane, two small balls, a torus, etc.). For more complicated configurations different approximate methods may be applied (see, e.g., [11 -13]).

The role of small distortions of the boundary surface is significant in experiments on measuring the Casimir force and for obtaining of restrictions on the hypothetical long-range interactions. The contribution of the distortions of the surfaces to the Casimir effect has not been investigated in detail up to now. The problem is that this can be done only approximately because the configuration of two parallel plates with distortions is too complicated for an exact calculation. In the papers [12, 14] the first order corrections to the Casimir force with respect to the amplitude of the distortions were obtained for the configuration of a parallelepiped. However, that result cannot be applied to parallel plates because the first order corrections vanish in the limit of large parallel plates. In the paper [15] the first nonzero corrections to the Casimir force were given for sinusoidal longitudinal distortions $A \sin \alpha x$ of plane plates. That corrections have the form $\sim \alpha A^2/d$, where d is the distance between the plates. That result cannot be true because it changes its sign under the transformation $\alpha \rightarrow -\alpha$, $A \rightarrow -A$, whereas the distortions are left unchanged.

In the present paper we investigate the contribution of small distortions of different types to the Casimir force between plane parallel plates. The periods of distortions are assumed to be much smaller than the sizes of the plates. In section 2 we discuss briefly the approximate method of calculating the Casimir force [13] which can be applied to distortions of arbitrary type. In section 3 the different types of periodic distortions are discussed. Among them there are longitudinal distortions of arbitrary profile and hillock like distortions. In section 4 the general expressions for the corrections to the Casimir force are obtained up to fourth order of perturbation theory with respect to the relative amplitude of the distortions. It is shown that the perturbative expansion starts from the second order and does not depend on the period of the distortions. This is in contrary to the results of ref. [15]. In section 4 some examples are represented for typical distortions of both longitudinal and hillock types. It is shown that the contribution of distortions with a typical amplitude of 0.1 relative to the distance between the plates reaches some tens percents. So it must be taken into account whenever a precision value of the Casimir force will be measured.

Throughout the paper we use units with $\hbar = c = 1$.

2 The approximate method of calculation of the Casimir force for arbitrary test bodies

In this section we present the outline of the method for approximate calculation of the Casimir force. According to this method the potential of the Casimir forces acting between two test bodies can be obtained by summation of the interatomic potentials over all atoms of that bodies with a subsequent multiplicative renormalization

$$U_R(d) = -\frac{CN_1N_2}{K} \int_{V_1} d\vec{r}_1 \int_{V_2} d\vec{r}_2 |\vec{r}_1 - \vec{r}_2|^{-7} . \quad (1)$$

In eq. (1) the integrations run over the volumes V_1 resp. V_2 of the test bodies, N_1 resp. N_2 are the numbers of atoms per unit volume, C is the constant of retarded van der Waals interatomic interaction, K is a special renormalization constant, d is the distance between the test bodies. Note, that even the simple summation of the interatomic potentials (i.e., eq. (1) without the correction factor K^{-1}) gives the proper dependence of U_R on d for three dimensional configurations (see, e.g., refs. [16-18]). But the values of the coefficients in such dependencies come out to be larger than their true values due to the screening effects.

The renormalization procedure gives us the possibility to take into account approximately the effects of screening of the distant layers of the test bodies matter by the nearest ones. The value of the constant K may be determined as the ratio of the Casimir force potentials between two infinite plane parallel plates obtained by the summation method (see eq. (1) without K) and by the exact calculation [4]. In obtaining this the thickness of the plates was assumed to be much larger than the distance between them. This constant may be written in the form [13]

$$K = \frac{CN_1N_2}{\Psi(\epsilon_1, \epsilon_2)} > 1 , \quad (2)$$

where $\epsilon_{1,2}$ are the static dielectric permeabilities of the plates materials. The exact representation for the function $\Psi(\epsilon_1, \epsilon_2)$ reads [4]:

$$\begin{aligned} \Psi(\epsilon_1, \epsilon_2) &= \frac{5}{16\pi^3} \int_0^\infty \int_1^\infty \frac{x^3}{p^2} \left\{ \left[\frac{(s_1 + p)(s_2 + p)}{(s_1 - p)(s_2 - p)} e^x - 1 \right]^{-1} + \right. \\ &\quad \left. \left[\frac{(s_1 + p\epsilon_1)(s_2 + p\epsilon_2)}{(s_1 - p\epsilon_1)(s_2 - p\epsilon_2)} e^x - 1 \right]^{-1} \right\} dp dx \\ &\equiv \frac{\pi}{24} \frac{(\epsilon_1 - 1)(\epsilon_2 - 1)}{(\epsilon_1 + 1)(\epsilon_2 + 1)} \varphi(\epsilon_1, \epsilon_2) , \end{aligned} \quad (3)$$

where $s_1 = (\epsilon_1 - 1 + p^2)^{1/2}$, $s_2 = (\epsilon_2 - 1 + p^2)^{1/2}$. It is instructive to remember the graphical representation of $\varphi(\epsilon_1, \epsilon_2)$ given in [4]. We show two limiting cases in

Fig. 1. The curve 1 corresponds to $\varphi(\epsilon_1, \epsilon_1)$ (i.e., to the interaction of two dielectric bodies with the same ϵ), the curve 2 corresponds to $\varphi(\epsilon_1, \infty)$, (i.e., to the interaction of a dielectric body with a metallic one).

As it was shown in [13], the relative error of the potential (1) with the renormalization constant (2), (3) is less than 4% for arbitrary perturbations of the surfaces of test bodies which are similar to planes. This estimation of the relative error was obtained in the following way. For the case of two infinite plane parallel plates our method gives the exact result by construction. The relative error of the method was shown to reach its largest value of 4% among the configurations of arbitrary shaped bodies over a plane for the configuration of a small ball over a plane [13]. Therefore the relative error of the Casimir force potential obtained by (1) is less than 4% for two parallel plates with small perturbations of arbitrary form.

Now it is possible to apply eq. (1) which in accordance with eq. (2) may be written as

$$U_R(d) = -\Psi(\epsilon_1, \epsilon_2) \int_{V_1} d\vec{r}_1 \int_{V_2} d\vec{r}_2 |\vec{r}_1 - \vec{r}_2|^{-7} \quad (4)$$

to the calculation of the Casimir force between plates with small distortions of several types. It should be noted that one can imagine to apply the perturbation method for Green functions (e.g, that of ref. [11]) in order to obtain the corresponding result for $U_R(d)$ in a more fundamental way. This will be a task of enormous efforts. But as long as the relative error is not required to be less than 4% it would only repeat the results of the much simpler formula (4).

3 Models of distortions for plane plates

Let us consider the surface of plane plates covered by periodic distortions. In general, the profile of such distortions may be described by a periodic function of two coordinates, corresponding to a double Fourier series:

$$f(x, y) = A \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} [C_{nm}^{(1)} \sin(n\alpha_x x) \sin(m\alpha_y y) + C_{nm}^{(2)} \sin(n\alpha_x x) \cos(m\alpha_y y) + C_{nm}^{(3)} \cos(n\alpha_x x) \sin(m\alpha_y y) + C_{nm}^{(4)} \cos(n\alpha_x x) \cos(m\alpha_y y)]. \quad (5)$$

In accordance with (5) the periods are denoted by $T_x = 2\pi/\alpha_x$, $T_y = 2\pi/\alpha_y$ and we have

$$f(a + nT_x, y + mT_y) = f(x, y). \quad (6)$$

The amplitude A of the distortions will be chosen such that $C_{nm}^{(i)} \leq 1$ holds.

The ansatz (5) is rather general and describes many types of distortions so that it is possible to use it in much cases of practical interest. It can easily be extended to nonperiodic and even random distortions.

Below the corrections to the Casimir force will be given in terms of the Fourier coefficients $C_{nm}^{(i)}$ by a general formula. Four examples will be calculated explicitly. In the first example we consider longitudinal distortions of square plates, as it was

done in ref. [19], which are described by eq. (5) with $\alpha_y = 0$, $C_{nm}^{(i)} = 0 (i = 1, 3, 4)$, $C_{nm}^{(2)} = \delta_{n1}\delta_{m0}$. As the second example we consider longitudinal distortions with a larger number of Fourier modes:

$$f^{(1)}(x, y) \equiv f^{(1)}(x) = A \left[\cos x + \sum_{k=1}^7 (-1)^{k+1} \left(\frac{7}{8}\right)^k \cos(k+1)x \right]. \quad (7)$$

It is shown in Fig. 2. The corresponding corrections to the Casimir force between the square plates with such distortions are given in section 5.

Let us now consider such distortions as hillocks. A great number of different hillocks on plane surfaces may be modeled with the help of a suitable function (5). As the third and the fourth examples we consider a simple sinusoidal profile

$$f^{(2)}(x, y) = A \sin(\alpha_x x) \sin(\alpha_y y) \quad (8)$$

and the surface corresponding to the function

$$f^{(3)}(x, y) = f^{(1)}(x) \sin(2\pi y), \quad (9)$$

where the function $f^{(1)}(x)$ is given by eq. (7) (see Fig. 3). These examples demonstrate the most essential types of periodic distortions on a plane surface.

For the calculation of corrections to the Casimir forces due to distortions, the Fourier expansions of powers of the functions, which describe the distortions, must be known. Below we give the corresponding formulas. With the notation $f^n(x, y) \equiv (f(x, y))^n$ we have:

$$f^2(x, y) = A^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [F_{nm}^{(1)} \sin(n\alpha_x x) \sin(m\alpha_y y) + F_{nm}^{(2)} \sin(n\alpha_x x) \cos(m\alpha_y y) + F_{nm}^{(3)} \sin(n\alpha_x x) \sin(m\alpha_y y) + F_{nm}^{(4)} \cos(n\alpha_x x) \cos(m\alpha_y y)]. \quad (10)$$

It should be noted that the mean value of $f(x, y)$ from eq. (5) is equal to zero by definition (the term $C_{00}^{(4)}$ is dropped) in accordance with the generally used determination of the distance d between the plates under consideration. The function (10) has the mean value $A^2 F_{00}^{(4)}$, which is different from zero. It is not difficult to obtain

$$F_{00}^{(4)} = \frac{1}{2} \sum_{n=1}^{\infty} [(C_{n0}^{(2)})^2 + (C_{n0}^{(4)})^2] + \frac{1}{4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [(C_{nm}^{(1)})^2 + (C_{nm}^{(2)})^2 + (C_{nm}^{(3)})^2 + (C_{nm}^{(4)})^2]. \quad (11)$$

The Fourier series for the function $f^3(x, y)$ may be written in the form

$$f^3(x, y) = A^3 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [G_{nm}^{(1)} \sin(n\alpha_x x) \sin(m\alpha_y y) + G_{nm}^{(2)} \sin(n\alpha_x x) \cos(m\alpha_y y) + G_{nm}^{(3)} \sin(n\alpha_x x) \sin(m\alpha_y y) + G_{nm}^{(4)} \cos(n\alpha_x x) \cos(m\alpha_y y)]. \quad (12)$$

Here we have two different cases. In dependence on the specific choice of the function (5), the mean value $A^3 G_{00}^{(4)}$ of the function (12) may be both equal to

zero (see functions (8) and (9)) and different from zero (see function (7)). For the coefficient $G_{00}^{(4)}$ we obtain

$$G_{00}^{(4)} = \frac{3}{4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [(C_{nm}^{(1)})^2 - (C_{nm}^{(2)})^2 - (C_{nm}^{(3)})^2 + (C_{nm}^{(4)})^2] C_{2n, 2m}^{(4)}. \quad (13)$$

A more detailed discussion of this question is given in Sect. 4.

Finally, let us write the general expression for the Fourier series of the function $f^4(x, y)$:

$$f^4(x, y) = A^4 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [H_{nm}^{(1)} \sin(n\alpha_x x) \sin(m\alpha_y y) + H_{nm}^{(2)} \sin(n\alpha_x x) \cos(m\alpha_y y) + H_{nm}^{(3)} \cos(n\alpha_x x) \sin(m\alpha_y y) + H_{nm}^{(4)} \cos(n\alpha_x x) \cos(m\alpha_y y)]. \quad (14)$$

The mean value $A^4 H_{00}^{(4)}$ of the function (14) differs from zero for an arbitrary function (5). This quantity may be written as

$$\begin{aligned} H_{00}^{(4)} = & \frac{3}{8} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \{ [(C_{n0}^{(2)} C_{n'0}^{(2)})^2 + (C_{n0}^{(4)} C_{n'0}^{(4)})^2] (2 - \delta_{nn'}) + 2(C_{n0}^{(2)} C_{n'0}^{(4)})^2 (2 - \delta_{nn'}) \} + \\ & + \frac{3}{64} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m'=1}^{\infty} \{ \sum_{i=1}^4 (C_{nm}^{(i)} C_{n'm'}^{(i)})^2 (1 - \frac{5}{6} \delta_{nn'} \delta_{mm'}) (2 + \delta_{nn'}) (2 + \delta_{mm'}) + \\ & + 2[(C_{nm}^{(1)} C_{n'm'}^{(2)})^2 + (C_{nm}^{(3)} C_{n'm'}^{(4)})^2] (2 + \delta_{nn'}) (2 - \delta_{mm'}) + \\ & + 2[(C_{nm}^{(1)} C_{n'm'}^{(3)})^2 + (C_{nm}^{(2)} C_{n'm'}^{(4)})^2] (2 - \delta_{nn'}) (2 + \delta_{mm'}) + \\ & + 2[(C_{nm}^{(1)} C_{n'm'}^{(4)})^2 + (C_{nm}^{(2)} C_{n'm'}^{(3)})^2] (2 - \delta_{nn'}) (2 - \delta_{mm'}) \}. \end{aligned} \quad (15)$$

4 Square Plates With Distortions: The General Case

Let us calculate the corrections to the Casimir force between the square plate P_1 with the side length $2L$ and the thickness D and the other plate P_2 which is parallel to P_1 and has the same side length and thickness. The surfaces of both plates are covered by distortions with the profile given by eq. (5). Generally speaking, the surfaces distortions of the plates P_1 and P_2 may be different. It is naturally to suppose, that $\alpha_{x_i}^{-1} \ll L$, $\alpha_{y_i}^{-1} \ll L$ and $A_i \ll d$ hold, where d is the distance between the plates counted from the mean value of the distortions (see Sect. 3). The index $i = 1, 2$ numerates the plates P_1 and P_2 respectively.

The renormalized potential of one atom at the distance z_2 over the first plate is given by

$$U_A(x_2, y_2, z_2) = -CN_1 \int_{-L}^L dx_1 \int_{-L}^L dy_1 \int_{-D}^{f_1(x_1, y_1)} dz_1 [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{-7/2}. \quad (16)$$

The corresponding coordinate system is shown in Fig. 4.

Let us calculate this expression as a series with respect to the parameter A_1/z_2 which is small due to $z_2 \geq d \gg A_1$. Carrying out these calculations one can neglect the corrections of order of z_2/L and z_2/D , i.e., assume that the thickness and the length of the sides of the first plate are infinitely large.

The result of such integration up to fourth order with respect to the small parameter A_1/z_2 may be written in the form

$$U_A(x_2, y_2, z_2) = \quad (17)$$

$$-\pi C N_1 \left\{ \frac{1}{10z_2^4} + \frac{1}{z_2^4} \Phi_1(x_2, y_2, z_2) \frac{A_1}{z_2} + \frac{1}{z_2^4} [F_{00,1}^{(4)} + \Phi_2(x_2, y_2, z_2)] \left(\frac{A_1}{z_2} \right)^2 + \right.$$

$$\left. + \frac{2}{z_2^4} [G_{00,1}^{(4)} + \Phi_3(x_2, y_2, z_2)] \left(\frac{A_1}{z_2} \right)^3 + \frac{7}{2z_2^4} [H_{00,1}^{(4)} + \Phi_4(x_2, y_2, z_2)] \left(\frac{A_1}{z_2} \right)^4 \right\}.$$

Here the quantities $F_{00,1}^{(4)}$, $G_{00,1}^{(4)}$, $H_{00,1}^{(4)}$ are given in the eqs. (11), (13), (15), respectively. The index 1 corresponds to the function $f_1(x_1, y_1)$. The functions $\Phi_i(x_2, y_2, z_2)$ in eq. (17) have the form of series

$$\Phi_i(x_2, y_2, z_2) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k,l=1}^2 \varphi_k(n\alpha_{x_1}x_2) \varphi_l(m\alpha_{y_1}y_2) R_{nm}^{kl}(z_2), \quad (18)$$

with the notation $\varphi_1(t) = \sin t$ and $\varphi_2(t) = \cos t$. The functions R_{nm}^{kl} depend on z_2 only.

The renormalized potential of the Casimir force between the plates may be calculated by the integration of eq. (17) over the volume V_2 of the second plate with the distortions $f_2(x_2, y_2)$ and by division of the obtained result by the renormalization constant K from eq. (2)

$$U_R = -\Psi(\epsilon_1, \epsilon_2) \int_{-L}^L dx_2 \int_{-L}^L dy_2 \int_{d+f(x_2, y_2)}^{d+D} dz_2 U_A(x_2, y_2, z_2). \quad (19)$$

The Casimir force between the plates is given by

$$F = -\frac{\partial U_R}{\partial d}. \quad (20)$$

The result of the calculations according to eq. (19), (20) is represented as a series in powers of the small parameter A_2/d . It is not difficult to see from eq. (18) that the integration of the functions $\Phi_i(x_2, y_2, z_2)$ over x_2, y_2 yields zero, so that one obtains the result

$$F = -\Psi(\epsilon_1, \epsilon_2) \frac{\pi}{10d^4} \left\{ 1 + 10 \left[F_{00,1}^{(4)} \left(\frac{A_1}{d} \right)^2 + F_{00,2}^{(4)} \left(\frac{A_2}{d} \right)^2 \right] + \right.$$

$$\left. + 20 \left[G_{00,1}^{(4)} \left(\frac{A_1}{d} \right)^3 - G_{00,2}^{(4)} \left(\frac{A_2}{d} \right)^3 \right] + 35 \left[H_{00,1}^{(4)} \left(\frac{A_1}{d} \right)^4 + H_{00,2}^{(4)} \left(\frac{A_2}{d} \right)^4 \right] \right\}. \quad (21)$$

Here the lower indices 1,2 numerate the quantities corresponding to the distortions of the plates P_1 and P_2 , respectively.

It should be noted that the intermediate results of calculations according to eqs. (19), (20) contain the interference addendums which are proportional to $A_1 A_2 / d^2$, $A_1^2 A_2 / d^3$, $A_1 A_2^2 / d^3$ and so on. It is not difficult to see that the integration of these addendums with respect to x_2 and y_2 will give zero provided the periods of the distortions of the first and the second plates are not equal to each other. But generally speaking, the description of the surface distortions by some periodical function (5) has the character of a model. In real cases, an exact coincidence of the distortion frequencies will not be realistic.

It is interesting to note that in the case of distortions on both plates instead of one the corrections of order of $(A/d)^2$ and $(A/d)^4$ increase whereas the corrections of order of $(A/d)^3$ may decrease. It should be remarked that the corrections of the third order may be different from zero only for some special cases (see. eq. (13)). It is interesting to note also that all corrections do not depend on the periods of the distortions. Some examples of corrections caused by concrete distortions are discussed in the Sect. 5.

5 Square Plates With Distortions: Some Examples

Let us use the eq. (21) for the calculation of the Casimir force between two plane plates with some special types of distortions.

At first, we consider the longitudinal distortions with arbitrary profile. In this case the quantity $F_{00,i}^{(4)}$ is given by the first sum in eq.(11), while the second, double sum equals zero. It is interesting to note that the results do not depend on the relative direction of the longitudinal distortions on the plates. But there is an essential dependence on their profile. So, for the simple sinusoidal profile [19], one has from eqs. (11), (13), (15)

$$F_{00,i}^{(4)} = \frac{1}{2}, \quad G_{00,i}^{(4)} = 0, \quad H_{00,i}^{(4)} = \frac{3}{8}. \quad (22)$$

Using eq. (21) for the calculation of the Casimir force and supposing $A_1 = A_2 \equiv A$ one has

$$F = -\Psi(\epsilon_1, \epsilon_2) \frac{\pi}{10d^4} \left[1 + 10 \left(\frac{A}{d} \right)^2 + \frac{105}{4} \left(\frac{A}{d} \right)^4 \right]. \quad (23)$$

In [19] only the first two terms of eq. (23) were obtained. Let us take for numerical calculations the realistic value of $A/d \approx 10^{-1}$. Then the contribution of the simple sinusoidal distortions to eq. (23) is 10%. Here, the contribution of the fourth order in A/d is negligible because it is less than the relative error of the method itself, i.e., less than 4% (see Sect. 2).

For the more complicated profile represented in Fig. 2 (see. eq. (7)), the corresponding constants in eq. (21)) are

$$F_{00,i}^{(4)} = 1.88, \quad G_{00,i}^{(4)} = 1.4, \quad H_{00,i}^{(4)} = 11.5. \quad (24)$$

The Casimir force between two plates with such distortions in accordance with eq. (21) turns out to be

$$F = -\Psi(\epsilon_1, \epsilon_2) \frac{\pi}{10d^4} \left[1 + 38 \left(\frac{A}{d} \right)^2 + 808 \left(\frac{A}{d} \right)^4 \right]. \quad (25)$$

For a value of $A/d \sim 10^{-1}$, the contribution of these distortions is 46%, including 8% from the fourth order. Generally speaking, the complication of the profile increases the corresponding corrections.

Let us consider now such distortions as hillocks. For the simplest sinusoidal hillocks given by eq. (8) it holds

$$F_{00,i}^{(4)} = \frac{1}{4}, \quad G_{00,i}^{(4)} = 0, \quad H_{00,i}^{(4)} = \frac{9}{64}. \quad (26)$$

The result is independent on the periods of the hillocks. So for the Casimir force between the plates with distortions given by eq. (8) we find

$$F = -\Psi(\epsilon_1, \epsilon_2) \frac{\pi}{10d^4} \left[1 + 5 \left(\frac{A}{d} \right)^2 + \frac{315}{32} \left(\frac{A}{d} \right)^4 \right]. \quad (27)$$

Such distortions cause corrections to the Casimir force of about 5%, which are half as much as the contributions of the sinusoidal longitudinal distortions given by eq. (23). For the more complicated hillocks shown in Fig. 3, the constants are

$$F_{00,i}^{(4)} = 0.94, \quad G_{00,i}^{(4)} = 0, \quad H_{00,i}^{(4)} = 5.65. \quad (28)$$

So the Casimir force is in this case

$$F = -\Psi(\epsilon_1, \epsilon_2) \frac{\pi}{10d^4} \left[1 + 18.8 \left(\frac{A}{d} \right)^2 + 396 \left(\frac{A}{d} \right)^4 \right]. \quad (29)$$

For the considered value of $A/d \sim 10^{-1}$ the contribution of such complicated hillocks to the total force is almost 23%. The complication of the profile also increases the corrections.

It should be remarked, that the observed increasing of the corrections together with the number of Fourier modes of the distortions does not mean a possible divergence of the method used. For any distortion profile, which can be expanded into a Fourier series, all expansions done here are convergent.

In all cases corrections, given by eq. (21), must be taken into account in precision experiments on Casimir force measurements.

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References

- [1] G. Plunien, B. Müller, W. Greiner, Phys. Rept. **134** (1986) 87.
- [2] V.M. Mostepanenko, N.N. Trunov, Sov. Phys. - Usp. (USA) **31** (1988) 965.
- [3] V.M. Mostepanenko, N.N. Trunov, *Casimir Effect and its Applications*. Moscow: Energoatomizdat 1990 (in Russian, English Edition in preparation in Oxford University Press).
- [4] E.M. Lifshitz, L.P. Pitaevsky, *Statistical Physics*, Oxford, Pergamon Press, 1980.
- [5] P. Hasenfratz, J. Kuti, Phys. Rept. **40** (1978) 75.
- [6] P. Candelas, S. Weinberg, Nucl. Phys. B**237** (1984) 397.
- [7] L.H. Ford, Phys. Rev. D**11** (1975) 3370.
- [8] V.M. Mostepanenko, I.Yu. Sokolov, Phys. Rev. D **47** (1993) 2882.
- [9] M. Krech and S. Dietrich, Phys. Rev. Lett. **66** (1991) 345; **67** (1991) 1055; Phys. Rev. A**46** (1992) 1886 and 1922.
- [10] E. Cheng, M.W. Cole, W.F. Salam, and J. Treiner, Phys. Rev. Lett. **67** (1991) 1007.
- [11] R. Balian, B. Duplantier, Ann. Phys. **104** (1977) 300.
- [12] Yu.B. Zayaev, A.Ya. Kazakov, V.M. Mostepanenko, N.N. Trunov, Sov. Phys. J. (USA) **31** (1988) 575.
- [13] V.M. Mostepanenko, I.Yu. Sokolov, Sov. Phys. Dokl. (USA) **33** (1988) 140.
- [14] Yu.B. Zayaev, V.M. Mostepanenko, Theor. Math. Phys. (USA) **79** (1989) 487.
- [15] S.K. Karepanov, M.Yu. Novikov, A.S. Sorin, Nuovo Cim. **100B** (1987) 411.
- [16] J. Mahanty, B.W. Ninham, *Dispersion Forces*, N.-Y., Academic, 1976.
- [17] P.W. Fowler, J.M. Hutson, Phys. Rev. B **33** (1986) 3724.
- [18] M.H. Hawton, W.J. Keeler, Phys. Rev. A **33** (1986) 3333.
- [19] M. Bordag, V.M. Mostepanenko, I.Yu. Sokolov, Preprint Leipzig University, NTZ 21/93, Leipzig, 1993.

Fig. 1. The dependence of the renormalization constant on the dielectric permeabilities.

Fig. 2. The profile of the longitudinal distortions given by eq. (7).

Fig. 3. The hillock-like distortions given by eqs. (7), (9).

Fig. 4. The coordinate system used in the calculation of the Casimir force. Here D is the thickness of the plates, $2L$ is the length of their sides, d is the distance between them. It should be noted, that the surface distortions of the plates are not represented here and that the distances are not to scale.